MA 265: Linear Algebra

EXAM II (Practice Problems)

April 3, 2008

NAME ________________________

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (20 pts) ____________	**9. (5 pts)** ____________
2. (10 pts) ____________	**10. (5 pts)** ____________
3. (5 pts) ____________	**11. (5 pts)** ____________
4. (5 pts) ____________	**12. (5 pts)** ____________
5. (5 pts) ____________	**13. (5 pts)** ____________
6. (5 pts) ____________	**14. (5 pts)** ____________
7. (5 pts) ____________	**15. (5 pts)** ____________
8. (5 pts) ____________	**16. (5 pts)** ____________

Total Points: ____________/100

1. Which of the following vector is in the span of $\{(1, 2, 0, 1), (1, 1, 1, 0)\}$

A. $(0, 1, -1, 1)$

B. $(1, 1, -1, 1)$

C. $(0, 0, -1, 1)$

D. $(0, 1, 0, 1)$

E. $(-1, 1, -1, 1)$

2. The value(s) of a **for which** $\begin{bmatrix} a^2 \\ -3a \\ -2 \end{bmatrix}$ **is in span** $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} \right\}$ **are**

A. $a = 1, -2$

B. $a = -1, 2$

C. $a = 1, 2$

D. any number

E. nothing

3. For what values of k is the vector $\mathbf{w} = (1, 3, k)$ in the subspace spanned by the vectors $\mathbf{w}_1 = (1, 2, 3),\ \mathbf{w}_2 = (1, -2, -1),\ \mathbf{w}_3 = (3, 7, 11)$?

A. $k = 2$.

B. $k = 3$.

C. $k = 7/2$.

D. There is no such k.

E. k can be any number.

4. Determine if the following vectors are Linearly Independent (L.I) or Linearly Dependent (L.D.)?

(a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \right\}$ **L.I.** **L.D.**

(b) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} \right\}$ **L.I.** **L.D.**

(c) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} \right\}$ **L.I.** **L.D.**

(d) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \right\}$ **L.I.** **L.D.**

(e) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ 0 \end{bmatrix} \right\}$ **L.I.** **L.D.**

5. **Find all the values of k for which the following vectors are linearly independent in $\mathbb{R}^4$ are $(1,1,0,-1),(1,k,1,1),(2,2,k,-2),(-1,1,1,k)$.**

A. $k \neq 0, 5, -1$

B. $k \neq 0, 3, -1$

C. $k \neq 0, \pm\sqrt{5}$

D. $k \neq 1, \pm\sqrt{7}$

E. **There is no such value.**

6. **Find all value(s) of k such that the vectors $\mathbf{V}_1 = 2t^2 + t + 1$, $\mathbf{V}_2 = 3t^2 + t - 5$, $\mathbf{V}_3 = kt + 13$ are linearly dependent.**

A. $k = 1$

B. $k = 2$

C. $k \neq 1$

D. $k \neq 2$

E. k **can be any value.**

7. Let W be the subspace of $\mathbb{R}^5$ defined by the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 0$, i.e., $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \in \mathbb{R}^5; x_1 + x_2 + x_3 + x_4 + x_5 = 0 \right\}$. We know the following two vectors are in W; $v_1 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \in W$. Find two other vectors $\{v_3, v_4\}$ so that $\{v_1, v_2, v_3, v_4\}$ forms a basis of W.

A. $\left\{ v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

B. $\left\{ v_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \\ 3 \end{bmatrix} \right\}$

C. $\left\{ v_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} -1 \\ -1 \\ 0 \\ -1 \\ 3 \end{bmatrix} \right\}$

D. $\left\{ v_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ 4 \end{bmatrix} \right\}$

E. $\left\{ v_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ -2 \end{bmatrix}, v_4 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ -3 \end{bmatrix} \right\}$

8. Let W be the subspace of $\mathbb{R}^4$ spanned by the following vectors $v_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 2 \\ -4 \\ -2 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 5 \end{bmatrix}, v_4 = \begin{bmatrix} -1 \\ 0 \\ -2 \\ -4 \end{bmatrix}$**. Find the dimension of W.**

A. 1

B. 2

C. 3

D. 4

E. 0

9. A basis for the subspace of P_3 consisting of all vectors of the from at^3+bt^2+ct+d with $c = a - 2d, b = 5a + 3d$ is given by

A. $\{t^3 - 5t^2 + t, 3t^2 + 2t + 1\}$.

B. $\{t^3 - 5t^2 - t, -3t^2 + 2t + 1\}$.

C. $\{t^3 + 5t^2 - t, 3t^2 + 2t - 1\}$.

D. $\{t^3 + 5t^2 + t, 3t^2 - 2t + 1\}$.

E. $\{t^3 + 5t^2 + t, 3t^2 + 2t + 1\}$.

10. Determine if the following sets of the vectors are bases for $\mathbb{R}^4$?

(a)	$[1,1,1,0],[2,2,0,0],[3,0,0,0]$.	**Yes**	**No**
(b)	$[1,1,1,0],[2,2,0,0],[3,0,0,0],[1,0,0,0]$.	**Yes**	**No**
(c)	$[1,1,1,0],[2,2,0,0],[3,0,0,0],[8,7,9,2]$.	**Yes**	**No**
(d)	$[1,1,1,0],[2,2,0,0],[3,0,0,0],[1,1,1,1]$.	**Yes**	**No**
(e)	$[1,1,1,0],[2,2,0,0],[3,0,0,0],[7,2,0,0],[3,5,7,1]$.	**Yes**	**No**

11. For what value(s) of a does the set $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} a \\ 0 \\ 1 \end{bmatrix} \right\}$ span $\mathbb{R}^3$?

A. $a = 1$

B. $a = -1$

C. **all** $a \neq 0$

D. **all** $a \neq 1$

E. **There is no such** a**.**

12. Find the dimensions of the given subspaces of $\mathbb{R}^3$.

(a) **All vectors of the form** $[a, b, c]$**, where** $a - 2b + 5c = 0$**.** **Dimension=______**

(b) **All vectors of the form** $[a, b, c]$**, wher** $a = 2b$ **and** $5c = 0$**.Dimension=______**

(c) **All vectors of the form** $[a - c, 3b + 2a, 4c - 2a + b]$**.** **Dimension=______**

(d) **All vectors of the form** $[a, b, c]$**, where** $a = b = c$**.** **Dimension=______**

(e) **All vectors of the form** $[a + c, 2a + 2c, a + b]$**.** **Dimension=______**

13. Determine if each of the following sets of the vectors forms a basis for $\mathbb{R}^4$?

(a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}.$ **Yes No**

(b) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}.$ **Yes No**

(c) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 7 \\ 9 \\ 2 \end{bmatrix} \right\}.$ **Yes No**

(d) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$ **Yes No**

(e) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 7 \\ 1 \end{bmatrix} \right\}.$ **Yes No**

14. **Let** $A = \begin{bmatrix} 1 & -2 & 1 & 2 & 1 \\ 5 & -3 & 2 & 0 & 2 \end{bmatrix}$. **The null space of** A **is**

A. **A 2-dimensional subspace of** $\mathbf{R}^3$.

B. **A 2-dimensional subspace of** $\mathbf{R}^4$.

C. **A 2-dimensional subspace of** $\mathbf{R}^5$.

D. **A 3-dimensional subspace of** $\mathbf{R}^5$.

E. **A 4-dimensional subspace of** $\mathbf{R}^5$.

15. **Let us consider the following matrix** $A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & -2 & 3 & 4 \\ 3 & -3 & 4 & 5 \end{bmatrix}$. **Which of the following sets of vectors is a basis of the null space of** A.

A. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

16. Find the dimension of the null space of the following matrix

$$\begin{bmatrix} 1001 & 1533 & 2687 & 5479 & 0 & 3008 \\ 0 & 0 & 2003 & 2004 & 2005 & 2006 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 2 & 2 \end{bmatrix}$$

A. 2

B. 3

C. 4

D. 5

E. 6

17. Find the coordinates of vector $v = [1,\ 2,\ 3]$ **in the basis consisting of vectors** $v_1 = [1,\ 1,\ 0], v_2 = [1,\ 0,\ 1]$ **and** $v_3 = [0,\ 1,\ 1]$**.**

A. $[1,\ 2,\ 3]$

B. $[3,\ 2,\ 1]$

C. $[1,\ 0,\ 1]$

D. $[0,\ 1,\ 2]$

E. $[0,\ 2,\ 1]$

18. Let A be a 3×3 matrix. Which of the following implies that A is non–singular?

A. The rank of A is **2**.

B. The nullity of A is **0**.

C. $Ax = 0$ has nontrivial solutions.

D. The determinant of A is **0**.

E. A has a row of 0's.

19. Let A be a 6×3 matrix whose null space is spanned by $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} \right\}$ then the rank of A is:

A. 1

B. 2

C. 3

D. 4

E. Insufficient information

20. Let A be a 7×3 matrix with rank$(A) = 2$. Which statement is correct?

A. The columns of A are linearly independent.

B. The rows of A are linearly independent.

C. The nullity of A is zero.

D. The rows of A span $\mathbb{R}^3$.

E. The columns of A are linearly dependent.

21. A basis for the column space of $A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 3 & 5 & 7 & 4 \\ 3 & -1 & 4 & 1 \end{bmatrix}$ **is**

A. $\left\{ \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ -3 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ -3 \end{bmatrix} \right\}$

22. Let V **be an inner product space. If** u, v **are vectors in** V **then which of the following is always true?**

A. $\frac{1}{4}\|u+v\|^2 = -\frac{1}{4}\|u-v\|^2 + (u, v)$

B. $\frac{1}{8}\|u+v\|^2 = -\frac{1}{4}\|u-v\|^2 + (u, v)$

C. $-\frac{1}{4}\|u+v\|^2 = \frac{1}{4}\|u-v\|^2 + (u, v)$

D. $-\frac{1}{8}\|u+v\|^2 = \frac{1}{4}\|u-v\|^2 + (u, v)$

E. $\frac{1}{4}\|u+v\|^2 = \frac{1}{4}\|u-v\|^2 + (u, v)$

23. **In $\mathbb{R}^3$ with the standard inner product, the cosine of the angle between** $u = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ **and** $v = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$ **is:**

A. $\dfrac{1}{6}$

B. $-\dfrac{1}{6}$

C. $\dfrac{1}{\sqrt{6}}$

D. $-\dfrac{1}{\sqrt{6}}$

E. $-\dfrac{1}{36}$

24. **Let V be the inner product space defined by** $\Big(f(t), g(t) \Big) = \int_0^1 f(t)g(t)dt$**, then the distance between the vectors t and t^2 is:**

A. 1

B. $\frac{1}{4}$

C. $\frac{1}{2}$

D. $\frac{1}{30}$

E. $\frac{1}{\sqrt{30}}$

25. Which of the following is an orthonormal basis of $\mathbb{R}^3$:

A. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

B. $\left\{ \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{6}} \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} \right\}$

D. $\left\{ \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{6}} \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}, \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \frac{1}{2} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

26. Let W be the null space of the matrix

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 1 & 3 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

What is the dimension of $W^{\perp}$?

A. 0

B. 1

C. 2

D. 3

E. 4

27. W **is the subspace of** $\mathbb{R}_4$ **spanned by**

$$\{(1,\ 0,\ 0,\ 1),\ (1,\ 0,\ 1,\ 0),\ (-1,\ 0,\ 0,\ 1)\}.$$

What is $\mathbf{proj}_W V$ **where** $V = (1\ 1\ 1\ 1)$**?**

A. $(\mathbf{1\ \ 1\ \ 1\ \ 0})$

B. $(\mathbf{0\ \ 1\ \ 1\ \ 1})$

C. $(\mathbf{1\ \ 1\ \ 0\ \ 1})$

D. $(\mathbf{1\ \ 0\ \ 1\ \ 1})$

E. $(\mathbf{1\ \ 1\ \ 1\ \ 1})$

28. If W **is a subspace of** $\mathbb{R}^4$ **spanned by** $\left\{\begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\\0\end{bmatrix}\right\}$**, then the distance from** $\mathbf{v} = \begin{bmatrix}1\\1\\0\\0\end{bmatrix}$ **to** W **is**

A. 0

B. 1

C. $\sqrt{2}$

D. 2

E. $\dfrac{1}{\sqrt{2}}$

29. Let W be the subspace of $\mathbb{R}_3$ with orthonormal basis

$$\{(1,\ 0,\ 0), (0,\ \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}})\}.$$

What is the distance from v to W where $v = (1,\ 2,\ 4)$?

A. $\mathbf{0}$

B. $\mathbf{1}$

C. $\sqrt{2}$

D. $\sqrt{3}$

E. $2\sqrt{2}$

30. Let W be the subspace of $\mathbb{R}^3$ spanned by $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$. For $V = \begin{bmatrix} 8 \\ 0 \\ 2 \end{bmatrix}$, find

$\text{proj}_W V$

A. $\begin{bmatrix} 9 \\ 4 \\ 5 \end{bmatrix}$

B. $\begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix}$

C. $\begin{bmatrix} 23/2 \\ 3 \\ 17/2 \end{bmatrix}$

D. $\begin{bmatrix} 19 \\ 8 \\ 10 \end{bmatrix}$

E. $\begin{bmatrix} 2 \\ 6 \\ 8 \end{bmatrix}$

31. The least square fit line $y = x_1 + x_2 t$ for the data

t_i	-2	-1	1	2
y_i	3	1	-2	4

A. is given by $y = 6 - t$,

B. is given by $y = 4 + 10t$,

C. is given by $y = \frac{3}{2} - \frac{1}{10}t$,

D. is given by $y = -\frac{1}{10} + \frac{3}{2}t$,

E. does not exist.

32. The least squares solution $\widehat{\mathbf{x}}$ of

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & -1 \end{bmatrix} \widehat{\mathbf{x}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \textbf{ is}$$

A. $\widehat{\mathbf{x}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

B. $\widehat{\mathbf{x}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

C. $\widehat{\mathbf{x}} = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$

D. $\widehat{\mathbf{x}} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

E. $\widehat{\mathbf{x}} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

33. **Let** $L\colon \mathbb{R}^2 \to \mathbb{R}^3$ **be a linear transformation such that** $L\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ **and** $L\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$**. Then** $L\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right)$ **is equal to**

A. $\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

C. $\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

E. $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$

34. **Let** $L\colon \mathbb{R}^3 \to \mathbb{R}^3$ **be a linear transformation for which** $L\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$**,** $L\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, L\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$**. Then** $L\left(\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}\right) =$

A. $\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$

B. $\begin{bmatrix} 7 \\ 6 \\ 10 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 7 \\ 12 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}$

E. Insufficient information

35. The linear transformation $L\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) = \begin{bmatrix} a_2 \\ a_1 \end{bmatrix}$ **is**

- **A. a 90^o counterclockwise rotation**
- **B. a 90^o clockwise rotation**
- **C. reflection through** $y = x$
- **D. reflection through** $y = -x$
- **E. reflection through the origin**