MA 265: Linear Algebra

EXAM II (Practice Problems)

April 3, 2008

NAME ________________________

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (20 pts) ____________

2. (10 pts) ____________

3. (5 pts) ____________

4. (5 pts) ____________

5. (5 pts) ____________

6. (5 pts) ____________

7. (5 pts) ____________

8. (5 pts) ____________

9. (5 pts) ____________

10. (5 pts) ____________

11. (5 pts) ____________

12. (5 pts) ____________

13. (5 pts) ____________

14. (5 pts) ____________

15. (5 pts) ____________

16. (5 pts) ____________

Total Points: ____________/100

1. Which of the following vector is in the span of $\{(1,2,0,1),(1,1,1,0)\}$

A. $\boxed{(0,1,-1,1)}$

B. $(1,1,-1,1)$

C. $(0,0,-1,1)$

D. $(0,1,0,1)$

E. $(-1,1,-1,1)$

2. The value(s) of a **for which** $\begin{bmatrix} a^2 \\ -3a \\ -2 \end{bmatrix}$ **is in span** $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} \right\}$ **are**

A. $a = 1, -2$

B. $a = -1, 2$

C. $\boxed{a = 1, 2}$

D. any number

E. nothing

3. For what values of k is the vector $\mathbf{w} = (1, 3, k)$ in the subspace spanned by the vectors $\mathbf{w}_1 = (1, 2, 3),\ \mathbf{w}_2 = (1, -2, -1),\ \mathbf{w}_3 = (3, 7, 11)$?

A. $k = 2$.

B. $k = 3$.

C. $k = 7/2$.

D. There is no such k.

E. $\boxed{k \text{ can be any number.}}$

4. Determine if the following vectors are Linearly Independent (L.I.) or Linearly Dependent (L.D.)?

(a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \right\}$ **L.I. L.D.**

(b) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} \right\}$ **L.I. L.D.**

(c) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} \right\}$ **L.I. L.D.**

(d) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \right\}$ **L.I. L.D.**

(e) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ 0 \end{bmatrix} \right\}$ **L.I. L.D.**

Answer: I, D, I, D, D

5. Find all the values of k for which the following vectors are linearly independent in $\mathbb{R}^4$ are $(1,1,0,-1),(1,k,1,1),(2,2,k,-2),(-1,1,1,k)$.

A. $k \neq 0, 5, -1$

B. $\boxed{k \neq 0, 3, -1}$

C. $k \neq 0, \pm\sqrt{5}$

D. $k \neq 1, \pm\sqrt{7}$

E. There is no such value.

6. Find all value(s) of k such that the vectors $\mathbf{V}_1 = 2t^2 + t + 1$, $\mathbf{V}_2 = 3t^2 + t - 5$, $\mathbf{V}_3 = kt + 13$ are linearly dependent.

A. $\boxed{k = 1}$

B. $k = 2$

C. $k \neq 1$

D. $k \neq 2$

E. k **can be any value.**

7. Let W be the subspace of $\mathbb{R}^5$ defined by the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 0$, i.e., $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \in \mathbb{R}^5; x_1 + x_2 + x_3 + x_4 + x_5 = 0 \right\}$. We know the following two vectors are in W; $v_1 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \in W$. Find two other vectors $\{v_3, v_4\}$ so that $\{v_1, v_2, v_3, v_4\}$ forms a basis of W.

A. $\left\{ v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

B. $\left\{ v_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \\ 3 \end{bmatrix} \right\}$

C. $\left\{ v_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} -1 \\ -1 \\ 0 \\ -1 \\ 3 \end{bmatrix} \right\}$

D. $\boxed{\left\{ v_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}, v_4 = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ 4 \end{bmatrix} \right\}}$

E. $\left\{ v_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ -2 \end{bmatrix}, v_4 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \\ -3 \end{bmatrix} \right\}$

8. Let W be the subspace of $\mathbb{R}^4$ spanned by the following vectors $v_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 2 \\ -4 \\ -2 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 5 \end{bmatrix}, v_4 = \begin{bmatrix} -1 \\ 0 \\ -2 \\ -4 \end{bmatrix}$. Find the dimension of W.

A. 1

B. $\boxed{2}$

C. 3

D. 4

E. 0

9. A basis for the subspace of P_3 consisting of all vectors of the from at^3+bt^2+ct+d with $c = a - 2d, b = 5a + 3d$ is given by

A. $\{t^3 - 5t^2 + t, 3t^2 + 2t + 1\}$.

B. $\{t^3 - 5t^2 - t, -3t^2 + 2t + 1\}$.

C. $\{t^3 + 5t^2 - t, 3t^2 + 2t - 1\}$.

D. $\boxed{\{t^3 + 5t^2 + t, 3t^2 - 2t + 1\}.}$

E. $\{t^3 + 5t^2 + t, 3t^2 + 2t + 1\}$.

10. Determine if the following sets of the vectors are bases for $\mathbb{R}^4$?

(a) $[1,1,1,0],[2,2,0,0],[3,0,0,0]$. **Yes** **No**

(b) $[1,1,1,0],[2,2,0,0],[3,0,0,0],[1,0,0,0]$. **Yes** **No**

(c) $[1,1,1,0],[2,2,0,0],[3,0,0,0],[8,7,9,2]$. **Yes** **No**

(d) $[1,1,1,0],[2,2,0,0],[3,0,0,0],[1,1,1,1]$. **Yes** **No**

(e) $[1,1,1,0],[2,2,0,0],[3,0,0,0],[7,2,0,0],[3,5,7,1]$. **Yes** **No**

Answer: N, N, Y, Y, N

11. For what value(s) of a does the set $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} a \\ 0 \\ 1 \end{bmatrix} \right\}$ **span $\mathbb{R}^3$?**

A. $a = 1$

B. $\boxed{\text{all } a \neq -1}$

C. all $a \neq 0$

D. all $a \neq 1$

E. There is no such a.

12. Find the dimensions of the given subspaces of $\mathbb{R}^3$.

(a) All vectors of the form $[a, b, c]$, where $a - 2b + 5c = 0$. Dimension=______

(b) All vectors of the form $[a, b, c]$, wher $a = 2b$ and $5c = 0$.Dimension=______

(c) All vectors of the form $[a - c, 3b + 2a, 4c - 2a + b]$. Dimension=______

(d) All vectors of the form $[a, b, c]$, where $a = b = c$. Dimension=______

(e) All vectors of the form $[a + c, 2a + 2c, a + b]$. Dimension=______

Answer: 2, 1, 3, 1, 2

13. Determine if each of the following sets of the vectors forms a basis for $\mathbb{R}^4$?

(a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}.$ **Yes No**

(b) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}.$ **Yes No**

(c) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 7 \\ 9 \\ 2 \end{bmatrix} \right\}.$ **Yes No**

(d) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$ **Yes No**

(e) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 7 \\ 1 \end{bmatrix} \right\}.$ **Yes No**

Answer: N, N, Y, Y, N

14. Let $A = \begin{bmatrix} 1 & -2 & 1 & 2 & 1 \\ 5 & -3 & 2 & 0 & 2 \end{bmatrix}$. The null space of A is

A. A 2-dimensional subspace of $\mathbf{R}^3$.

B. A 2-dimensional subspace of $\mathbf{R}^4$.

C. A 2-dimensional subspace of $\mathbf{R}^5$.

D. $\boxed{\textbf{A 3-dimensional subspace of } \mathbf{R}^5\textbf{.}}$

E. A 4-dimensional subspace of $\mathbf{R}^5$.

15. Let us consider the following matrix $A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & -2 & 3 & 4 \\ 3 & -3 & 4 & 5 \end{bmatrix}$. Which of the following sets of vectors is a basis of the null space of A.

A. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix} \right\}$

C. $\boxed{\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix} \right\}}$

D. $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

16. Find the dimension of the null space of the following matrix

$$\begin{bmatrix} 1001 & 1533 & 2687 & 5479 & 0 & 3008 \\ 0 & 0 & 2003 & 2004 & 2005 & 2006 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 2 & 2 \end{bmatrix}$$

A. **2**

B. $\boxed{\mathbf{3}}$

C. **4**

D. **5**

E. **6**

17. Find the coordinates of vector $v = [1,\ 2,\ 3]$ **in the basis consisting of vectors** $v_1 = [1,\ 1,\ 0], v_2 = [1,\ 0,\ 1]$ **and** $v_3 = [0,\ 1,\ 1]$**.**

A. $[1,\ 2,\ 3]$

B. $[3,\ 2,\ 1]$

C. $[1,\ 0,\ 1]$

D. $\boxed{[0,\ 1,\ 2]}$

E. $[0,\ 2,\ 1]$

18. Let A be a 3×3 matrix. Which of the following implies that A is non–singular?

- **A. The rank of A is 2.**
- **B. $\boxed{\textbf{The nullity of } A \textbf{ is 0.}}$**
- **C. $Ax = 0$ has nontrivial solutions.**
- **D. The determinant of A is 0.**
- **E. A has a row of 0's.**

19. Let A be a 6×3 matrix whose null space is spanned by $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} \right\}$ then the rank of A is:

- **A. $\boxed{\mathbf{1}}$**
- **B. 2**
- **C. 3**
- **D. 4**
- **E. Insufficient information**

20. Let A be a 7×3 matrix with rank$(A) = 2$. Which statement is correct?

- **A. The columns of A are linearly independent.**
- **B. The rows of A are linearly independent.**
- **C. The nullity of A is zero.**
- **D. The rows of A span $\mathbb{R}^3$.**
- **E. $\boxed{\textbf{The columns of } A \textbf{ are linearly dependent.}}$**

21. A basis for the column space of $A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 3 & 5 & 7 & 4 \\ 3 & -1 & 4 & 1 \end{bmatrix}$ is

A. $\left\{ \begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ -3 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right\}$

D. $\boxed{\left\{ \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \right\}}$

E. $\left\{ \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ -3 \end{bmatrix} \right\}$

22. Let V be an inner product space. If u, v are vectors in V then which of the following is always true?

A. $\frac{1}{4}\|u+v\|^2 = -\frac{1}{4}\|u-v\|^2 + (u,v)$

B. $\frac{1}{8}\|u+v\|^2 = -\frac{1}{4}\|u-v\|^2 + (u,v)$

C. $-\frac{1}{4}\|u+v\|^2 = \frac{1}{4}\|u-v\|^2 + (u,v)$

D. $-\frac{1}{8}\|u+v\|^2 = \frac{1}{4}\|u-v\|^2 + (u,v)$

E. $\boxed{\frac{1}{4}\|u+v\|^2 = \frac{1}{4}\|u-v\|^2 + (u,v)}$

23. In $\mathbb{R}^3$ with the standard inner product, the cosine of the angle between $u = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $v = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$ is:

A. $\frac{1}{6}$

B. $\boxed{-\frac{1}{6}}$

C. $\frac{1}{\sqrt{6}}$

D. $-\frac{1}{\sqrt{6}}$

E. $-\frac{1}{36}$

24. Let V be the inner product space defined by $\Big(f(t), g(t)\Big) = \int\limits_0^1 f(t)g(t)dt$, then the distance between the vectors t and t^2 is:

A. 1

B. $\frac{1}{4}$

C. $\frac{1}{2}$

D. $\frac{1}{30}$

E. $\boxed{\frac{1}{\sqrt{30}}}$

25. Which of the following is an orthonormal basis of $\mathbb{R}^3$:

A. $\left\{\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}-1\\-1\\2\end{bmatrix}, \begin{bmatrix}-1\\1\\0\end{bmatrix}\right\}$

B. $\left\{\frac{1}{\sqrt{5}}\begin{bmatrix}1\\2\\0\end{bmatrix}, \frac{1}{\sqrt{5}}\begin{bmatrix}2\\-1\\0\end{bmatrix}, \frac{1}{\sqrt{6}}\begin{bmatrix}-1\\2\\1\end{bmatrix}\right\}$

C. $\left\{\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}-1\\-1\\2\end{bmatrix}, \begin{bmatrix}2\\-2\\0\end{bmatrix}\right\}$

D. $\boxed{\left\{\frac{1}{\sqrt{3}}\begin{bmatrix}1\\1\\1\end{bmatrix}, \frac{1}{\sqrt{6}}\begin{bmatrix}-1\\-1\\2\end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix}-1\\1\\0\end{bmatrix}\right\}}$

E. $\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \frac{1}{2}\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$

26. Let W be the null space of the matrix

$$\begin{bmatrix}1 & 0 & 2 & 0 & 1\\0 & 0 & 0 & 1 & 3\\0 & 1 & 3 & 0 & 2\\0 & 0 & 0 & 0 & 1\end{bmatrix}.$$

What is the dimension of $W^{\perp}$?

A. 0

B. 1

C. 2

D. 3

E. $\boxed{4}$

27. **W is the subspace of $\mathbb{R}_4$ spanned by**

$$\{(1,\ 0,\ 0,\ 1),\ (1,\ 0,\ 1,\ 0),\ (-1,\ 0,\ 0,\ 1)\}.$$

What is $\mathbf{proj}_W V$ where $V = (1\ 1\ 1\ 1)$?

A. $\mathbf{(1\ \ 1\ \ 1\ \ 0)}$

B. $\mathbf{(0\ \ 1\ \ 1\ \ 1)}$

C. $\mathbf{(1\ \ 1\ \ 0\ \ 1)}$

D. $\boxed{\mathbf{(1\ \ 0\ \ 1\ \ 1)}}$

E. $\mathbf{(1\ \ 1\ \ 1\ \ 1)}$

28. **If W is a subspace of $\mathbb{R}^4$ spanned by** $\left\{\begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\0\\0\end{bmatrix}\right\}$**, then the distance from**

$\mathbf{v} = \begin{bmatrix}1\\1\\0\\0\end{bmatrix}$ **to W is**

A. 0

B. $\boxed{1}$

C. $\sqrt{2}$

D. 2

E. $\dfrac{1}{\sqrt{2}}$

29. Let W be the subspace of $\mathbb{R}_3$ with orthonormal basis

$$\{(1,\ 0,\ 0),(0,\ \frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}})\}.$$

What is the distance from v to W where $v = (1,\ 2,\ 4)$?

A. **0**

B. **1**

C. $\boxed{\ \sqrt{2}}$

D. $\sqrt{3}$

E. $2\sqrt{2}$

30. Let W be the subspace of $\mathbb{R}^3$ spanned by $\left\{\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}\right\}$. For $V = \begin{bmatrix}8\\0\\2\end{bmatrix}$, find

proj$_W V$

A. $\begin{bmatrix}9\\4\\5\end{bmatrix}$

B. $\boxed{\begin{bmatrix}6\\2\\4\end{bmatrix}}$

C. $\begin{bmatrix}23/2\\3\\17/2\end{bmatrix}$

D. $\begin{bmatrix}19\\8\\10\end{bmatrix}$

E. $\begin{bmatrix}2\\6\\8\end{bmatrix}$

31. The least squares fit line $y = x_1 + x_2 t$ for the data

t_i	-2	-1	1	2
y_i	3	1	-2	4

- **A. is given by $y = 6 - t$,**
- **B. is given by $y = 4 + 10t$,**
- **C.** $\boxed{\textbf{is given by } y = \frac{3}{2} - \frac{1}{10}t,}$
- **D. is given by $y = -\frac{1}{10} + \frac{3}{2}t$,**
- **E. does not exist.**

32. The least squares solution $\widehat{\mathbf{x}}$ of

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & -1 \end{bmatrix} \widehat{\mathbf{x}} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \textbf{ is}$$

- **A.** $\widehat{\mathbf{x}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- **B.** $\widehat{\mathbf{x}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
- **C.** $\widehat{\mathbf{x}} = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$
- **D.** $\boxed{\widehat{\mathbf{x}} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}}$
- **E.** $\widehat{\mathbf{x}} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

33. Let $L\colon \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that $L\left(\begin{bmatrix}1\\-1\end{bmatrix}\right) = \begin{bmatrix}1\\1\\-1\end{bmatrix}$ and $L\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\1\\1\end{bmatrix}$. Then $L\left(\begin{bmatrix}2\\1\end{bmatrix}\right)$ is equal to

A. $\begin{bmatrix}-2\\1\\1\end{bmatrix}$

B. $\begin{bmatrix}1\\2\\2\end{bmatrix}$

C. $\boxed{\begin{bmatrix}2\\2\\1\end{bmatrix}}$

D. $\begin{bmatrix}-1\\2\\1\end{bmatrix}$

E. $\begin{bmatrix}1\\3\\1\end{bmatrix}$

34. Let $L\colon \mathbb{R}^3 \to \mathbb{R}^3$ **be a linear transformation for which** $L\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}-2\\3\\1\end{bmatrix}$, $L\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\2\\2\end{bmatrix}, L\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}3\\1\\3\end{bmatrix}$**. Then** $L\left(\begin{bmatrix}1\\0\\3\end{bmatrix}\right) =$

A. $\begin{bmatrix}1\\0\\3\end{bmatrix}$

B. $\boxed{\begin{bmatrix}7\\6\\10\end{bmatrix}}$

C. $\begin{bmatrix}1\\7\\12\end{bmatrix}$

D. $\begin{bmatrix}1\\7\\6\end{bmatrix}$

E. Insufficient information

35. The linear transformation $L\left(\begin{bmatrix}a_1\\a_2\end{bmatrix}\right) = \begin{bmatrix}a_2\\a_1\end{bmatrix}$ **is**

A. a 90^o **counterclockwise rotation**

B. a 90^o **clockwise rotation**

C. $\boxed{\textbf{reflection through } y = x}$

D. reflection through $y = -x$

E. reflection through the origin