MA 271: Several Variable Calculus

EXAM I

Sep. 28, 2017

NAME ____________________ Class Meet Time ____________________

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (10 pts) ________	**2. (10 pts)** ________
3. (10 pts) ________	**4. (10 pts)** ________
5. (10 pts) ________	**6. (10 pts)** ________
7. (10 pts) ________	**8. (10 pts)** ________
9. (10 pts) ________	**10. (10 pts)** ________
11. (10 pts) ________	**12. (10 pts)** ________

Total Points: ________/120

1. Determine convergence or divergence for the given sequences or series. Fill in the blanks.

(a) $\lim_{n\to\infty} \frac{1}{n}$ ____________________ **(converge, diverge)**

Answer: converge

(b) $\lim_{n\to\infty} \sqrt[n]{2}$ ____________________ **(converge, diverge)**

Answer: converge

(c) $\sum_{n=1}^{\infty} \frac{1}{n}$ ____________________ **(converge, diverge)**

Answer: diverge

(d) $\sum_{n=1}^{1000} \frac{1}{n}$ ____________________ **(converge, diverge)**

Answer: converge

(e) $\sum_{n=1}^{\infty} \frac{1}{n^n}$ ____________________ **(converge, diverge)**

Answer: converge

2. True or False (False means not always true or the formula does not make sense). For three-dimensional vectors a, b and c

(i) if $\mathbf{b} = \mathbf{c}$, then $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ ____________________ **(T, F)**

(ii) $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$ ____________________ **(T, F)**

(iii) $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$ ____________________ **(T, F)**

(iv) $\mathbf{a} \times \mathbf{a} = |\mathbf{a}|^2$ ____________________ **(T, F)**

(v) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$ ____________________ **(T, F)**

Answer: T, F, T, F, F

3. (a) The intersection of the surface $y+4=(x-2)^2+(z+2)^2$ and the yz-plane is ____________________ (a straight line, two straight lines, a circle, a parabola or a hyperbola.)

Answer: parabola

(b) Let x be a nonzero real number, what is the value of m if

$$\sum_{n=1}^{\infty} x^{n-6} = \sum_{n=m}^{\infty} x^n$$

$m =$ ____________________

Answer: -5

4. Find a vector $\mathbf{a} \neq \mathbf{0}$, and vectors $\mathbf{b}$ and $\mathbf{c}$ such that

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c} \quad \text{but} \quad \mathbf{b} \neq \mathbf{c}.$$

(You need to specify a, b, c , and calculate $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \cdot \mathbf{c}$)

Your Answer:

$\mathbf{a} =$ ____________________

$\mathbf{b} =$ ____________________

$\mathbf{c} =$ ____________________

$\mathbf{a} \cdot \mathbf{b} =$ ____________________

$\mathbf{a} \cdot \mathbf{c} =$ ____________________

Answer: $a = \mathbf{i}, b = \mathbf{j}, c = \mathbf{k}$

5. Evaluate the limit:

$$\lim_{n\to\infty}(1+\frac{5}{n})^{3n} =$$

Note: Show your work! **answer:** $\boxed{}$

Answer: e^{15}

6. Let L be the tangent line to the curve S where the parametric equation of the curve S is given by

$$\begin{aligned} x &= 2\cos(t) + \sin(2t) \\ y &= 2\sin(t) + \cos(2t) \\ z &= 3t \end{aligned}$$

at the point $(2, 1, 0)$. Find a parametric equation of the tangent line L?

answer: $\boxed{}$ **Answer:** $x(t) = 2 + 2t; y(t) = 1 + 2t; z(t) = 3t$

7. The plane S passes through the points $(1, 2, 3)$, $(3, 2, 1)$ and $(-3, 0, 3)$. Find the equation for S.

Note: Show your work! **answer:**

Answer: $x - 2y + z = 0$

8. A particle starts at the origin with initial velocity $\vec{i} + \vec{j} - \vec{k}$. Its acceleration is $\vec{a}(t) = 6t\ \vec{i} + 2\ \vec{j} - 6t\ \vec{k}$. Find its position at $t = 2$.

Note: Show your work! **answer:**

Answer: $10\ \vec{i} + 6\ \vec{j} - 10\ \vec{k}$

9. The position of an object is given by $\mathbf{r}(t) = \cos(t^2)\mathbf{i} + \sin(t^2)\mathbf{j} + \sqrt{3}\,t^2\mathbf{k},\ t \geq 0$**. with length scale in meters. At what time** t **has the object traveled** 18 **meters? (The object started traveling at** $t = 0$**)**

Note: Show your work! **answer:**

Answer: 3

10. Let $\mathbf{u} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ **and** $\mathbf{v} = 2\mathbf{j} - 3\mathbf{k}$**. What is** $\mathbf{Proj_v u} =$

Note: Show your work! **answer:**

Answer: $-\frac{10}{13}\mathbf{j} + \frac{15}{13}\mathbf{k}$

11. Find all the x such that the series

$$\sum_{n=1}^{\infty}(-1)^n \frac{(n+1)(2x+7)^n}{n^2}$$

converges. Note: Show your work! **answer:** $\boxed{}$

Answer: $-4 < x \leq -3$

12. Find the curvature of the curve defined by $\vec{r}(t) = (\sin(3t))\vec{i} + (\cos(3t))\vec{j} + (4t)\vec{k}$ at $t = 2$. Recall: $\kappa = |\frac{dT}{ds}| = |\frac{dT}{dt}|/|\mathbf{v}|$

Note: Show your work! **answer:** $\boxed{}$

Answer: $\frac{9}{25}$