MA 271: Several Variable Calculus

EXAM I

Sep. 28, 2017

NAME _____ Class Meet Time _____

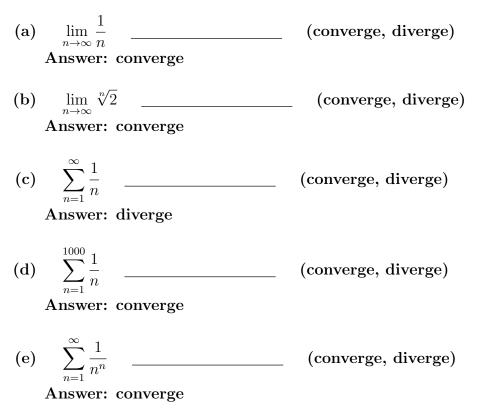
NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (10 pts)	2. (10 pts)
3. (10 pts)	4. (10 pts)
5. (10 pts)	6. (10 pts)
7. (10 pts)	8. (10 pts)
9. (10 pts)	10. (10 pts)
11. (10 pts)	12. (10 pts)

Total Points: _____/120

1. Determine convergence or divergence for the given sequences or series. Fill in the blanks.



2. True or False (False means not always true or the formula does not make sense). For three-dimensional vectors a, b and c

(i) if $\mathbf{b} = \mathbf{c}$, then $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ (T, F) (ii) $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{a}$ (T, F) (iii) $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$ (T, F) (iv) $\mathbf{a} \times \mathbf{a} = |\mathbf{a}|^2$ (T, F) (v) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{b}) \times \mathbf{c}$ (T, F) Answer: T, F, T, F, F 3. (a) The intersection of the surface $y + 4 = (x - 2)^2 + (z + 2)^2$ and the *yz*-plane is (a straight line, two straight lines, a circle, a parabola or a hyperbola.) Answer: parabola

(b) Let x be a nonzero real number, what is the value of m if

$$\sum_{n=1}^{\infty} x^{n-6} = \sum_{n=m}^{\infty} x^n$$

m = ______Answer: -5

4. Find a vector $a \neq 0$, and vectors b and c such that

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$$
 but $\mathbf{b} \neq \mathbf{c}$.

(You need to specify a, b, c , and calculate $a \cdot b$ and $a \cdot c$)

Your Answer: $\mathbf{a} = _$ _____ $\mathbf{b} = _$ _____ $\mathbf{c} = _$ _____ $\mathbf{a} \cdot \mathbf{b} = _$ _____ $\mathbf{a} \cdot \mathbf{c} = _$ _____ Answer: $a = \mathbf{i}, b = \mathbf{j}, c = \mathbf{k}$

inswer: $a = \mathbf{I}, b = \mathbf{J}, c = \mathbf{K}$

5. Evaluate the limit:

$$\lim_{n\to\infty}(1+\frac{5}{n})^{3n}=$$

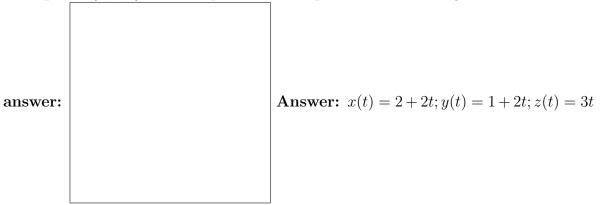
Note: Show your work!

Answer: e^{15}

6. Let L be the tangent line to the curve S where the parametric equation of the curve S is given by

$$x = 2\cos(t) + \sin(2t)$$
$$y = 2\sin(t) + \cos(2t)$$
$$z = 3t$$

at the point (2,1,0). Find a parametric equation of the tangent line L?



answer:

7. The plane S passes through the points (1,2,3), (3,2,1) and (-3,0,3). Find the equation for S.

Note: Show your work!

answer:

Answer: x - 2y + z = 0

8. A particle starts at the origin with initial velocity $\vec{i} + \vec{j} - \vec{k}$. Its acceleration is $\vec{a}(t) = 6t \ \vec{i} + 2 \ \vec{j} - 6t \ \vec{k}$. Find its position at t = 2.

Note: Show your work!

answer:

Answer: $10 \ \vec{i} + 6 \ \vec{j} - 10 \ \vec{k}$

9. The position of an object is given by $\mathbf{r}(t) = \cos(t^2)\mathbf{i} + \sin(t^2)\mathbf{j} + \sqrt{3}t^2\mathbf{k}, t \ge 0.$ with length scale in meters. At what time t has the object traveled 18 meters? (The object started traveling at t = 0) Note: Show your work! answer:

Answer: 3

10. Let $\mathbf{u} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{v} = 2\mathbf{j} - 3\mathbf{k}$. What is $\operatorname{Proj}_{\mathbf{v}} \mathbf{u} =$ Note: Show your work! answer:

Answer: $-\frac{10}{13}\mathbf{j}+\frac{15}{13}\mathbf{k}$

11. Find all the x such that the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{(n+1)(2x+7)^n}{n^2}$$

converges. Note: Show your work!

answer:

Answer: $-4 < x \le -3$

12. Find the curvature of the curve defined by $\vec{r}(t) = (\sin(3t))\vec{i} + (\cos(3t))\vec{j} + (4t)\vec{k}$ at t = 2. Recall: $\kappa = |\frac{dT}{ds}| = |\frac{dT}{dt}|/|\mathbf{v}|$

Note: Show your work!

answer:

Answer: $\frac{9}{25}$