

MA 271: Several Variable Calculus

EXAM II

Nov. 2, 2017

NAME _____ Class Meet Time _____

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

- | | |
|--------------------|--------------------|
| 1. (10 pts) _____ | 2. (10 pts) _____ |
| 3. (10 pts) _____ | 4. (10 pts) _____ |
| 5. (10 pts) _____ | 6. (10 pts) _____ |
| 7. (10 pts) _____ | 8. (10 pts) _____ |
| 9. (10 pts) _____ | 10. (10 pts) _____ |
| 11. (10 pts) _____ | 12. (10 pts) _____ |

Total Points: _____/120

1. Find L_1 and L_2 where $L_1 = \lim_{(x,y) \rightarrow (3,4)} \frac{5x}{\sqrt{x^2 + y^2}}$ and $L_2 = \lim_{(x,y) \rightarrow (0,0)} \frac{5x}{\sqrt{x^2 + y^2}}$. State the reason if any of them does not exist.

$$L_1 = \underline{\hspace{2cm}} \quad L_2 = \underline{\hspace{2cm}}$$

Answer: $L_1 = 3$ and $L_2 = \text{undefined}$.

2. Find $f_x(0,0)$ and $f_y(0,0)$ when

$$f(x, y) = \begin{cases} \frac{x^2 + y^2}{x^2 + y}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

State the reason if any of them does not exist.

$$f_x(0,0) = \underline{\hspace{2cm}} \quad f_y(0,0) = \underline{\hspace{2cm}}$$

Answer: $f_x(0,0) = \text{undefined}$ and $f_y(0,0) = 1$.

3. Find

$$\left(\frac{\partial w}{\partial z}\right)_y$$

at $(x, y, z) = (1, -1, -3)$ if $w = x^2 + y^2 + z^2$ and $xz + y \ln x - x^2 + 4 = 0$.

Answer: _____

Answer: $-\frac{17}{3}$

4. Find the unit vector(s) such that the directional derivative(s) of $f(x, y) = x^3 e^{-4y}$ at the point $(1, 0)$ is 0.

Answer: _____

Answer: $\left(-\frac{4}{5}, -\frac{3}{5}\right), \left(\frac{4}{5}, \frac{3}{5}\right)$

5. Use the degree two Taylor polynomial of $\ln(x)$ centered at $x_0 = 1$ to estimate the value of

$$I = \int_{0.7}^{1.3} \ln(x) dx.$$

The approximate value of I is _____.

Answer: -0.009

6. For what values of the constant k will the Second Derivative Test guarantee that $f(x, y) = x^2 + kxy + y^2$ has a minimum at $(0, 0)$?

Answer: _____

Answer: $|k| < 2$

7. Find the equation of the plane that is tangent to the surface $e^z = x^2y - xy^2$ at the point $(3, 2, \ln(6))$.

Answer: _____

Answer: $8(x - 3) - 3(y - 2) - 6(z - \ln(6)) = 0$

8. Find $\frac{\partial z}{\partial u}$ if $z = x^2 + xy^3$ where $x = uv^2 + w^3$ and $y = u + ve^w$ at $u = -1, v = 2, w = 0$.

Answer: _____

Answer: -40

9. Find absolute maximum and minimum values of

$$f(x, y) = x^2 + y^2 - x + y$$

on the disc $x^2 + y^2 \leq 8$.

Answer: max = _____, min = _____

Answer: max=12, min=1/2

10. Find the largest product the positive numbers x , y and z can have if

$$x + y + z^2 = 5.$$

Answer: _____

Answer: 4

11. Evaluate

$$\int_0^4 \int_{\sqrt{x}}^2 9\sqrt{1+y^3} dydx.$$

Answer: _____

Answer: 52

12. Find the second order Taylor approximation for e^{xy} near the point (0, 1).

Answer: _____

Answer: $1 + x + \frac{x^2}{2} + x(y - 1)$