MA 271: Several Variable Calculus

EXAM II (practice)

NAME ______________________ Lecture Time ______________________

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (5 pts) ___________	**7. (5 pts)** ___________
2. (5 pts) ___________	**8. (5 pts)** ___________
3. (5 pts) ___________	**9. (5 pts)** ___________
4. (5 pts) ___________	**10. (5 pts)** ___________
5. (5 pts) ___________	**11. (5 pts)** ___________
6. (5 pts) ___________	**12. (5 pts)** ___________

Total Points: ___________

1. Find the second degree Taylor polynomial of $f(x) = \dfrac{1-\cos(2x)}{3x^2}$ with center $x_0 = 0$.

Answer: $\frac{2}{3} - \frac{2}{9}x^2$

2. Find the second degree Taylor polynomial of $f(x) = \sqrt{x}$ with center $x_0 = 4$.
Answer: $2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$

3. If $L = \lim_{(x,y,z)\to(0,0,0)} \dfrac{x^2+2y^2-3zy}{\sqrt{x^2+y^2+z^2}}$, then

A. $L = 1$

B. $L = -2$

C. $L = -3$

D. $\boxed{L = 0}$

E. the limit does not exist

4. If $L = \lim_{(x,y,z)\to(0,3,4)} \dfrac{x+5y-5z}{\sqrt{x^2+y^2+z^2}}$**, then**

A. $L = -3$

B. $L = -2$

C. $\boxed{L = -1}$

D. $L = 0$

E. the limit does not exist

5. If $L = \lim_{(x,y,z)\to(0,0,0)} \dfrac{x+2y-3z}{\sqrt{x^2+y^2+z^2}}$**, then**

A. $L = 1$

B. $L = -2$

C. $L = -3$

D. $L = 0$

E. $\boxed{\textbf{the limit does not exist}}$

6. If $f(x,y) = \ln(x+2y^2)$**, then the partial derivative** f_{xy} **equals**

A. $\dfrac{-2x}{(x+2y^2)^2}$

B. $\boxed{\dfrac{-4y}{(x+2y^2)^2}}$

C. $\dfrac{4xy}{(x+2y^2)^2}$

D. $\dfrac{-8xy}{(x+2y^2)^2}$

E. $\dfrac{4(x^2-y^2)}{(x+2y^2)^2}$

7. Find $\dfrac{\partial z}{\partial y}$ **at** $(-2, 2, 2)$ **if** $z(x, y)$ **is defined by the equation**

$$xe^y + ye^z = 0$$

A. -1

B. $-\frac{1}{2}$

C. 0

D. $\boxed{\frac{1}{2}}$

E. 1

8. Find $\dfrac{\partial z}{\partial y}$ **at** $(1, \ln 2, \ln 3)$ **if** $z(x, y)$ **is defined by the equation**

$$xe^y + ye^z + 2\ln x - 2 - 3\ln 2 = 0.$$

A. $2 + \ln 2$

B. $\frac{4}{3\ln 2}$

C. $-\frac{5}{3\ln 2+1}$

D. $\boxed{-\frac{5}{3\ln 2}}$

E. 1

9. Let

$$f(x, y) = \begin{cases} \frac{y^2}{x^2+y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Find $f_x(0, 0)$ **and** $f_y(0, 0)$**.**

Answer: $f_x(0, 0) = 0$ **and** $f_y(0, 0)$ **does not exist.**

10. Let

$$f(x, y) = \begin{cases} 2x + 3y + 1, & xy \neq 0 \\ 0, & xy = 0 \end{cases}$$

Find $f_x(0, 0)$ **and** $f_x(0, 1)$**.**

Answer: $f_x(0, 0) = 0$ **and** $f_x(0, 1)$ **does not exist.**

11. **Suppose** $z = f(x, y)$**, where** $x = e^t$ **and** $y = t^2 + 3t + 2$**. Given that** $\frac{\partial z}{\partial x} = x - y$ **and** $\frac{\partial z}{\partial y} = -x$**, find** $\frac{dz}{dt}$ **when** $t = 0$**.**

A. -6

B. $\boxed{-4}$

C. 6

D. 9

E. 15

12. **Find the directional derivative of the function** $f(x, y, z) = x^2 y^2 z^6$ **at the point** $(1, 1, 1)$ **in the direction of the vector** $\langle 2, 1, -2 \rangle$**.**

A. -6

B. $\boxed{-2}$

C. 0

D. 2

E. 6

13. Find the direction in which the function $z = x^2 + 3xy - \frac{1}{2}\, y^2$ is increasing most rapidly at $(-1, -1)$.

A. $3i$

B. $5\vec{i} + 2\vec{j} - \vec{k}$

C. $\boxed{-5\vec{i} - 2\vec{j}}$

D. $2\vec{i} - 5\vec{j}$

E. $\sqrt{29}$

14. Consider the function $f(x, y) = 2x^2 - 3xy + y^2$. Find two unit vectors such that the directional derivative of f at the point $(1, 1)$ in these two directions is 1.

Answer: $(1, 0)$ **and** $(0, -1)$

15. By using a linear approximation of $f(x, y) = \sqrt{x^2 + y}$ at $(4, 9)$, compute the approximate value of $f(5, 8)$.

A. 5.2

B. 5.3

C. 5.5

D. $\boxed{5.7}$

E. 5.9

16. The volume of a right circular cone with base radius r and height h is $V = \frac{\pi}{3}r^2h$. Suppose the radius is measured to be $6m \pm .2m$, and the height is measured to be $12m \pm .3m$. The volume calculated use differentials is $a \pm b\ m^3$. What are the values of a and b?

Answer: $a = 144\pi, b = 13.2\pi$

17. Find a equation for the tangent plane of

$$\cos(\pi x) - x^2y + e^{xz} + yz = 4 \quad \textbf{at} \quad (0,1,2)$$

Answer: $2x + 2y + z - 4 = 0$

18. Find a parametric equation for the line passing through $P = (5, 2, 0)$, and normal to the tangent plane of

$$y^2 + z^2 = 4$$

at P.

A. $x = 0, y = t, z = 0$

B. $x = 5, y = 4t, z = 3t$

C. $x = 5t, y = 2t, z = 3t$

D. $\boxed{x = 5, y = 4t + 2, z = 0}$

E. $x = 5t + 5, y = 2t + 2, z = 3t$

19. For the function $f(x, y) = x^3 + 2y^2 + xy - 2x + 5y$, the point $(-1, -1)$ yields

A. **a local minimum**

B. **a local maximum**

C. $\boxed{\textbf{a saddle point}}$

D. $\nabla f(-1, -1) \neq 0$

E. **The Second Derivative Test gives no information at $(-1, -1)$**

20. The function $f(x, y) = y\sin(x)$ has

A. **infinitely many local maximum points.**

B. **infinitely many local minimum points.**

C. $\boxed{\text{infinitely many saddle points.}}$

D. **exactly one local minimum point and one maximum point.**

E. **no critical point.**

21. The max and min values of $f(x, y, z) = xyz$ on the surface $2x^2 + 2y^2 + z^2 = 2$ are

A. $\pm \dfrac{\sqrt{2}}{9}$

B. $\pm \dfrac{\sqrt{3}}{9}$

C. $\boxed{\pm \dfrac{\sqrt{6}}{9}}$

D. $\pm \dfrac{2\sqrt{2}}{9}$

E. $\pm \dfrac{2\sqrt{3}}{9}$

22. If we use the method of Lagrange multipliers to find the maximum of $f(x, y) = 2x^2 - y^2 - y$ subject to the constraint $x^2 + y^2 = 1$, the Lagrange multipliers λ that we find are:

A. $\boxed{\lambda = 2}$

B. $\lambda = 0$

C. $\lambda = -1$

D. $\lambda = 2$ **and** $\lambda = -1$

E. $\lambda = 0$ **and** $\lambda = -1$

23. Find the minimum value of $x^2+y^2+z^2$ subject to the constraint $2x+y-z-6=0$.

A. $\frac{25}{6}$

B. 2

C. 4

D. $\boxed{6}$

E. 16

24. A rectangular box is to have volume 48 cubic feet, and is made of three different grades of material. The material for the front and back costs \$1 per square foot, the material for the top and bottom costs \$2 per square foot, and the material for the two ends costs \$3 per square foot. What are the dimensions of the box of minimal cost? Answer: 2 by 4 by 6

25. Find $\left(\frac{\partial w}{\partial y}\right)_x$, that is with x and y independent, at $(w,x,y,z)=(4,2,1,-1)$ if

$$w = x^2y^2 + yz - z^3, \quad x^2+y^2+z^2 = 6$$

A. -1

B. 1

C. 3

D. $\boxed{5}$

E. 7

26. Find cubic approximation of $f(x, y) = \dfrac{1}{1 - x - y + xy}$ **near the origin.**

Answer: $1 + x + y + x^2 + xy + y^2 + x^3 + x^2y + xy^2 + y^3$

27. Find cubic approximation of $f(x, y) = \dfrac{1}{1 + x - xy}$ **near (0, 1)**

Answer: $1 + x * (y - 1)$

28. Evaluate $\displaystyle\int_0^{\pi} \int_x^{\pi} \frac{\sin(y)}{y} dy\, dx$**.**

A. -1

B. 0

C. 1

D. $\boxed{2}$

E. $\dfrac{\pi^2}{2}$