

MA 271 PRACTICE PROBLEMS

1. If the line ℓ is parallel to the vector $\vec{v} = 2\vec{i} - 3\vec{j} + 7\vec{k}$, and contains the point $(2, 1, -3)$, then its vector equation is

- A. $\vec{r} = (1 + 2t)\vec{i} - 3t\vec{j} + (-2 + 7t)\vec{k}$ B. $\vec{r} = (2 + t)\vec{i} - 3\vec{j} + (7 - 2t)\vec{k}$
 C. $\vec{r} = (2 + 2t)\vec{i} + (1 - 3t)\vec{j} + (-3 + 7t)\vec{k}$ D. $\vec{r} = (2 + 2t)\vec{i} + (-3 + t)\vec{j} + (7 - 3t)\vec{k}$
 E. $\vec{r} = (2 + t)\vec{i} + \vec{j} + (7 - 3t)\vec{k}$

2. Find parametric equations of the line containing the points $(1, -1, 0)$ and $(-2, 3, 5)$.

- A. $x = 1 - 3t, y = -1 + 4t, z = 5t$ B. $x = t, y = -t, z = 0$
 C. $x = 1 - 2t, y = -1 + 3t, z = 5t$ D. $x = -2t, y = 3t, z = 5t$
 E. $x = -1 + t, y = 2 - t, z = 5$

3. Find an equation of the plane that contains the point $(1, -1, -1)$ and has normal vector $\frac{1}{2}\vec{i} + 2\vec{j} + 3\vec{k}$.

- A. $x - y - z + \frac{9}{2} = 0$ B. $x + 4y + 6z + 9 = 0$ C. $\frac{x-1}{\frac{1}{2}} = \frac{y+1}{2} = \frac{z+1}{3}$
 D. $x - y - z = 0$ E. $\frac{1}{2}x + 2y + 3z = 1$

4. Find an equation of the plane that contains the points $(1, 0, -1)$, $(-5, 3, 2)$, and $(2, -1, 4)$.

- A. $6x - 11y + z = 5$ B. $6x + 11y + z = 5$ C. $11x - 6y + z = 0$
 D. $\vec{r} = 18\vec{i} - 33\vec{j} + 3\vec{k}$ E. $x - 6y - 11z = 12$

5. Find parametric equations of the line tangent to the curve $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$ at the point $(2, 4, 8)$

- A. $x = 2 + t, y = 4 + 4t, z = 8 + 12t$ B. $x = 1 + 2t, y = 4 + 4t, z = 12 + 8t$
 C. $x = 2t, y = 4t, z = 8t$ D. $x = t, y = 4t, z = 12t$ E. $x = 2 + t, y = 4 + 2t, z = 8 + 3t$

6. The position function of an object is

$$\vec{r}(t) = \cos t\vec{i} + 3 \sin t\vec{j} - t^2\vec{k}$$

Find the velocity, acceleration, and speed of the object when $t = \pi$.

- | | Velocity | Acceleration | Speed |
|----|------------------------------------|---------------------------|----------------------|
| A. | $-\vec{i} - \pi^2\vec{k}$ | $-3\vec{j} - 2\pi\vec{k}$ | $\sqrt{1 + \pi^4}$ |
| B. | $\vec{i} - 3\vec{j} + 2\pi\vec{k}$ | $-\vec{i} - 2\vec{k}$ | $\sqrt{10 + 4\pi^2}$ |
| C. | $3\vec{j} - 2\pi\vec{k}$ | $-\vec{i} - 2\vec{k}$ | $\sqrt{9 + 4\pi^2}$ |
| D. | $-3\vec{j} - 2\pi\vec{k}$ | $\vec{i} - 2\vec{k}$ | $\sqrt{9 + 4\pi^2}$ |
| E. | $\vec{i} - 2\vec{k}$ | $-3\vec{j} - 2\pi\vec{k}$ | $\sqrt{5}$ |

7. A smooth parametrization of the semicircle which passes through the points $(1, 0, 5)$, $(0, 1, 5)$ and $(-1, 0, 5)$ is
- A. $\vec{r}(t) = \sin t\vec{i} + \cos t\vec{j} + 5\vec{k}, 0 \leq t \leq \pi$ B. $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + 5\vec{k}, 0 \leq t \leq \pi$
 C. $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + 5\vec{k}, \frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$ D. $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + 5\vec{k}, 0 \leq t \leq \frac{\pi}{2}$
 E. $\vec{r}(t) = \sin t + \cos t\vec{j} + 5\vec{k}, \frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$
8. The length of the curve $\vec{r}(t) = \frac{2}{3}(1+t)^{\frac{3}{2}}\vec{i} + \frac{2}{3}(1-t)^{\frac{3}{2}}\vec{j} + t\vec{k}, -1 \leq t \leq 1$ is
- A. $\sqrt{3}$ B. $\sqrt{2}$ C. $\frac{1}{2}\sqrt{3}$ D. $2\sqrt{3}$ E. $\sqrt{2}$
9. The level curves of the function $f(x, y) = \sqrt{1 - x^2 - 2y^2}$ are
- A. circles B. lines C. parabolas D. hyperbolas E. ellipses
10. The level surface of the function $f(x, y, z) = z - x^2 - y^2$ that passes through the point $(1, 2, -3)$ intersects the (x, z) -plane ($y = 0$) along the curve
- A. $z = x^2 + 8$ B. $z = x^2 - 8$ C. $z = x^2 + 5$ D. $z = -x^2 - 8$
 E. does not intersect the (x, z) -plane
11. Match the graphs of the equations with their names:
- (1) $x^2 + y^2 + z^2 = 4$ (a) paraboloid
 (2) $x^2 + z^2 = 4$ (b) sphere
 (3) $x^2 + y^2 = z^2$ (c) cylinder
 (4) $x^2 + y^2 = z$ (d) double cone
 (5) $x^2 + 2y^2 + 3z^2 = 1$ (e) ellipsoid
- A. 1b, 2c, 3d, 4a, 5e B. 1b, 2c, 3a, 4d, 5e C. 1e, 2c, 3d, 4a, 5b
 D. 1b, 2d, 3a, 4c, 5e E. 1d, 2a, 3b, 4e, 5c
12. Suppose that $w = u^2/v$ where $u = g_1(t)$ and $v = g_2(t)$ are differentiable functions of t . If $g_1(1) = 3$, $g_2(1) = 2$, $g_1'(1) = 5$ and $g_2'(1) = -4$, find $\frac{dw}{dt}$ when $t = 1$.
- A. 6 B. $33/2$ C. -24 D. 33 E. 24
13. If $w = e^{uv}$ and $u = r + s$, $v = rs$, find $\frac{\partial w}{\partial r}$.
- A. $e^{(r+s)rs}(2rs + r^2)$ B. $e^{(r+s)rs}(2rs + s^2)$ C. $e^{(r+s)rs}(2rs + r^2)$
 D. $e^{(r+s)rs}(1 + s)$ E. $e^{(r+s)rs}(r + s^2)$.

31. The area of one leaf of the three-leaved rose bounded by the graph of $r = 5 \sin 3\theta$ is
- A. $\frac{5\pi}{6}$ B. $\frac{25\pi}{12}$ C. $\frac{25\pi}{6}$ D. $\frac{5\pi}{3}$ E. $\frac{25\pi}{3}$.
32. Find the area of the portion of the plane $x + 3y + 2z = 6$ that lies in the first octant.
- A. $3\sqrt{11}$ B. $6\sqrt{7}$ C. $6\sqrt{14}$ D. $3\sqrt{14}$ E. $6\sqrt{11}$.
33. A solid region in the first octant is bounded by the surfaces $z = y^2$, $y = x$, $y = 0$, $z = 0$ and $x = 4$. The volume of the region is
- A. 64 B. $\frac{64}{3}$ C. $\frac{32}{3}$ D. 32 E. $\frac{16}{3}$.
34. An object occupies the region bounded above by the sphere $x^2 + y^2 + z^2 = 32$ and below by the cone $z = \sqrt{x^2 + y^2}$. The mass density at any point of the object is equal to its distance from the xy plane. Set up a triple integral in rectangular coordinates for the total mass m of the object.
- A. $\int_{-4}^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{-\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z \, dz \, dy \, dx$ B. $\int_{-4}^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z \, dz \, dy \, dx$
- C. $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z \, dz \, dy \, dx$ D. $\int_0^4 \int_0^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z \, dz \, dy \, dx$
- E. $\int_{-4}^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} xy \, dz \, dy \, dx$.
35. Do problem 34 in spherical coordinates.
- A. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho^3 \cos \varphi \sin \varphi \, d\rho \, d\varphi \, d\theta$ B. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho \cos \varphi \sin \varphi \, d\rho \, d\varphi \, d\theta$
- C. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho^3 \sin^2 \varphi \, d\rho \, d\varphi \, d\theta$ D. $\int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^{\sqrt{32}} \rho^3 \cos \varphi \sin \varphi \, d\rho \, d\varphi \, d\theta$
- E. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho \cos \varphi \, d\rho \, d\varphi \, d\theta$.
36. The double integral $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2(x^2 + y^2)^3 \, dy \, dx$ when converted to polar coordinates becomes
- A. $\int_0^\pi \int_0^1 r^9 \sin^2 \theta \, dr \, d\theta$ B. $\int_0^{\frac{\pi}{2}} \int_0^1 r^8 \sin^2 \theta \, dr \, d\theta$ C. $\int_0^\pi \int_0^1 r^8 \sin \theta \, dr \, d\theta$
- D. $\int_0^{\frac{\pi}{2}} \int_0^1 r^8 \sin \theta \, dr \, d\theta$ E. $\int_0^{\frac{\pi}{2}} \int_0^1 r^9 \sin^2 \theta \, dr \, d\theta$.
37. Which of the triple integrals converts $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 dz \, dy \, dx$ from rectangular to cylindrical coordinates?
- A. $\int_0^\pi \int_0^2 \int_r^2 r \, dz \, dr \, d\theta$ B. $\int_0^{2\pi} \int_0^2 \int_r^2 r \, dz \, dr \, d\theta$ C. $\int_0^{2\pi} \int_{-2}^2 \int_r^2 r \, dz \, dr \, d\theta$
- D. $\int_0^\pi \int_0^2 \int_r^2 r \, dz \, dr \, d\theta$ E. $\int_0^{\frac{2\pi}{2}} \int_{-2}^2 \int_r^2 r \, dz \, dr \, d\theta$.
38. If D is the solid region above the xy -plane that is between $z = \sqrt{4 - x^2 - y^2}$ and $z = \sqrt{1 - x^2 - y^2}$, then $\iiint_D \sqrt{x^2 + y^2 + z^2} \, dV =$
- A. $\frac{14\pi}{3}$ B. $\frac{16\pi}{3}$ C. $\frac{15\pi}{2}$ D. 8π E. 15π .

39. Determine which of the vector fields below are conservative, i. e. $\vec{F} = \text{grad } f$ for some function f .
1. $\vec{F}(x, y) = (xy^2 + x)\vec{i} + (x^2y - y^2)\vec{j}$.
 2. $\vec{F}(x, y) = \frac{x}{y}\vec{i} + \frac{y}{x}\vec{j}$.
 3. $\vec{F}(x, y, z) = ye^z\vec{i} + (xe^z + e^y)\vec{j} + (xy + 1)e^z\vec{k}$.
- A. 1 and 2 B. 1 and 3 C. 2 and 3 D. 1 only E. all three
40. Let \vec{F} be any vector field whose components have continuous partial derivatives up to second order, let f be any real valued function with continuous partial derivatives up to second order, and let $\nabla = \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}$. Find the incorrect statement.
- A. $\text{curl}(\text{grad } f) = \vec{0}$ B. $\text{div}(\text{curl } \vec{F}) = 0$ C. $\text{grad}(\text{div } \vec{F}) = 0$
D. $\text{curl } \vec{F} = \nabla \times \vec{F}$ E. $\text{div } \vec{F} = \nabla \cdot \vec{F}$
41. A wire lies on the xy -plane along the curve $y = x^2$, $0 \leq x \leq 2$. The mass density (per unit length) at any point (x, y) of the wire is equal to x . The mass of the wire is
- A. $(17\sqrt{17} - 1)/12$ B. $(17\sqrt{17} - 1)/8$ C. $17\sqrt{17} - 1$
D. $(\sqrt{17} - 1)/3$ E. $(\sqrt{17} - 1)/12$
42. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = y\vec{i} + x^2\vec{j}$ and C is composed of the line segments from $(0, 0)$ to $(1, 0)$ and from $(1, 0)$ to $(1, 2)$.
- A. 0 B. $\frac{2}{3}$ C. $\frac{5}{6}$ D. 2 E. 3
43. Evaluate the line integral
- $$\int_C x dx + y dy + xy dz$$
- where C is parametrized by $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + \cos t\vec{k}$ for $-\frac{\pi}{2} \leq t \leq 0$.
- A. 1 B. -1 C. $\frac{1}{3}$ D. $-\frac{1}{3}$ E. 0
44. Are the following statements true or false?
1. The line integral $\int_C (x^3 + 2xy)dx + (x^2 - y^2)dy$ is independent of path in the xy -plane.
 2. $\int_C (x^3 + 2xy)dx + (x^2 - y^2)dy = 0$ for every closed oriented curve C in the xy -plane.
 3. There is a function $f(x, y)$ defined in the xy -plane, such that $\text{grad } f(x, y) = (x^3 + 2xy)\vec{i} + (x^2 - y^2)\vec{j}$.
- A. all three are false B. 1 and 2 are false, 3 is true C. 1 and 2 are true, 3 is false
D. 1 is true, 2 and 3 are false E. all three are true
45. Evaluate $\int_C y^2 dx + 6xy dy$ where C is the boundary curve of the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$, in the counterclockwise direction.
- A. 0 B. 4 C. 8 D. 16 E. 32

46. If C goes along the x -axis from $(0, 0)$ to $(1, 0)$, then along $y = \sqrt{1 - x^2}$ to $(0, 1)$, and then back to $(0, 0)$ along the y -axis, then $\int_C xy \, dy =$
- A. $-\int_0^1 \int_0^{\sqrt{1-x^2}} y \, dy \, dx$ B. $\int_0^1 \int_0^{\sqrt{1-x^2}} y \, dy \, dx$ C. $-\int_0^1 \int_0^{\sqrt{1-x^2}} x \, dy \, dx$
D. $\int_0^1 \int_0^{\sqrt{1-x^2}} x \, dy \, dx$ E. 0
47. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, if $\vec{F}(x, y) = (xy^2 - 1)\vec{i} + (x^2y - x)\vec{j}$ and C is the circle of radius 1 centered at $(1, 2)$ and oriented counterclockwise.
- A. 2 B. π C. 0 D. $-\pi$ E. -2
48. Green's theorem yields the following formula for the area of a simple region R in terms of a line integral over the boundary C of R , oriented counterclockwise. Area of $R = \iint_R dA =$
- A. $-\int_C y \, dx$ B. $\int_C y \, dx$ C. $\int_C x \, dx$ D. $\frac{1}{2} \int_C y \, dx - x \, dy$ E. $-\int_C x \, dy$
49. Evaluate the surface integral $\iint_S x \, d\sigma$ where S is the part of the plane $2x + y + z = 4$ in the first octant.
- A. $8\sqrt{6}$ B. $\frac{8}{3}\sqrt{6}$ C. $\frac{8}{3}\sqrt{14}$ D. $\frac{\sqrt{14}}{3}$ E. $\frac{\sqrt{10}}{3}$
50. If S is the part of the paraboloid $z = x^2 + y^2$ with $z \leq 4$, \vec{n} is the unit normal vector on S directed upward, and $\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k}$, then $\iint_S \vec{F} \cdot \vec{n} \, d\sigma =$
- A. 0 B. 8π C. 4π D. -4π E. -8π
51. If $\vec{F}(x, y, z) = \cos z\vec{i} + \sin z\vec{j} + xy\vec{k}$, S is the complete boundary of the rectangular solid region bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = \frac{\pi}{2}$, and \vec{n} is the outward unit normal on S , then $\iint_S \vec{F} \cdot \vec{n} \, d\sigma =$
- A. 0 B. $\frac{1}{2}$ C. 1 D. $\frac{\pi}{2}$ E. 2
52. If $\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k}$, S is the unit sphere $x^2 + y^2 + z^2 = 1$ and \vec{n} is the outward unit normal on S , then $\iint_S \vec{F} \cdot \vec{n} \, d\sigma =$
- A. -4π B. $\frac{2\pi}{3}$ C. 0 D. $\frac{4\pi}{3}$ E. 4π

53. Evaluate the limit $\lim_{n \rightarrow \infty} \left[1 + \frac{(-1)^n}{n} \right]$.
 A. 0 B. 1 C. -1 D. 2 E. limit does not exist

54. Evaluate the limit $\lim_{n \rightarrow \infty} \left(\sqrt[n]{n} + \frac{1}{n!} \right)$.
 A. 0 B. 1 C. e D. $1/e$ E. limit does not exist

55. If $s = \sum_{n=0}^{\infty} \frac{2^{n+1}}{3^n}$, then $s =$
 A. 3 B. 6 C. 9 D. 2 E. $4/3$

56. If $L = \sum_{n=1}^{\infty} \frac{1}{2^n} + \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$, then $L =$
 A. $1/3$ B. $2/3$ C. 1 D. $4/3$ E. $5/3$

57. $\sum_{n=1}^{\infty} \frac{1}{(n^2 + 1)^p}$ converges when
 A. $p > 1$ B. $p \leq 1$ C. $p \geq 1$ D. $p > \frac{1}{2}$ E. $p \leq \frac{1}{2}$

58. $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n} \right)^p$ converges for
 A. $p \leq 1$ B. $p > 1$ C. $p < 0$ D. $p > 0$ E. no values of p

59. Which of the following series converge conditionally?

$$(i) \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \quad (ii) \sum_{n=2}^{\infty} \frac{(-1)^n n}{\ln n} \quad (iii) \sum_{n=1}^{\infty} \frac{(-1)^n n}{e^n}$$

A. only (ii) B. only (i) and (iii) C. only (i) and (ii) D. all three E. none of them

60. Which of the following series converge?

$$(i) \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{\frac{1}{4}}} \quad (ii) \sum_{n=1}^{\infty} \frac{n!}{1 \cdot 3 \cdot 5 \cdots (2n-1)} \quad (iii) \sum_{n=1}^{\infty} \frac{4}{3} \left(\frac{1}{2} \right)^n$$

A. only (ii) B. only (i) and (iii) C. only (i) and (ii) D. all three E. none of them

61. The interval of convergence for the power series $\sum_{n=2}^{\infty} \frac{3^n x^n}{n \ln n}$ is

A. $-\frac{1}{3} \leq x < \frac{1}{3}$ B. $-\frac{1}{3} < x \leq \frac{1}{3}$ C. $0 \leq x \leq \frac{1}{3}$ D. $-1 \leq x \leq 2$ E. $-1 < x < 1$

62. Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{nx^n}{2^n}$$

A. $-\frac{1}{2} < x < \frac{1}{2}$ B. $-2 < x < 2$ C. $-2 \leq x \leq 2$ D. $-2 < x \leq 2$ E. $-\infty < x < \infty$

63. The fourth term of the Maclaurin series for $\frac{x^2+3}{x-1}$ is

A. $-x^3$ B. $3x^3$ C. $-3x^3$ D. $-4x^3$ E. $4x^3$

64. The first three nonzero terms of the Maclaurin series for $f(x) = (1 - x^2) \sin x$ are

A. $x - \frac{5}{6}x^3 + \frac{31}{150}x^5$ B. $1 - \frac{3}{2}x^2 + \frac{13}{24}x^4$ C. $x - \frac{7}{6}x^3 + \frac{31}{150}x^5$ D. $x^2 - \frac{7}{6}x^3 + \frac{1}{25}x^5$ E. $x - \frac{7}{6}x^3 + \frac{21}{120}x^5$

65. The fourth term of the Taylor series for $f(x) = \ln x$ centered at $a = 2$ is

- A. $\frac{1}{6}(x-2)^3$ B. $\frac{1}{12}(x-2)^3$ C. $\frac{1}{24}(x-2)^3$ D. $-\frac{1}{3}(x-2)^3$ E. $-(x-2)^3$

ANSWERS

1-C, 2-A, 3-B, 4-B, 5-A, 6-D, 7-B, 8-D, 9-E, 10-B, 11-A, 12-E, 13-B, 14-B,
15-D, 16-C, 17-E, 18-D, 19-A, 20-C, 21-E, 22-A, 23-A, 24-B, 25-E, 26-B, 27-C,
28-B, 29-E, 30-A, 31-B, 32-D, 33-B, 34-B, 35-A, 36-E, 37-B, 38-C, 39-B, 40-C,
41-A, 42-D, 43-D, 44-E, 45-D, 46-B, 47-D, 48-A, 49-B, 50-E, 51-A, 52-E, 53-B
54-B, 55-B, 56-E, 57-D, 58-E, 59-E, 60-D, 61-A, 62-B, 63-D, 64-E, 65-C