THIS EXAM IS CLOSED TO BOOKS AND NOTES. NO CALCULATORS ARE ALLOWED!
Use the back of the **previous** page if more space is needed!

MA303 EXAMINATION I (Practice)

Name_____ ID #_____ Section #_____

There are 11 Problems on this booklet. For All problems, mark the answers clearly.

Points awarded

1. (5 pts) _____
2. (5 pts) _____
3. (5 pts) _____
4. (5 pts) _____
5. (5 pts) _____
6. (5 pts) _____
7. (5 pts) _____
8. (5 pts) _____
9. (5 pts) _____
10. (5 pts) _____

Total Points: _____

1. (5 points) Determine the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{n}{2^n}(x-3)^n$$

a) 3

b) 2

c) 1

d) .5

e) ∞

2. (5 points) The Taylor series of $\dfrac{1}{1-x}$ about $x_0 = 2$ is

a) $-1 + (x-2) - (x-2)^2 + \cdots$

b) $-1 + (x-2) - \dfrac{1}{2}(x-2)^2 + \cdots$

c) $1 + (x-2) + \dfrac{1}{2}(x-2)^2 + \cdots$

d) $1 + x + \dfrac{1}{2}x^2 + \cdots$

e) $1 + \dfrac{1}{2}x^2 + \dfrac{1}{24}x^4 + \cdots$

3. (5 points) Determine $y''(1)$ and $y'''(1)$ if $y(x)$ satisfies

$$x^2y'' + (1+x)y' + 3(\ln(x))y = 0; \quad y(1) = 2, \quad y'(1) = 0.$$

a) $y''(1) = 0; \quad y'''(1) = 6$

b) $y''(1) = 1; \quad y'''(1) = -6$

c) $y''(1) = 1; \quad y'''(1) = 6$

d) $y''(1) = 0; \quad y'''(1) = 1$

e) $y''(1) = 0; \quad y'''(1) = -6$

4. (5 points) In finding the power series solution $\sum_{n=0}^{\infty} a_n x^n$ of

$$y'' + xy' + 2y = 0,$$

what is the recurrence relation?

a) $a_{n+2} = -a_n/(n+1), \quad n = 0, 1, 2, \cdots$

b) $a_{n+2} = a_n/(n+2), \quad n = 0, 1, 2, \cdots$

c) $(n+2)a_{n+2} - a_{n+1} - a_n = 0, \quad n = 0, 1, 2, \cdots$

d) $a_{n+2} = a_n/((n+2)(n+1)), \quad n = 0, 1, 2, \cdots$

e) $a_2 = -a_0/2; \quad (n+2)(n+1)a_{n+2} - n(n+1)a_{n+1} + a_n = 0, \quad n = 1, 2, \cdots$

5. (5 points) Let $y(x) = \sum_{n=0}^{\infty} a_n x^n$. The recurrence relation reads

$$a_2 = -a_0/2, \quad a_{n+2} = \frac{n}{n+2} a_{n+1} - \frac{a_n}{(n+2)(n+1)}, \quad n = 1, 2, 3, \cdots .$$

If $a_0 = 1, a_1 = 2$, the first four terms of the series solution is

a) $y(x) = 1 + 2x + 3x^2 + 4x^3 + \cdots$.

b) $y(x) = 1 + 2x - \frac{1}{2}x^2 - 4x^3 + \cdots$.

c) $y(x) = 1 + 2x - \frac{1}{2}x^2 - \frac{1}{2}x^3 + \cdots$.

d) $y(x) = 1 + 2x - \frac{1}{2}x^2 - \frac{1}{4}x^3 + \cdots$.

e) $y(x) = 1 + 2x - 3x^2 - 4x^3 + \cdots$.

6. (5 points) The general solution of

$$x^2 y'' - 3xy' + 4y = 0, \quad x \neq 0$$

is

a) $y(x) = c_1 e^{2x} + c_2 x e^{2x}$

b) $y(x) = c_1 e^{4x} + c_2 x e^{-x}$

c) $y(x) = c_1 |x|^{-4} + c_2 |x|$

d) $y(x) = c_1 |x| + c_2 |x| \ln |x|$

e) $y(x) = c_1 |x|^2 + c_2 |x|^2 \ln |x|$

7. (5 points) What is the value of $y(3)$ if $y(x)$ is the solution of

$$x^2y'' + 4xy' + 2y = 0, \quad y(1) = 1, \quad y'(1) = 2$$

a) -2

b) -1

c) 0

d) 1

e) 2

8. (5 points) Classify the singular points of the given equation

$$x^2(2 - x^2)y'' + (2/x)y' + 4y = 0.$$

The regular singular points are _____.
The irregular singular points are _____.

9. (5 points) What is the real part of the complex number 3^{4+5i}?

10. (5 points) Determine the radius of convergence of the Taylor series of $\dfrac{1}{x(x-3)}$ about $x_0 = 1$

a) 2

b) 0

c) .5

d) 1

e) ∞

11.

EXAMINATION I (Practice) (Answer)

1. B
2. A
3. E
4. A
5. C
6. E
7. D
8. (a) $\pm\sqrt{2}$, (b) 0
9. $3^4 \cos(5 \ln(3))$
10. D