MA 36600
MIDTERM II EXAM (Practice)

NAME ________________________

1. NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

2. No partial credit will be given. For All problems, show all your work **and** write (or mark) the answers clearly.

Points awarded

1. (5 pts) ____________
2. (5 pts) ____________
3. (5 pts) ____________
4. (5 pts) ____________
5. (5 pts) ____________
6. (5 pts) ____________
7. (5 pts) ____________
8. (5 pts) ____________
9. (5 pts) ____________
10. (5 pts) ____________

Total Points: ____________/50

1. Find the general solution of

$$y'' + 2y' + 2y = 5e^{-2t}\sin(t)$$

A. $y = c_1e^{-t}\cos(t) + c_2e^{-t}\sin(t) + e^{-2t}(2\cos(t) - \sin(t))$

B. $y = c_1e^{-t}\cos(t) + c_2e^{-t}\sin(t) + e^{-2t}(-2\cos(t) - \sin(t))$

C. $y = c_1e^{t}\cos(t) + c_2e^{t}\sin(t) + e^{-2t}(2\cos(t) + \sin(t))$

D. $y = c_1e^{-t}\cos(t) + c_2e^{-t}\sin(t) + e^{-t}(2\cos(t) + \sin(t))$

E. $y = c_1e^{-t}\cos(t) + c_2e^{-t}\sin(t) + e^{-2t}(2\cos(t) + \sin(t))$

2. Transform the following third-order equation

$$y'''(t) - 3ty' + \sin(2t)y = 7e^{-t}$$

into a first-order system

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}' = A\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + \mathbf{f}(t)$$

by setting $u_1(t) = y(t)$, $u_2(t) = y'(t)$ and $u_3(t) = y''(t)$. Find the matrix A.

A. $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\sin(2t) & 3t & 0 \end{pmatrix}$

B. $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \sin(2t) & -3t & 0 \end{pmatrix}$

C. $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 3t & \sin(2t) & 0 \end{pmatrix}$

D. $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 3t & \sin(2t) \end{pmatrix}$

E. $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\sin(2t) & 3t & 7e^{-t} \end{pmatrix}$

3. If $y'' + 2y' + 5y = 0,\ y(0) = 1,\ y'(0) = 2$, then $y(\frac{\pi}{2}) =$

A. 1.

B. 0.

C. e^{π}.

D. $e^{\frac{\pi}{2}}$.

E. $-e^{-\frac{\pi}{2}}$.

4. Find all values of k for which the equilibrium point $(0,0)$ of the system

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} -2 & k \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

is a saddle point.

A. $k > 0$

B. $k > 1$

C. $k < 0$

D. $k < 1$

E. $k \geq 1$

5. Find the solution of the linear system

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \quad \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

A. $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \frac{1}{2}e^{3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \frac{1}{2}e^{t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

B. $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \frac{1}{2}e^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{2}e^{-t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

C. $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \frac{1}{2}e^{3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{2}e^{t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

D. $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

E. $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \frac{1}{2}e^{-3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \frac{1}{2}e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

6. Find the solution of the linear system

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \quad \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$

A. $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \frac{1}{2}e^{3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + \frac{1}{2}e^{t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

B. $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \frac{5}{2}e^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{2}e^{-t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

C. $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = e^{-t}\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

D. $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = e^{-3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

E. $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = e^{-3t}\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

7. (6 points) Solve

$$y'' - 4y' - 5y = 4e^{-2t}, \quad y(0) = 0, \quad y'(0) = -1$$

A. $y = -\frac{4}{7}e^{5t} + \frac{4}{7}e^{-2t}$

B. $y = -\frac{1}{2}e^{5t} + \frac{1}{2}e^{-t} + \frac{4}{7}te^{-2t}$

C. $y = -\frac{1}{7}e^{-5t} - \frac{3}{7}e^{t} + \frac{4}{7}e^{-2t}$

D. $y = -\frac{1}{14}e^{5t} - \frac{1}{2}e^{-t} + \frac{4}{7}e^{-2t}$

E. $y = -\frac{1}{5}e^{5t} - \frac{1}{5}e^{-t} + \frac{2}{5}e^{-2t}$

8. (6 points) Find the general solution of

$$y'' - y' = t.$$

A. $c_1 + c_2 t - t^3 - 2t^2$

B. $c_1 + c_2 e^t - \frac{1}{3}t^2 - \frac{1}{2}t$

C. $c_1 e^{-t} + c_2 e^t - \frac{1}{3}t^2 - \frac{1}{2}t$

D. $c_1 + c_2 e^t - \frac{1}{2}t - 1$

E. $c_1 + c_2 e^t - \frac{1}{2}t^2 - t$

9. (6 points) Given $y_1(t) = t^{-1}$ is a solution of

$$t^2y'' + 6ty' + 4y = 0.$$

Find another solution $y_2(t)$ which is linearly independent of $y_1(t)$.

A. $t\ln(t)$

B. $t^{-1}\ln(t)$

C. t^{-2}

D. t^{-3}

E. t^{-4}

10. Using variation of parameters to find the general solution of

$$y'' + 4y' + 4y = t^{-2}e^{-2t}$$

A. $c_1e^{-2t} + c_2te^{-2t} + e^{-2t}\log(t)$

B. $c_1e^{-2t} + c_2e^{-2t} - e^{-2t}$

C. $c_1e^{-2t} + c_2te^{-2t} - e^{-2t}t$

D. $c_1e^{-2t} + c_2te^{-2t} - te^{-2t}\log(t)$

E. $c_1e^{-2t} + c_2te^{-2t} - e^{-2t}\log(t)$

11. Given $A = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 4 & -2 & -3 \end{pmatrix}$ has eigenvalues -1, $-1+i$, $-1-i$ and corresponding eigenvector

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1+i \\ i \\ 2 \end{pmatrix}, \quad \begin{pmatrix} 1-i \\ -i \\ 2 \end{pmatrix}.$$

Find the **real** (meaning: does not involve complex numbers) general solution of $y' = Ay$.

12. Find the **general solution** (simplify as much as you can) of

$$z'' + 4z = 5e^{(-2+3i)t}.$$