

MA 36600  
MIDTERM II EXAM (Practice)

NAME \_\_\_\_\_

1. NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.
2. No partial credit will be given. For All problems, show all your work **and** write (or mark) the answers clearly.

**Points awarded**

1. (5 pts) \_\_\_\_\_

2. (5 pts) \_\_\_\_\_

3. (5 pts) \_\_\_\_\_

4. (5 pts) \_\_\_\_\_

5. (5 pts) \_\_\_\_\_

6. (5 pts) \_\_\_\_\_

7. (5 pts) \_\_\_\_\_

8. (5 pts) \_\_\_\_\_

9. (5 pts) \_\_\_\_\_

10. (5 pts) \_\_\_\_\_

**Total Points:** \_\_\_\_\_/50

1. Find the general solution of

$$y'' + 2y' + 2y = 5e^{-2t} \sin(t)$$

- A.  $y = c_1 e^{-t} \cos(t) + c_2 e^{-t} \sin(t) + e^{-2t}(2 \cos(t) - \sin(t))$   
B.  $y = c_1 e^{-t} \cos(t) + c_2 e^{-t} \sin(t) + e^{-2t}(-2 \cos(t) - \sin(t))$   
C.  $y = c_1 e^t \cos(t) + c_2 e^t \sin(t) + e^{-2t}(2 \cos(t) + \sin(t))$   
D.  $y = c_1 e^{-t} \cos(t) + c_2 e^{-t} \sin(t) + e^{-t}(2 \cos(t) + \sin(t))$   
E.  $y = c_1 e^{-t} \cos(t) + c_2 e^{-t} \sin(t) + e^{-2t}(2 \cos(t) + \sin(t))$

2. Transform the following third-order equation

$$y'''(t) - 3ty' + \sin(2t)y = 7e^{-t}$$

into a first-order system

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}' = A \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + \mathbf{f}(t)$$

by setting  $u_1(t) = y(t)$ ,  $u_2(t) = y'(t)$  and  $u_3(t) = y''(t)$ . Find the matrix  $A$ .

A.  $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\sin(2t) & 3t & 0 \end{pmatrix}$

B.  $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \sin(2t) & -3t & 0 \end{pmatrix}$

C.  $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 3t & \sin(2t) & 0 \end{pmatrix}$

D.  $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 3t & \sin(2t) \end{pmatrix}$

E.  $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\sin(2t) & 3t & 7e^{-t} \end{pmatrix}$

3. If  $y'' + 2y' + 5y = 0$ ,  $y(0) = 1$ ,  $y'(0) = 2$ , then  $y(\frac{\pi}{2}) =$

- A. 1.
- B. 0.
- C.  $e^\pi$ .
- D.  $e^{\frac{\pi}{2}}$ .
- E.  $-e^{-\frac{\pi}{2}}$ .

4. Find all values of  $k$  for which the equilibrium point  $(0, 0)$  of the system

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} -2 & k \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

is a saddle point.

- A.  $k > 0$
- B.  $k > 1$
- C.  $k < 0$
- D.  $k < 1$
- E.  $k \geq 1$

5. Find the solution of the linear system

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \quad \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

A.  $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \frac{1}{2}e^{3t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \frac{1}{2}e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

B.  $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \frac{1}{2}e^{-3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{2}e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

C.  $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \frac{1}{2}e^{3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{2}e^t \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

D.  $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

E.  $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \frac{1}{2}e^{-3t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \frac{1}{2}e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

6. Find the solution of the linear system

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}, \quad \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$

A.  $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \frac{1}{2}e^{3t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \frac{1}{2}e^t \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

B.  $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \frac{5}{2}e^{-3t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{1}{2}e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

C.  $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = e^{-t} \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

D.  $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = e^{-3t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

E.  $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = e^{-3t} \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

7. (6 points) Solve

$$y'' - 4y' - 5y = 4e^{-2t}, \quad y(0) = 0, \quad y'(0) = -1$$

A.  $y = -\frac{4}{7}e^{5t} + \frac{4}{7}e^{-2t}$

B.  $y = -\frac{1}{2}e^{5t} + \frac{1}{2}e^{-t} + \frac{4}{7}te^{-2t}$

C.  $y = -\frac{1}{7}e^{-5t} - \frac{3}{7}e^t + \frac{4}{7}e^{-2t}$

D.  $y = -\frac{1}{14}e^{5t} - \frac{1}{2}e^{-t} + \frac{4}{7}e^{-2t}$

E.  $y = -\frac{1}{5}e^{5t} - \frac{1}{5}e^{-t} + \frac{2}{5}e^{-2t}$

8. (6 points) Find the general solution of

$$y'' - y' = t.$$

A.  $c_1 + c_2t - t^3 - 2t^2$

B.  $c_1 + c_2e^t - \frac{1}{3}t^2 - \frac{1}{2}t$

C.  $c_1e^{-t} + c_2e^t - \frac{1}{3}t^2 - \frac{1}{2}t$

D.  $c_1 + c_2e^t - \frac{1}{2}t - 1$

E.  $c_1 + c_2e^t - \frac{1}{2}t^2 - t$

9. (6 points) Given  $y_1(t) = t^{-1}$  is a solution of

$$t^2 y'' + 6ty' + 4y = 0.$$

Find another solution  $y_2(t)$  which is linearly independent of  $y_1(t)$ .

A.  $t \ln(t)$

B.  $t^{-1} \ln(t)$

C.  $t^{-2}$

D.  $t^{-3}$

E.  $t^{-4}$

10. Using variation of parameters to find the general solution of

$$y'' + 4y' + 4y = t^{-2}e^{-2t}$$

A.  $c_1 e^{-2t} + c_2 t e^{-2t} + e^{-2t} \log(t)$

B.  $c_1 e^{-2t} + c_2 e^{-2t} - e^{-2t}$

C.  $c_1 e^{-2t} + c_2 t e^{-2t} - e^{-2t} t$

D.  $c_1 e^{-2t} + c_2 t e^{-2t} - t e^{-2t} \log(t)$

E.  $c_1 e^{-2t} + c_2 t e^{-2t} - e^{-2t} \log(t)$

11. Given  $A = \begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 4 & -2 & -3 \end{pmatrix}$  has eigenvalues  $-1$ ,  $-1 + i$ ,  $-1 - i$  and corresponding eigenvector

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1+i \\ i \\ 2 \end{pmatrix}, \quad \begin{pmatrix} 1-i \\ -i \\ 2 \end{pmatrix}.$$

Find the **real** (meaning: does not involve complex numbers) general solution of  $y' = Ay$ .

$$\text{Answer: } y = c_1 e^{-t} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + c_2 e^{-t} \begin{pmatrix} \cos(t) - \sin(t) \\ -\sin(t) \\ 2 \cos(t) \end{pmatrix} + c_3 e^{-t} \begin{pmatrix} \sin(t) + \cos(t) \\ \cos(t) \\ 2 \sin(t) \end{pmatrix}$$

12. (6 points) Find the **general solution** (simplify as much as you can) of

$$z'' + 4z = 5e^{(-2+3i)t}.$$

$$\text{Answer: } c_1 \cos(2t) + c_2 \sin(2t) + \frac{5it \sin(3)}{4e^2} + \frac{5t \cos(3)}{4e^2}$$