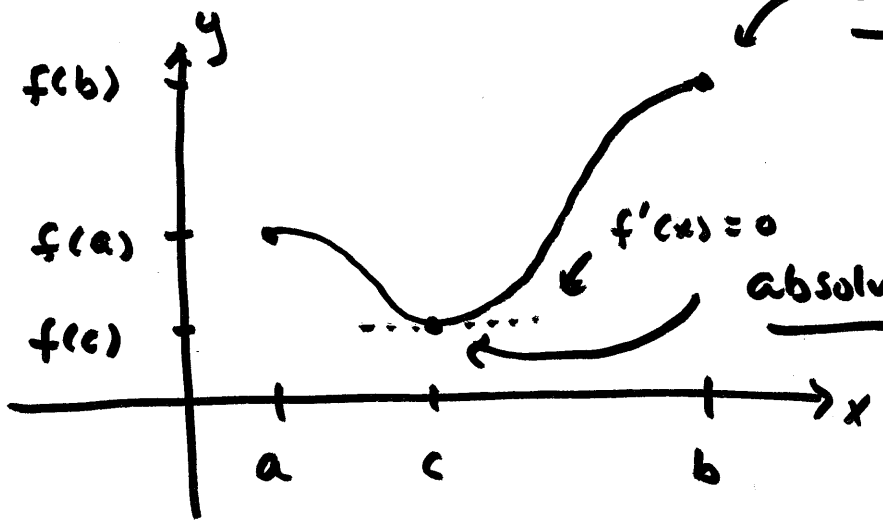
4.1 Maximum and Minimum Values

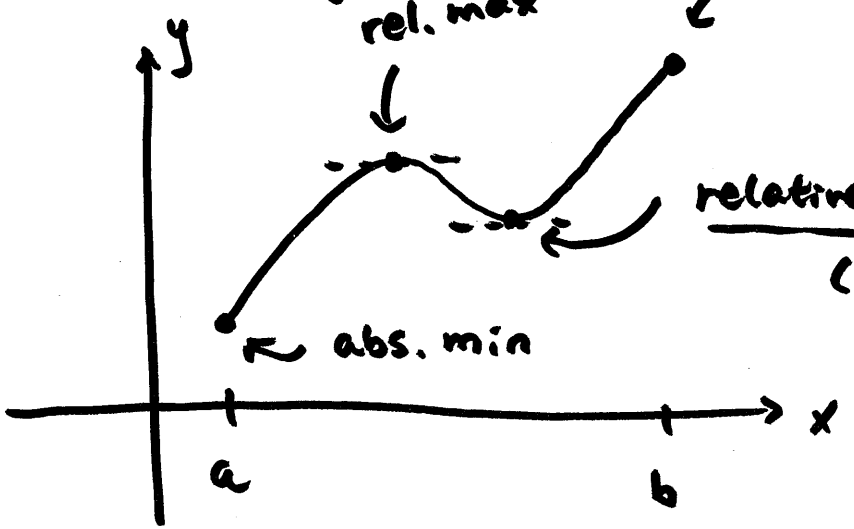
find the max/min values of $f(x)$ on the interval $a \le x \le b$
where $f(x)$ is continuous



absolute (global) maximum
(highest within $[a, b]$)

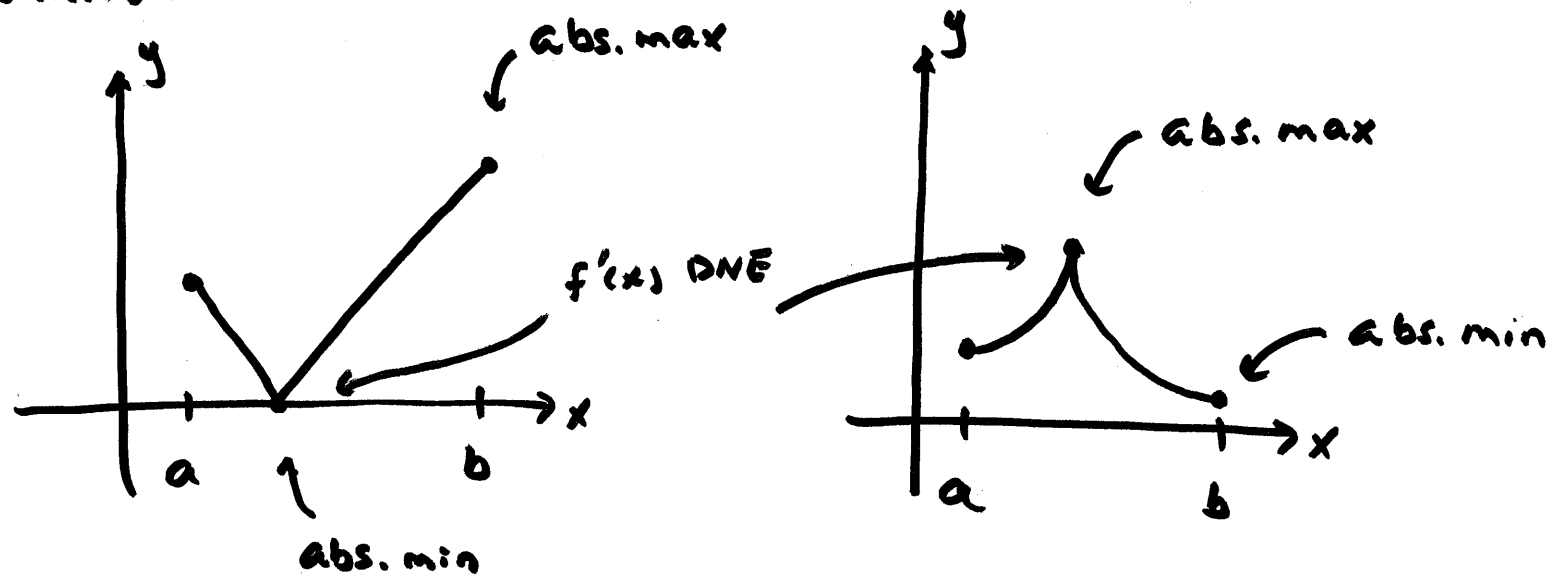
absolute minimum

another possibility:



(lowest in the neighborhood)

other possibilities:



so, to find the abs. max/min, we need to look at:

- end points of the interval
- $f'(x) = 0$
- $f'(x)$ DNE

find the x values of each, then compare f(x) at these xs to find the biggest/smallest

example Find the abs. max/min of $f(x) = 2x^3 - 6x^2 - 18x + 4$ on $[-2, 4]$

find where $f'(x) = 0$ and $f'(x)$ DNE

$$f'(x) = 6x^2 - 12x - 18 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$x = -1$, $x = 3$

"critical numbers" ($f' = 0$)

$f'(x) = 6x^2 - 12x - 18$ is a polynomial, which is defined for all real x, so $f'(x)$ DNE doesn't happen.

potential locations of abs. max/min: $x = -2, -1, 3, 4$

compare y values $\quad f(x) = 2x^3 - 6x^2 - 18x + 4$

$f(-2) = 2(-2)^3 - 6(-2)^2 - 18(-2) + 4 = 0$

$f(-1) = 14 \longrightarrow$ abs. max @ $(-1, 14)$

$f(3) = -50 \longrightarrow$ abs. min @ $(3, -50)$

$f(4) = -36$

<u>example</u>: $f(x) = x + \frac{1}{x}$ on $[1, 4]$

$= x + x^{-1}$

$f'(x) = 0$: $\quad f'(x) = 1 - x^{-2} = 0$

$1 - \frac{1}{x^2} = 0$

$1 = \frac{1}{x^2}$

$x^2 = 1$

$x = \cancel{-1}, \quad x = +1$

throw it out because it is outside $[+1, 4]$

$f'(x)$ DNE : $f'(x) = 1 - \frac{1}{x^2}$

is undefined at $x \neq 0$

throw away because it is outside $[1, 4]$

the only critical number is $x = 1$ (which is also the left boundary)

compare: $f(1) = 2 \rightarrow$ abs. min @ $(1, 2)$

$f(4) = \frac{17}{4} \rightarrow$ abs. max @ $(4, \frac{17}{4})$

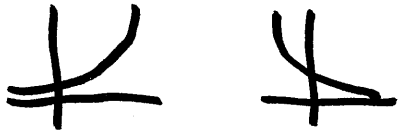
example $f(x) = (x)(e^{-x^2/32})$ on $[-2, 8]$

prod. rule $f'(x) = (x)(e^{-x^2/32})(-\frac{x}{16}) + e^{-x^2/32}(1)$

$$f'(x) = e^{-x^2/32}\left(-\frac{1}{16}x^2 + 1\right)$$

$f'(x) = 0$: $\underbrace{e^{-x^2/32} = 0}$ or $\underbrace{-\frac{1}{16}x^2 + 1 = 0}$

NEVER, because an exponential function doesn't have x-intercepts



$x^2 - 16 = 0$

$x \neq -4$, $x = 4$

because it's outside $[-2, 8]$

$f'(x)$ DNE : NEVER, because it is a product of exponential and polynomial, which are defined for all real x.

Compare: $f(-2) = (-2)e^{-(-2)^2/32} = -1.76 \rightarrow$ abs. min @ $(-2, -1.76)$

$f(4) = 2.43 \rightarrow$ abs. max @ $(4, 2.43)$

$f(8) = 1.68$

Example $f(x) = x - \ln(9x)$ on $\left[\frac{1}{2}, 2\right]$

$$f'(x) = 1 - \frac{1}{9x} \cdot 9 = 1 - \frac{1}{x}$$

$f'(x) = 0$: $x = 1$

$f'(x)$ DNE : $x \neq 0$

outside $\left[\frac{1}{2}, 2\right]$

AND also because $f(0)$ is not defined so $f(0)$ cannot be a max/min

Compare:

$f\left(\frac{1}{2}\right) = \frac{1}{2} - \ln\left(\frac{9}{2}\right) = -1.004$

$f(1) = -1.197 \rightarrow$ abs. min @ $(1, -1.197)$

$f(2) = -0.890 \rightarrow$ abs. max @ $(2, -0.890)$