MA 161 Exam 1

Tuesday, 2/1/2022, 6:30 pm

Recitation Instructor	Location
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The domain of the function $\dfrac{\sqrt[4]{6+5x}}{x^2-9}$ is

A. $x \neq \pm 3$

B. $\left[-\frac{6}{5}, 3\right) \cup (3, \infty)$

C. $\left(-\infty, \frac{6}{5}\right]$

D. $(-\infty, -3) \cup \left(-3, \frac{6}{5}\right)$

E. $(3, \infty)$

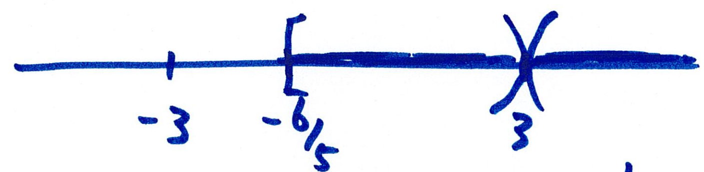
no division by zero

no negative under even root

$x^2 - 9 \neq 0 \rightarrow x \neq -3, \; x \neq 3$

$6 + 5x \geq 0$

$5x \geq -6 \rightarrow x \geq -6/5$



$[-6/5, 3) \; (3, \infty)$

9) The graph of $f(x) = 3^{x-4} + 1$ is obtained from the graph of $g(x) = 3^x$ by

A) Shifting horizontally 4 units left and vertically 1 unit down.

B) Reflecting it about the x-axis and shifting vertically 1 unit up.

C) Shifting vertically 4 units up and horizontally 1 unit left.

D) Shifting horizontally 4 units to the right and vertically 1 unit up.

E) Shifting vertically 4 units down and horizontally 1 unit right.

$f(x) = 3^{x-4} + 1$

→ change x : horizontal shift

-4 → RIGHT 4 units

↳ change to y : vertical shift

+1 → UP 1 unit

The inverse of the function $f(x) = \dfrac{3x-2}{2x+5}$ is $f^{-1}(x) =$

A. $\dfrac{5x-2}{3-2x}$

B. $\dfrac{2x-5}{3-2x}$

C. $\dfrac{2x+3}{5-2x}$

D. $\dfrac{5x+2}{3-2x}$

E. $\dfrac{3x-2}{3-5x}$

$$y = \frac{3x-2}{2x+5}$$

interchange x and y

$$x = \frac{3y-2}{2y+5}$$

solve for y

$$(x)(2y+5) = 3y-2$$

$$2xy + 5x = 3y - 2$$

$$2xy - 3y = -2 - 5x$$

$$y(2x-3) = -2-5x$$

$$y = \frac{-2-5x}{2x-3} = \frac{-(5x+2)}{2x-3} = \frac{5x+2}{-(2x-3)} = \frac{5x+2}{3-2x}$$

For $-1 \leq x \leq 1$, $\tan(\sin^{-1} x)$ equals

A. $\sqrt{1-x^2}$

B. $\dfrac{\sqrt{1-x^2}}{x}$

C. $\dfrac{x}{\sqrt{1-x^2}}$

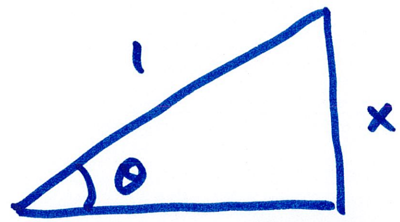
D. $x\sqrt{1-x^2}$

E. $\dfrac{1}{\cos x}$

$\tan(\underbrace{\sin^{-1} x}_{\theta})$

$\theta = \sin^{-1} x \longleftrightarrow x = \boxed{\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{x}{1}}$

draw triangle



$a = \sqrt{1-x^2}$

Solve for a: Pythagorean Theorem

$1^2 = a^2 + x^2$

$a^2 = 1 - x^2$

$a = \sqrt{1-x^2}$

$\tan(\theta) = \dfrac{\text{opp}}{\text{adj}} = \dfrac{x}{\sqrt{1-x^2}}$

If $f(x) = \left(2 + \frac{3}{x}\right)$, then the expression $\frac{f(a+h) - f(a-h)}{2h} =$

A. $1 + \frac{3}{a^2 - h^2}$

B. $2 + \frac{6}{a^2 - h^2}$

C. $\frac{1}{h} - \frac{1}{2(a^2 - h^2)}$

D. $-\frac{3}{a^2 - h^2}$

E. $-\frac{6}{a^2 - h^2}$

$$f(a+h) = 2 + \frac{3}{a+h}$$

$$f(a-h) = 2 + \frac{3}{a-h}$$

$$f(a+h) - f(a-h) = 2 + \frac{3}{a+h} - \left(2 + \frac{3}{a-h}\right)$$

$$= 2 + \frac{3}{a+h} - 2 - \frac{3}{a-h}$$

$$= \frac{3}{a+h} - \frac{3}{a-h} = \frac{3}{a+h} \cdot \frac{a-h}{a-h} - \frac{3}{a-h} \cdot \frac{a+h}{a+h}$$

$$= \frac{3a-3h}{a^2-h^2} - \frac{3a+3h}{a^2-h^2} = \frac{3a-3h-(3a+3h)}{a^2-h^2} = \frac{3a-3h-3a-3h}{a^2-h^2}$$

$$= \frac{-6h}{a^2-h^2}$$

$$\frac{f(a+h)-f(a-h)}{2h} = \frac{-6h}{a^2-h^2} \cdot \frac{1}{2h} = \frac{-3}{a^2-h^2}$$

$$\lim_{x\to 2} \frac{2x - x^2}{x^2 - x - 2} =$$

A. $-\frac{2}{3}$

B. $\frac{2}{3}$

C. 0

D. ∞

E. $-\infty$

try $x=2$: $\frac{2(2)-(2)^2}{(2)^2-(2)-2} = \frac{0}{0}$

factor & cancel

rationalization

reverse order : change sign

$$\lim_{x\to 2} \frac{x(2-x)}{(x-2)(x+1)} = \lim_{x\to 2} \frac{-x\,(\cancel{x-2})}{(\cancel{x-2})(x+1)}$$

$$= \lim_{x\to 2} \frac{-x}{x+1}$$

try $x=2$ again

$$= \frac{-2}{2+1} = \frac{-2}{3}$$

$$\lim_{x \to 3} \frac{x-3}{\sqrt{x}-\sqrt{3}}$$

a. $= 2\sqrt{3}$

b. $= \frac{\sqrt{3}}{3}$

c. $= \frac{2\sqrt{3}}{3}$

d. $= \sqrt{3}$

e. does not exist

try $x=3$: $\frac{3-3}{\sqrt{3}-\sqrt{3}} = \frac{0}{0}$

roots are involved: rationalize

$$\lim_{x \to 3} \frac{x-3}{\sqrt{x}-\sqrt{3}} \cdot \frac{\sqrt{x}+\sqrt{3}}{\sqrt{x}+\sqrt{3}}$$

$$= \lim_{x \to 3} \frac{(x-3)(\sqrt{x}+\sqrt{3})}{(\sqrt{x}-\sqrt{3})(\sqrt{x}+\sqrt{3})} = \lim_{x \to 3} \frac{\cancel{(x-3)}(\sqrt{x}+\sqrt{3})}{\cancel{(x-3)}}$$

$$= \lim_{x \to 3} \sqrt{x}+\sqrt{3}$$

try $x=3$ again

$$= \sqrt{3}+\sqrt{3} = 2\sqrt{3}$$

$$\lim_{x\to 2^-} \frac{x+1}{x-2} =$$

A. ∞

B. $-\infty$

C. 0

D. 3

E. 1

try $x=2$: $\frac{3}{0} \to \infty$ or $-\infty$

depending on the sign of the ratio

means a small number can be positive or negative

$x \to 2^-$ means x is a number close to 2 but less than 2

let $x = 1.9999$

then $\frac{x+1}{x-2} = \frac{2.9999}{-0.0001}$ overall ratio is negative

so $\lim_{x\to 2^-} \frac{x+1}{x-2} = -\infty$

Spring 2020

7. Find all x in $[0, 2\pi]$ such that $2\sin x\cos x + \cos x = 0$

$2\sin x \cos x = -\cos x$

if you divide by $\cos x$

$2\sin x = -1$

this will only give part of the solutions

means $\cos x \neq 0$

but $\cos x$ could be zero on $[0, 2\pi]$

right way: $2\sin x\cos x + \cos x = 0$

$(\cos x)(2\sin x + 1) = 0$

$\cos x = 0$ or $2\sin x + 1 = 0$

$\sin x = -1/2$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

Fall 2021 #8 ignore (derivative question)

Spring 2019 #9

$$\lim_{x \to 2} \frac{\overbrace{\cos\left(\frac{x^2-4}{\pi}\right)}^{\to \text{ goes } 1}(x-2)}{\sqrt{x^2+12}-4}$$

try $x=2$: $\dfrac{\cos(0)\,(0)}{\sqrt{16}-4} \to \dfrac{0}{0} = ?$

the $\cos\left(\frac{x^2-4}{\pi}\right)$ goes to 1, does not affect limit

focus on

$$\lim_{x \to 2} \frac{x-2}{\sqrt{x^2+12}-4} \cdot \frac{\sqrt{x^2+12}+4}{\sqrt{x^2+12}+4}$$

$$= \lim_{x \to 2} \frac{(x-2)(\sqrt{x^2+12}+4)}{\left(\sqrt{x^2+12}\right)^2-(4)^2} = \lim_{x \to 2} \frac{(x-2)(\sqrt{x^2+12}+4)}{x^2+12-16}$$

$$= \lim_{x \to 2} \frac{(x-2)(\ \cdots\ )}{x^2-4} = \lim_{x \to 2} \frac{\cancel{(x-2)}(\sqrt{x^2+12}+4)}{(x+2)\cancel{(x-2)}}$$

$$= \lim_{x \to 2} \frac{\sqrt{x^2+12}+4}{x+2} = \frac{\sqrt{16}+4}{4} = \frac{8}{4} = 2$$