If $y = (x^2+1)\tan x$, then $\frac{dy}{dx} =$ A. $2x\tan x + (x^2+1)\sec^2 x$ B. $2x\sec^2 x$ C. $2x\tan x + (x^2+1)\tan x$ D. $2x\tan x + 2x\sec^2 x$ E. $2x\tan x$

product rule

$y = (x^2+1)\tan x$

$\frac{dy}{dx} = (x^2+1)\sec^2 x + \tan x\,(2x)$

If $h(x) = \begin{cases} x^2 + a, & \text{for } x < -1 \\ x^3 - 8 & \text{for } x \geq -1 \end{cases}$ determine all values of a so that h is continuous for all values of x.

A. $a = -1$ B. $a = -8$ C. $a = -9$ D. $a = -10$ E. There are no values of a.

continuous at $x = b$ $\rightarrow$ $f(b)$ defined

$\lim\limits_{x \to b} f(x)$ exists

$f(b) = \lim\limits_{x \to b} f(x)$

$x^2 + a$ and $x^3 - 8$ are polynomials, so defined and continuous on their own subdomains.

only place to check is at $x = -1$

$h(-1)$ defined? yes, $h(x) = x^3 - 8$ $x \geq -1$ so $h(-1) = (-1)^3 - 8$
$= -9$

$\lim\limits_{x \to -1} h(x)$ exists? Check if $\lim\limits_{x \to -1^-} h(x) = \lim\limits_{x \to -1^+} h(x)$

$$\left.\begin{aligned} \lim_{x \to -1^-} h(x) &= \lim_{x \to -1^-} x^2 + a = 1 + a \\ \lim_{x \to -1^+} h(x) &= \lim_{x \to -1^+} x^3 - 8 = -9 \end{aligned}\right\}$$

they match if $1 + a = -9$

or $a = -10$

lastly, is $h(-1) = \lim_{x \to -1} h(x)$ with $a = -10$?

$h(-1) = -9$. $\lim_{x \to -1} h(x) = 1 + a = 1 - 10 = -9$ so, yes.

Evaluate $\lim_{x\to 0^+} x\cos(\frac{1}{x})$. (Hint: $-1 \le \cos(\frac{1}{x}) \le 1$ for all $x \ne 0$.) A. 0 B. 1 C. -1 D. $\frac{\pi}{2}$ E. Does not exist

think of Squeeze Theorem

if $g(x) \le f(x) \le h(x)$ for all x except possibly at $x = a$

then $\lim_{x\to a} g(x) \le \lim_{x\to a} f(x) \le \lim_{x\to a} h(x)$

$-1 \le \cos(\frac{1}{x}) \le 1$

multiply by x

$-x \le x\cos(\frac{1}{x}) \le x$

$\lim_{x\to 0^+} -x \le \lim_{x\to 0^+} x\cos(\frac{1}{x}) \le \lim_{x\to 0^+} x$

$0 \le \lim_{x\to 0^+} x\cos(\frac{1}{x}) \le 0$ so limit is 0

If $f(x) = \frac{1}{x+3}$, then $\lim\limits_{x\to 1} \frac{f(x)-f(1)}{x-1} =$ A. $\frac{1}{4}$ B. $\frac{1}{16}$ C. $-\frac{1}{16}$ D. $-\frac{1}{4}$ E. Does not exist

two ways: 1) treat as a regular limit problem

$f(x) = \frac{1}{x+3}$

$f(1) = \frac{1}{4}$

$$\lim_{x\to 1} \frac{\frac{1}{x+3} - \frac{1}{4}}{x-1} = \lim_{x\to 1} \frac{\frac{4}{4(x+3)} - \frac{x+3}{4(x+3)}}{x-1}$$

$$= \lim_{x\to 1} \frac{\frac{4-(x+3)}{4(x+3)}}{x-1} = \lim_{x\to 1} \frac{\frac{1-x}{4(x+3)}}{x-1}$$

$$= \lim_{x\to 1} \frac{1-x}{4(x+3)} \cdot \frac{1}{x-1} = \lim_{x\to 1} \frac{-\cancel{(x-1)}}{4(x+3)} \cdot \frac{1}{\cancel{x-1}}$$

$$= \lim_{x\to 1} \frac{-1}{4(x+3)} = -\frac{1}{16}$$

2) recognize that $f'(a) = \lim\limits_{x\to a} \frac{f(x)-f(a)}{x-a}$ here, $a=1$

so just calculate $f'(1)$

$f(x) = (x+3)^{-1}$ $\quad f'(x) = -(x+3)^{-2} = \frac{-1}{(x+3)^2}$

$f'(1) = \frac{-1}{16}$

If $f(x) = \frac{1-x}{1+x}$, then $f'(1) =$ A. -1 B. $-\frac{1}{2}$ C. 0 D. $\frac{1}{2}$ E. 1

quotient rule

$$f'(x) = \frac{(1+x)(-1) - (1-x)(1)}{(1+x)^2}$$

$$f'(1) = \frac{(2)(-1) - (0)(1)}{(2)^2} = \frac{-2}{4} = -\frac{1}{2}$$

Find $f''(x)$ if $f(x) = \frac{1-x}{1+x}$ A. $\frac{4}{(1+x)^3}$ B. $\frac{-4}{(1+x)^3}$ C. $-\frac{4x}{(1+x)^3} + \frac{2}{(1+x)^2}$ D. $\frac{2(1+x)^2-2x(1+x)}{(1+x)^4}$ E. -1

$$f'(x) = \frac{(1+x)(-1) - (1-x)(1)}{(1+x)^2}$$

Simplify

$$= \frac{-1 - x - 1 + x}{(1+x)^2} = \frac{-2}{(1+x)^2}$$

Chain Rule

$$f''(x) = \cancel{\frac{(1+x)^2 \frac{d}{dx}(2x) - (2x)\frac{d}{dx}(1+x)^2}{(1+x)^4}}$$

$$= \cancel{\frac{(1+x)^2(2) - (2x)(2)(1+x)(1)}{(1+x)^4}}$$

$$f'(x) = \frac{-2}{(1+x)^2} = -2(1+x)^{-2}$$

$$f''(x) = (-2)(-2)(1+x)^{-3}(1)$$

$$= 4(1+x)^{-3} = \frac{4}{(1+x)^3}$$

If $y = \ln(1 - x^2) + \sin^2 x$, then $\frac{dy}{dx} =$ A. $\frac{1}{1-x^2} + \cos^2 x$ B. $\frac{1}{1-x^2} + 2\sin x \cos x$ C. $\frac{1}{1-x^2} + 2\sin x$ D. $\frac{-2x}{1-x^2} + \cos^2 x$ E. $\frac{-2x}{1-x^2} + 2\sin x \cos x$

Lots of Chain Rule here

$$\frac{d}{dx} \ln(u) = \frac{1}{u} \frac{du}{dx}$$

$$\frac{d}{dx} u^n = n u^{n-1} \frac{du}{dx}$$

$$y = \ln(1-x^2) + (\sin x)^2$$

$$\frac{dy}{dx} = \frac{1}{1-x^2} \frac{d}{dx}(1-x^2) + 2(\sin x) \frac{d}{dx}(\sin x)$$

$$= \frac{-2x}{1-x^2} + 2 \sin x \cos x$$

Assume that y is defined implicitly as a differentiable function of x by the equation $xy^2 - x^2 + y + 5 = 0$. Find $\frac{dy}{dx}$ at $(-2, 1)$. A. 9 B. $\frac{-5}{3}$ C. 1 D. 2 E. $\frac{5}{3}$

$$\frac{d}{dx}(\underbrace{xy^2}_{\text{product rule}} - x^2 + y + 5) = \frac{d}{dx}(0)$$

$$\underbrace{(x)(2)(y)\frac{dy}{dx} + (y^2)(1)} - 2x + \frac{dy}{dx} + 0 = 0$$

$$2xy\frac{dy}{dx} + \frac{dy}{dx} = 2x - y^2$$

$$\frac{dy}{dx}(2xy + 1) = 2x - y^2$$

$$\frac{dy}{dx} = \frac{2x - y^2}{2xy + 1}$$

now we have $\frac{dy}{dx}$, we can use $(\overset{x}{-2}, \overset{y}{1})$

$$= \frac{2(-2) - (1)^2}{2(-2)(1) + 1} = \frac{-5}{-3} = \frac{5}{3}$$

absolute max/min

Find the maximum and minimum values of the function $f(x) = 3x^2+6x-10$ on the $\boxed{\text{interval } -2 \le x \le 2.}$
A. max is 14, min is -10. B. max is -10, min is -13 C. max is 14, min is -13 D. no max, min is -10
E. max is 14, no min.

abs. max/min: find where $f'(x)=0$ or f' DNE
then compare $f(x)$ at ↑ and end points.

$f(x) = 3x^2+6x-10$

$f'(x) = 6x+6 = 0 \rightarrow x = -1$ check if this is inside here, we keep it

original function NOT f'

now compare $f(-2) = 3(-2)^2+6(-2)-10 = -10$

$f(-1) = 3(-1)^2+6(-1)-10 = -13$ min

$f(2) = 3(2)^2+6(2)-10 = 14$ max

For a differentiable function $f(x)$ it is known that $f(3) = 5$ and $f'(3) = -2$. Use a linear approximation to get the approximate value of $f(3.02)$. A. 6.02 B. 5.02 C. 5.04 D. 3 E. 4.96.

$\boxed{\text{linear approx near } x=a: \quad f(x) = f(a) + f'(a)(x-a)}$

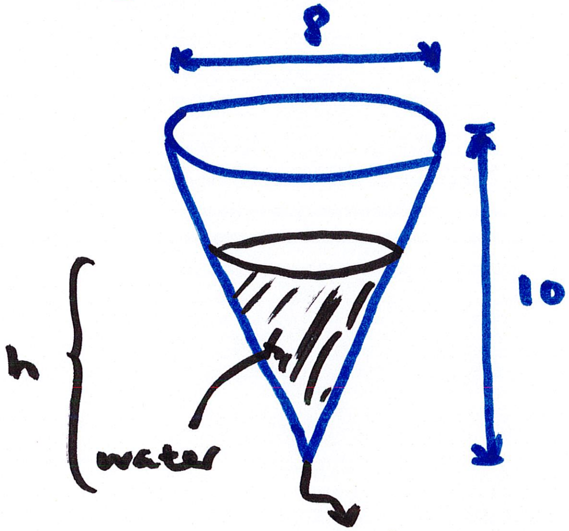
here, we have $f(3)$ and $f'(3)$ so we use $a=3$

$f(x) = f(3) + f'(3)(x-3)$

$f(x) = 5 - 2(x-3)$

$f(3.02) = 5 - 2(3.02-3) = 5 - 2(0.02) = 5 - 0.04 = 4.96$

Water is withdrawn from a conical reservoir, 8 feet in diameter and 10 feet deep (vertex down) at the constant rate of 5 ft^3/min. How fast is the water level falling when the depth of the water in the reservoir is 5 ft? ($V = \frac{1}{3}\pi r^2 h$). A. $\frac{15}{16\pi}$ ft/min B. $\sqrt{\frac{3}{\pi}}$ ft/min C. $\frac{2}{\pi}$ ft/min D. $5\sqrt[3]{3/4\pi}$ ft/min E. $\frac{5}{4\pi}$ ft/min.



water goes out at 5 ft^3/min

$V = \frac{1}{3}\pi r^2 h$

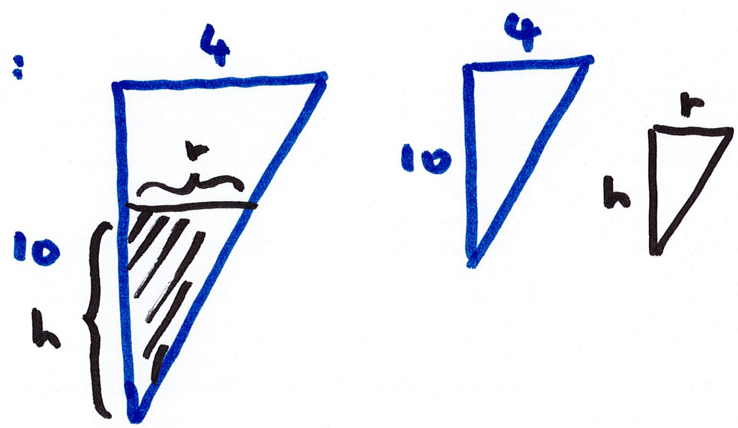
we know: $\frac{dV}{dt} = -5$ ft^3/min

find: $\frac{dh}{dt}$ when $h = 5$

two variables: r, h

we know and want to know more about h, so we want to get rid of r

Similar triangles:



they are similar (same shape)

$\frac{r}{4} = \frac{h}{10}$ $r = \frac{4}{10}h = \frac{2}{5}h$

Sub into V equation

$V = \frac{1}{3}\pi r^2 h$ sub in $r = \frac{2}{5}h$

$= \frac{1}{3}\pi\left(\frac{2}{5}h\right)^2 h$

$= \frac{1}{3}\pi \cdot \frac{4}{25}h^2 \cdot h = \frac{4}{75}\pi h^3$

$V = \frac{4}{75}\pi h^3$

want this

$\frac{dv}{dt} = \frac{4}{75}\pi \cdot 3h^2 \cdot \frac{dh}{dt}$

-5 $\quad$ 5

$\frac{dh}{dt} = \frac{\frac{dv}{dt}}{\frac{4}{75}\pi \cdot 3h^2} = \frac{-5}{\frac{4}{25}\pi(5)^2} = \frac{-5}{4\pi}$

so water level is falling at $\frac{5}{4\pi}$ ft/min