

Find the area of the region between the graph of $y = \frac{1}{1+x^2}$ and the x -axis, from $x = -\sqrt{3}$ to $x = 1$.

- A. $\frac{\pi}{2}$ B. $\frac{3\pi}{4}$ C. $\frac{15\pi}{12}$ D. $\frac{\pi}{3}$ E. $\frac{7\pi}{12}$

$\int_a^b f(x) dx \rightarrow$ area of region bounded by $f(x)$ and x -axis from $x=a$ to $x=b$

here, $\int_{-\sqrt{3}}^1 \underbrace{\frac{1}{1+x^2}}_{f(x)} dx$
 $f(x) \rightarrow F(x) = ?$

$$= \tan^{-1}(x) \Big|_{-\sqrt{3}}^1$$
$$= \tan^{-1}(1) - \tan^{-1}(-\sqrt{3})$$
$$= \frac{\pi}{4} - -\frac{\pi}{3}$$

$$= \frac{\pi}{4} + \frac{\pi}{3} = \frac{7\pi}{12}$$

FTC 2: $\int_a^b f(x) dx = F(b) - F(a)$

where $F'(x) = f(x)$

$F(x)$ is antiderivative of $f(x)$

$$\theta = \tan^{-1}(-\sqrt{3})$$

$$\begin{aligned}\tan \theta &= -\sqrt{3} \\ &= -\frac{\sqrt{3}/2}{1/2}\end{aligned}$$

$$\theta = -\frac{\pi}{3}$$

$$\int_0^1 \frac{e^x}{1+e^x} dx = \quad \text{(A) } \ln \frac{1+e}{2} \quad \text{B. } \ln(1+e) \quad \text{C. } \frac{1}{2} \quad \text{D. } 1 - \ln 2 \quad \text{E. } e$$

$$\int_0^1 \frac{e^x}{1+e^x} dx$$

e^x

 $1+e^x$

e^x vs. $1+e^x$

find one part whose derivative is a constant multiple of the other part

rule of thumb: the more complicated part

here, $\frac{d}{dx}(1+e^x) = e^x \rightarrow$ exactly the other part

so, let $u = 1+e^x$

$$\frac{du}{dx} = e^x \quad du = e^x dx$$

old upper limit: $x=1 \rightarrow u=1+e^x \rightarrow u=1+e$

old lower limit: $x=0 \rightarrow u=1+e^x \rightarrow u=1+e^0 = 1+1 = 2$

$$\int_0^1 \frac{1}{1+e^x} e^x dx = \int_2^{1+e} \frac{1}{u} du = \ln|u| \Big|_2^{1+e} = \ln(1+e) - \ln(2)$$

do NOT go back to x

$$= \ln \frac{1+e}{2}$$

$1+e$
 \int_0^1
 2

$e^x dx$
 du

$\frac{1}{u}$

Compute $\int_{\ln \frac{\pi}{4}}^{\ln \frac{\pi}{2}} e^x \cos e^x dx$.

$$\int_{\ln \frac{\pi}{4}}^{\ln \frac{\pi}{2}} e^x \cos e^x dx$$

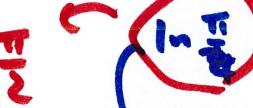
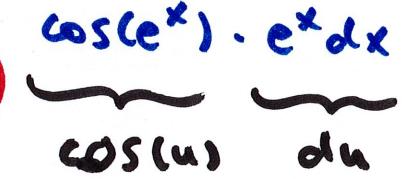
- A. 1
- B. $\frac{\sqrt{3} + 1}{2}$
- C. $1 - \sqrt{2}$
- D. $\frac{\sqrt{3} - 1}{2}$
- E. $\frac{2 - \sqrt{2}}{2}$

look at parts: e^x , $\cos e^x$

neither has a derivative that is a constant multiple of the other

but if we focus on the two e^x , notice

if $u = e^x$ then we can substitute cleanly

| | | |
|-----------------------|-------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------|
| $u = e^x$ | $\frac{\pi}{2}$  | $\cos(e^x) \cdot e^x dx$  |
| $\frac{du}{dx} = e^x$ | $\ln \frac{\pi}{4}$  | $\cos(u) du$ |
| $du = e^x dx$ | π  | |

adjust limits: $x = \ln \frac{\pi}{2} \rightarrow u = e^x = e^{\ln \frac{\pi}{2}} = \frac{\pi}{2}$

$x = \ln \frac{\pi}{4} \rightarrow u = e^x = e^{\ln \frac{\pi}{4}} = \frac{\pi}{4}$

new integral: $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos(u) du = \sin(u) \Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} = \sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{\pi}{4}\right) = 1 - \frac{1}{\sqrt{2}} = 1 - \frac{\sqrt{2}}{2} = \frac{2 - \sqrt{2}}{2}$

Compute $\int_0^{\frac{\pi}{2}} \left(\frac{d}{dx} (x^4 \sin(x)) \right) dx$.

A. $\frac{\pi}{2}$

B. 0

C. 1

D. $\frac{\pi^3}{8}$

E. $\frac{\pi^4}{16}$

$$f(x) = \frac{d}{dx} (x^4 \sin(x))$$

$F(x)$ is antideriv. of $\frac{d}{dx} (x^4 \sin(x))$

$$\text{so } F(x) = x^4 \sin(x)$$

$$\text{FTC 2: } \int_a^b f(x) dx = F(b) - F(a)$$

where $F' = f(x)$

or F is antideriv. of $f(x)$

$$\int_0^{\frac{\pi}{2}} \frac{d}{dx} (x^4 \sin(x)) dx = x^4 \sin(x) \Big|_0^{\frac{\pi}{2}}$$

$$= \left(\frac{\pi}{2}\right)^4 \sin\left(\frac{\pi}{2}\right) - 0$$

$$= \frac{\pi^4}{16}$$

what if deriv. is on the outside?

$$\frac{d}{dx} \left[\int_0^{\pi/2} x^4 \sin(x) dx \right]$$

FtC 1: $\frac{d}{dx} \int_a^x f(t) dt = f(x)$

one has to be
a non-constant

↳ area between $x^4 \sin(x)$ and x -axis

from $x=0$ to $x=\pi/2$

which is always a number (constant)

$$\frac{d}{dx} (\text{constant}) = 0$$

S2018 #4

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{2x}\right)^{3x} \xrightarrow{x \rightarrow \infty} 1^\infty \quad \text{indeterminate form}$$

need to turn it into $\frac{0}{0}$ or $\frac{\infty}{\infty}$ then use l'Hospital's Rule

$$\lim_{x \rightarrow \infty} \underbrace{\left(1 + \frac{1}{2x}\right)^{3x}}_y \quad \text{so we want } \lim_{x \rightarrow \infty} y$$

$$y = \left(1 + \frac{1}{2x}\right)^{3x}$$

$$\ln y = \ln \left(1 + \frac{1}{2x}\right)^{3x}$$

$$= 3x \cdot \ln \left(1 + \frac{1}{2x}\right) = \frac{\ln \left(1 + \frac{1}{2x}\right)}{\frac{1}{3x}} \xrightarrow{x \rightarrow \infty} \frac{\ln(1) = 0}{0} \rightarrow \frac{0}{0}$$

$$\lim_{x \rightarrow \infty} \ln y = \lim_{x \rightarrow \infty} \frac{\ln \left(1 + \frac{1}{2x}\right)}{\frac{1}{3x}}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{1}{1+\frac{1}{2x}} \frac{d}{dx} \left(1 + \frac{1}{2x}\right)}{\frac{d}{dx} \left(\frac{1}{3x}\right)} = \lim_{x \rightarrow \infty} \frac{\frac{1}{1+\frac{1}{2x}} \cdot \frac{1}{2} \cdot \cancel{-\frac{1}{x^2}}}{\frac{1}{3} \cdot \cancel{-\frac{1}{x^2}}}$$

$$\begin{aligned}\frac{d}{dx} \left(\frac{1}{2x}\right) &= \frac{1}{2} \frac{d}{dx} \left(\frac{1}{x}\right) \\ &= \frac{1}{2} \cdot -\frac{1}{x^2}\end{aligned}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{1}{2} \cdot \frac{1}{1+\frac{1}{2x}}}{\frac{1}{3}} \stackrel{0/0 \text{ as } x \rightarrow \infty}{=} \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{3}{2}$$

$$\lim_{x \rightarrow \infty} \ln y = \frac{3}{2} \quad \text{but we want } \lim_{x \rightarrow \infty} y$$

$$y = e^{\ln y}$$

$$\text{so } \lim_{x \rightarrow \infty} y = \lim_{x \rightarrow \infty} e^{\ln y} = e^{\frac{3}{2}}$$

S19 # P

find $f'(1)$ if $f(x) = \ln\left(\frac{x}{x^2+2}\right)$

one way to do this: $f'(x) = \frac{1}{\frac{x}{x^2+2}} \cdot \underbrace{\frac{d}{dx}\left(\frac{x}{x^2+2}\right)}_{\text{quotient rule}}$

it's ok but might get messy

another way: $f(x) = \ln\left(\frac{x}{x^2+2}\right) = \underbrace{\ln(x)}_{\text{each has simple deriv.}} - \underbrace{\ln(x^2+2)}$

$$f'(x) = \frac{1}{x} - \frac{1}{x^2+2} \cdot \frac{d}{dx}(x^2+2)$$

$$= \frac{1}{x} - \frac{2x}{x^2+2}$$

$$f'(1) = \frac{1}{1} - \frac{2}{1+2} = 1 - \frac{2}{3} = \frac{1}{3}$$

$$f(x) = \int_3^{\tan x} \sqrt{t^2 + 6t} dt$$

$$\text{find } f'(\frac{\pi}{4})$$

deriv. of area function:

$$\text{FTC 1 : } \frac{d}{dx} \int_a^x f(t) dt = f(x)$$

if x is not just x , use
Chain Rule

$$f(x) = \int_3^{\tan x} \sqrt{t^2 + 6t} dt$$

let $u = \tan x$ then

$$f(u) = \int_3^u \sqrt{t^2 + 6t} dt$$

$$\text{Chain Rule: } \frac{df}{dx} = \frac{df}{du} \frac{du}{dx}$$

$$\frac{df}{dx} = \underbrace{\left(\frac{d}{du} \int_3^u \sqrt{t^2 + 6t} dt \right)}_{\text{FTC 1}} \left(\frac{d}{dx} \tan x \right)$$

$$= (\sqrt{u^2 + 6u}) (\sec^2 x) = (\sqrt{\tan x + 6 \tan x}) (\sec^2 x)$$

$$\text{at } x = \frac{\pi}{4} \quad \tan x = 1$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\text{so } f'(\frac{\pi}{4}) = \sqrt{1+6} \cdot (\sqrt{2})^2 = 2\sqrt{7}$$