

## 1.1 + 1.2 Review of Functions

(there are 3 parts to the first HW)

function :  $y = f(x)$

↑  
output      input

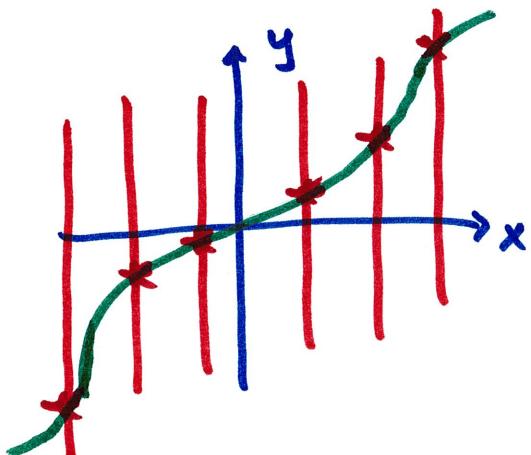
for example,  $f(x) = 2x^2 - x + 2$

$$f(1) = 2(1)^2 - (1) + 2 = 3$$

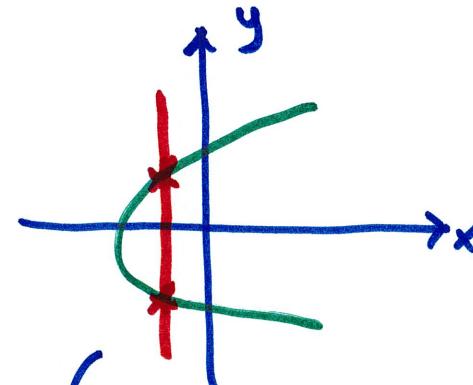
$$f(0) = 2(0)^2 - 0 + 2 = 2$$

$$f(x-3) = 2(x-3)^2 - (x-3) + 2$$

graphs of functions must pass the vertical line test



Each vertical line  
must intersect the  
graph at most once



↳ Cross more than once  
NOT graph of a function

Domain : all acceptable input values

acceptable :  $f(x) = \frac{3}{x-2}$

$$f(1) = \frac{3}{1-2} = \frac{3}{-1} = -3 \text{ number out}$$

unacceptable :  $f(x) = \frac{3}{x-2}$

$$f(2) = \frac{3}{2-2} = \frac{3}{0} \text{ undefined NOT a number}$$

no division by zero

no even roots of negative number

etc

Range : all possible output values

example

$$f(x) = \sqrt{x-5}$$

domain: no even root of negative numbers

$$\text{so, } x-5 \geq 0$$

$$x \geq 5 \quad \text{or} \quad [5, \infty) \quad \text{or} \quad \begin{array}{c} \boxed{5} \\ \hline \end{array}$$

range: the smallest possible value of  $x-5$  within the domain is 0 (when  $x=5$ )

so, the smallest  $\sqrt{x-5}$  can be is 0

there is not maximum to  $x-5$  within the domain

so there is no max to  $\sqrt{x-5}$  either

therefore, the range of  $\sqrt{x-5}$  is

$$[0, \infty)$$

Composition of functions  $\rightarrow$  using a function as input of a function

$$f(x) = x^2 \quad g(x) = \frac{1}{x-1}$$

$$f(g(x)) = (f \circ g)(x)$$

$$= \underbrace{f\left(\frac{1}{x-1}\right)}_{\text{change all } x} = \left(\frac{1}{x-1}\right)^2$$

change all  $x$

$$\text{in } f(x) \text{ to } \frac{1}{x-1}$$

$$g(f(x)) = (g \circ f)(x)$$

$$= \underbrace{g(x^2)}_{\text{change all } x} = \frac{1}{x^2-1}$$

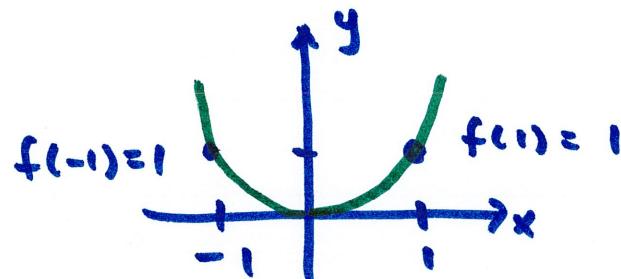
change all  $x$

$$\text{in } g(x) \text{ to } x^2$$

Symmetry : if  $f(-x) = f(x)$  then  $f(x)$  is even  
and its graph has y-axis symmetry

example:  $f(x) = x^2$

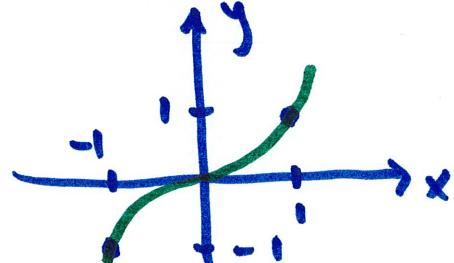
$$f(-x) = (-x)^2 = x^2 = f(x)$$



if  $f(-x) = -f(x)$  then  $f(x)$  is odd  
and its graph has origin symmetry

example :  ~~$f(x) = x$~~   $f(x) = x^3$

$$f(-x) = (-x)^3 = -x^3 = -(x^3) = -f(x)$$



graph transformation :  $y = f(x) + c$

shift vertically by  $c$

$y = f(x+b)$

shifts horizontally  
by  $b$

$b > 0 \rightarrow$  shift LEFT

$b < 0 \rightarrow$  shift RIGHT

$y = af(x)$

vertical stretch or compression  
(Stretch if  $a > 1$ )

(Compression if  $0 < a < 1$ )

horizontal stretch/compress

(Stretch if  $0 < b < 1$ )

(Compress if  $b > 1$ )

$y = -f(x)$

vertical reflection

$y = f(-x)$

horizontal reflection

