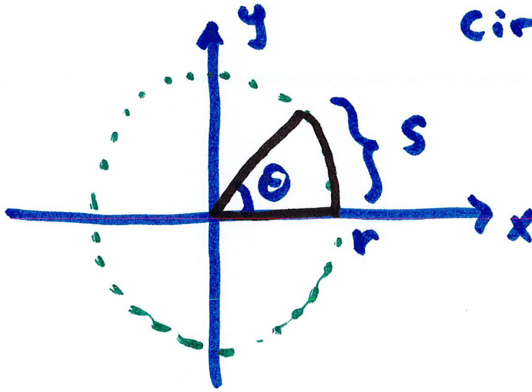
1.4 Trig Functions and Their Inverses

two common ways to measure an angle: degrees, radians

a whole circle is $360°$ or 2π radians

what is a radian?



circle radius r

circular segment w/ included angle θ

s: arc length

↳ $s = r\theta$

the angle θ such that $s = r$ (arc length = radius) is called a radian $\approx 57°$

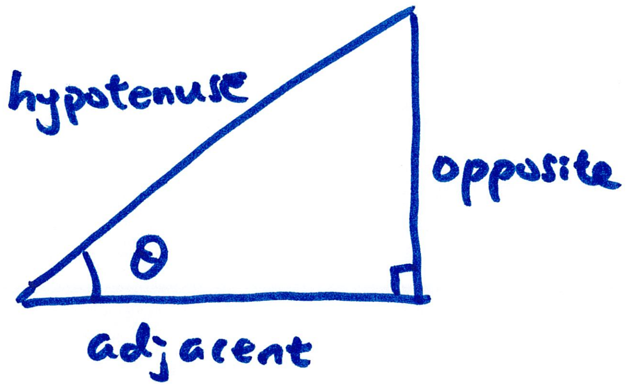
conversion: $360° = 2\pi$ rad

so, $1 \text{ rad} = \dfrac{360°}{2\pi}$ → $1 \text{ rad} = \dfrac{180}{\pi}$ deg

$1° = \dfrac{2\pi}{360}$ rad → $1 \text{ deg} = \dfrac{\pi}{180}$ rad

In calculus, ALWAYS use radians!

basic trig functions : sine, cosine, tangent, cosecant, secant, cotangent



$$\sin\theta = \frac{opp}{hyp} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{hyp}{opp}$$

$$\cos\theta = \frac{adj}{hyp} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{hyp}{adj}$$

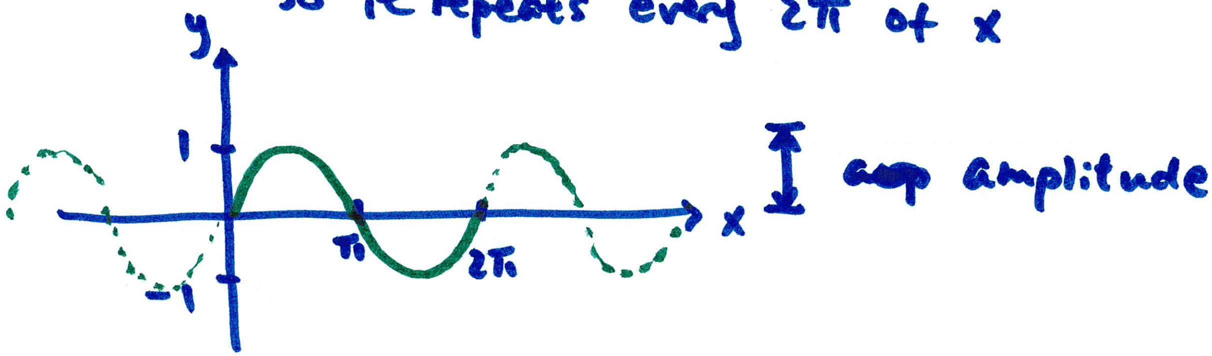
$$\tan\theta = \frac{opp}{adj} = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{adj}{opp} = \frac{\cos\theta}{\sin\theta}$$

these are periodic function : $f(x+p) = f(x)$

period

for example, $f(x) = \sin(x)$ has a period of 2π

so it repeats every 2π of x



we can transform them, too (vertical shift, stretch, etc)
horizontal

$$f(x) = a\sin(bx + c) + d$$

a: vertical stretch/compress (changes amplitude)

b: frequency (horiz. stretch/compress) Period $= \frac{2\pi}{b}$

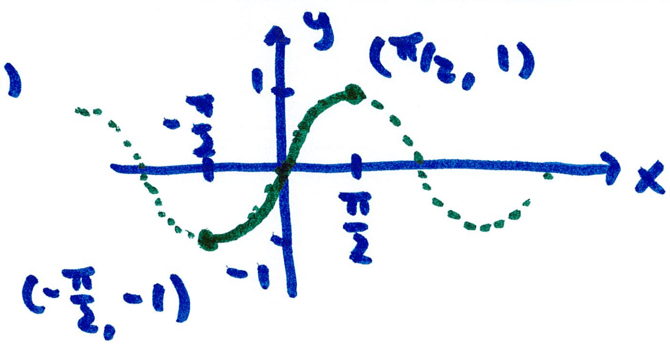
c: horiz. shift phase shift $= \frac{c}{b}$

d: vertical shift

Inverses of sin(x), cos(x), tan(x)

→ not one-to-one, must restrict domain

sin(x)



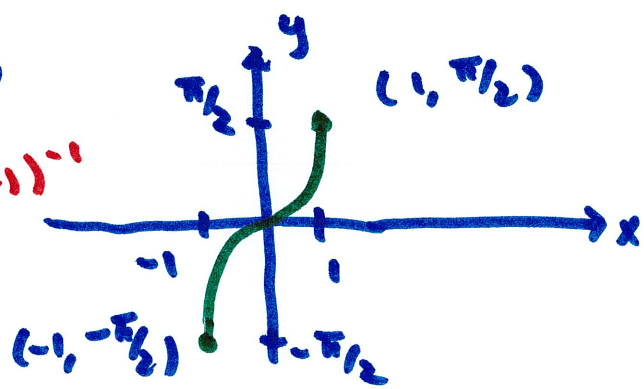
sin(x): restrict domain to $[-\frac{\pi}{2}, \frac{\pi}{2}]$

range: $[-1, 1]$

$\sin^{-1}(x)$

NOT $(\sin(x))^{-1}$

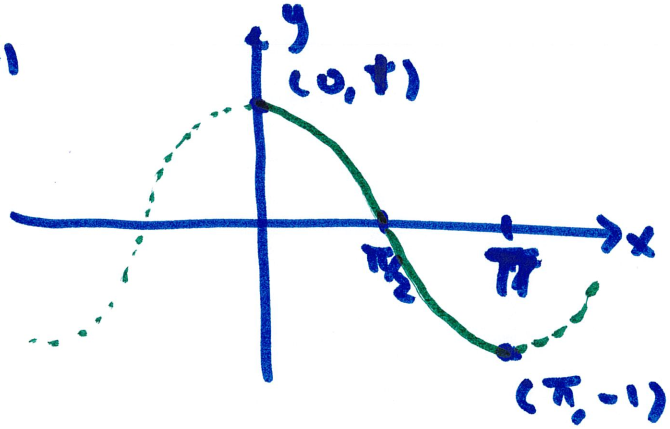
$\neq \frac{1}{\sin x}$



$\sin^{-1}(x)$: domain $[-1, 1]$

range $[-\frac{\pi}{2}, \frac{\pi}{2}]$

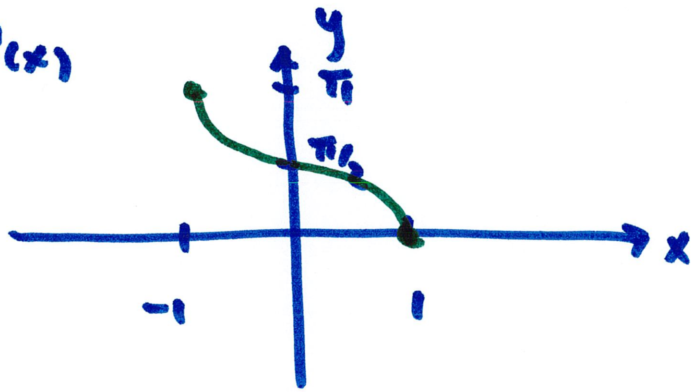
$\cos(x)$



$\cos(x)$: domain $[0, \pi]$

range $[-1, 1]$

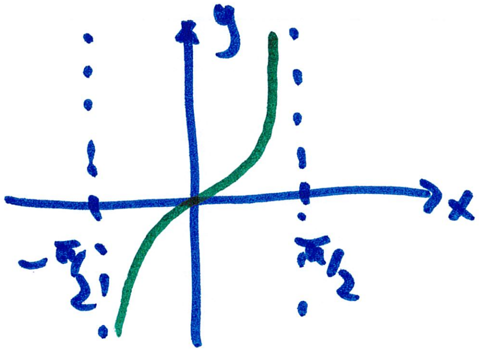
$\cos^{-1}(x)$



$\cos^{-1}(x)$: domain $[-1, 1]$

range $[0, \pi]$

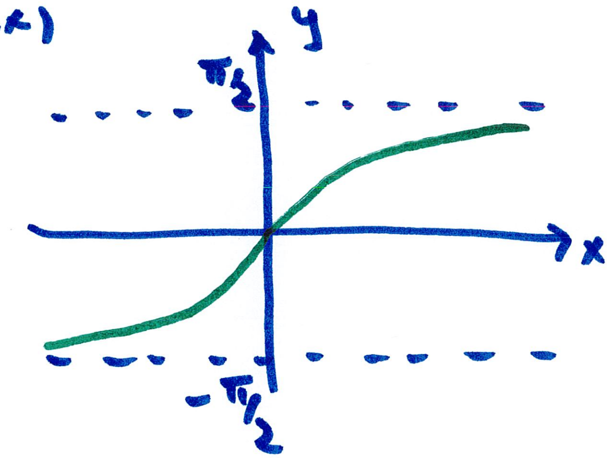
$\tan(x)$



domain: $(-\pi/2, \pi/2)$

range: $(-\infty, \infty)$

$\tan^{-1}(x)$



domain: $(-\infty, \infty)$

range: $(-\pi/2, \pi/2)$

$\sin^{-1}(x)$ has range of $[-\pi/2, \pi/2]$ so $\sin^{-1}(x)$ can only give back an angle in quadrant IV or quadrant I

for example, $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ means to find an angle whose sine is $\frac{\sqrt{2}}{2}$ and is in $[-\pi/2, \pi/2]$

quadrants IV, I

what angles have sine equal to $\frac{\sqrt{2}}{2}$?

$\frac{\pi}{4}$, $\cancel{\frac{3\pi}{4}}$

but since $\sin^{-1}(x)$ can only give a QI or QIV angle, we must discard $\frac{3\pi}{4}$ since it is in QII

so, $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ <u>only</u>

Similarly, $\cos^{-1}(x)$ has range $[0, \pi]$ (QI, QII)

for example, $\cos^{-1}\left(\frac{1}{2}\right) = \pi/3$ and NOT $-\pi/3$ since $-\pi/3$ is NOT in QI or QII example $\cos^{-1}\left(\underbrace{\cos\left(\frac{4\pi}{3}\right)}_{\text{call this } x}\right) = ?$

first, find x: $x = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$

work on the unit circle !!!!

$\cos^{-1}\left(-\frac{1}{2}\right) \to$ find an angle whose cosine is $-\frac{1}{2}$ that is in the interval $[0, \pi]$

(QI, QII)

$= \boxed{\frac{2\pi}{3}}$

example

$\sin^{-1}(\underbrace{\sin(\tfrac{\pi}{4})})$

call this x : $x = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

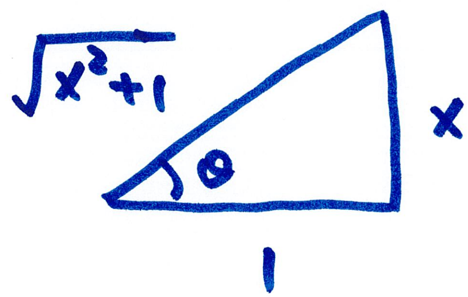
$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$ $\rightarrow$ find angles whose sine is $\frac{\sqrt{2}}{2}$ and in the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ (QI, QIV)

$= \boxed{\frac{\pi}{4}}$

example Rewrite $\cos(\tan^{-1}(x))$ w/o trig functions

$\cos(\underbrace{\tan^{-1}(x)}_{\text{call this } \theta})$ work inside-out

$\theta = \tan^{-1}(x) \rightarrow \underbrace{\tan(\theta) = x = \frac{x}{1} = \frac{\text{opp}}{\text{adj}}}_{\text{build a triangle}}$



$\frac{x}{1} = \frac{\text{opp}}{\text{adj}}$

opp $= x$

adj $= 1$

$\cos(\theta) = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{x^2+1}}$

so, $\boxed{\cos(\tan^{-1}(x)) = \frac{1}{\sqrt{x^2+1}}}$