4.4 Graphing Functions (part 1)

Sketching Guidelines

Domain

Symmetry → not very useful, ok to skip

Increasing / Decreasing Intervals
Relative max/min

} deal with f'

Concave up/down intervals
Inflection points

} deal with f''

Asymptotes

Intercepts

finally, use the above info to sketch a graph

example $f(x) = x^3 - 12x^2 + 36x$

Domain: $(-\infty, \infty)$ because $f(x)$ is a polynomial

Symmetry: skip

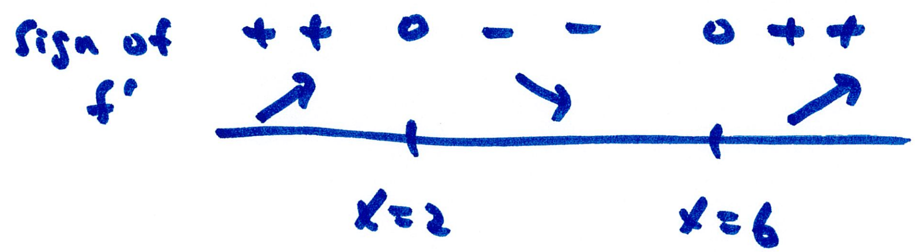
Inc/dec intervals: $f'(x) = 3x^2 - 24x + 36$

$= 3(x^2 - 8x + 12)$

$= 3(x-6)(x-2)$

$f' = 0 \rightarrow x = 6,\ x = 2$

f' DNE $\rightarrow$ never, since f' is polynomial



inc: $(-\infty, 2), (6, \infty)$

dec: $(2, 6)$

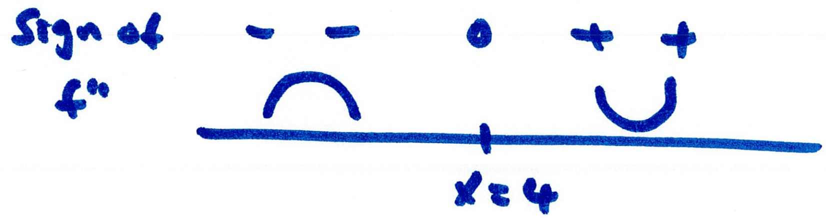
Relative max/min: rel. max at $x = 2$, $y = f(2) = 32 \rightarrow (2, 32)$

rel. min at $x = 6$, $y = f(6) = 0 \rightarrow (6, 0)$

concave up/down intervals: $f'' = 6x - 24$

$f'' = 0 \rightarrow x = 4$

f'' DNE $\rightarrow$ never



CU : $(4, \infty)$

CD : $(-\infty, 4)$

Inflection pts: at $x = 4$, & $y = f(4) = 16 \rightarrow (4, 16)$

Asymptotes: none, $f(x)$ is a polynomial

Intercepts: x-intercept @ $y = 0$

$f(x) = x^3 - 12x^2 + 36x = 0$

$0 = x(x^2 - 12x + 36)$

$0 = x(x-6)(x-6)$

$x = 0, \; x = 6$

y-intercept @ $x = 0$ $\quad y = 0$

points we know: x-ints: $x = 0, x = 6$

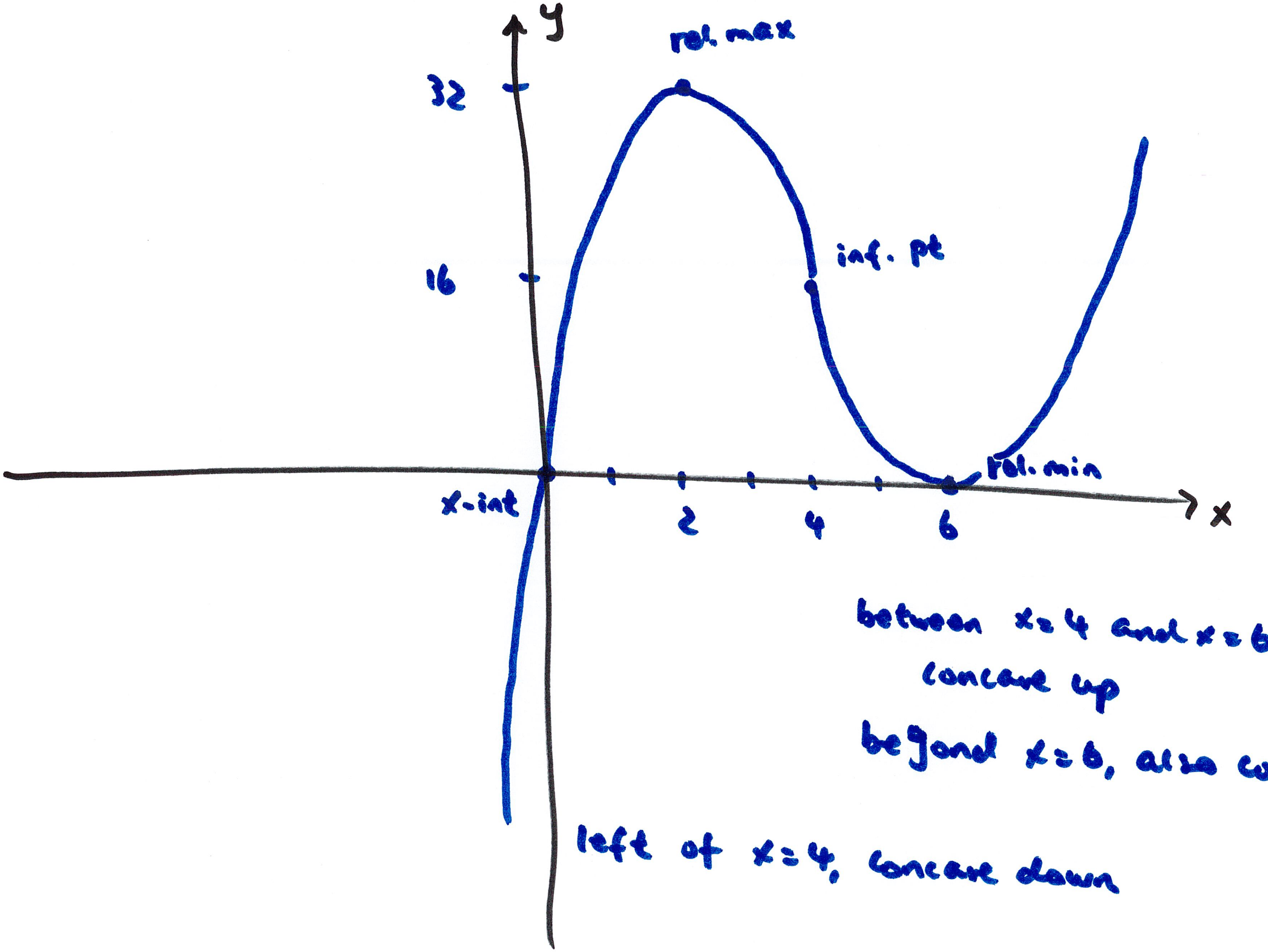
y-int: $y = 0$

rel. max: $(2, 32)$

rel. min: $(6, 0)$

inf. pt: $(4, 16)$

graph: start with known points then use concavity or inc/dec info to fill in in between



between $x=4$ and $x=6$
concave up

beyond $x=6$, also concave up

left of $x=4$, concave down

example $f(x) = \dfrac{x}{x^2-16}$

Domain: $x \neq 4$, $x \neq -4$ (these are vertical asymptotes denom $= 0$ numer $\neq 0$)

$$f' = \cdots = -\frac{x^2+16}{(x^2-16)^2}$$

$f' = 0 \rightarrow x^2+16 = 0 \rightarrow$ no solutions

f' DNE $\rightarrow x^2-16=0 \rightarrow x=-4,\ x=4$

sign of f'	$-\ -$	DNE	$-\ -$	DNE	$-\ -$
	$\searrow$	$x=-4$	$\searrow$	$x=4$	$\searrow$

dec: $(-\infty,-4), (-4,4), (4,\infty)$

inc: none

Rel max/min: none, since there are no f' sign changes

CU / CD intervals: $\quad f'' = \dots = \dfrac{2x(x^2+48)}{(x^2-16)^3}$

$f'' = 0 \rightarrow 2x(x^2+48) = 0 \rightarrow x = 0$

($x^2+48=0$ has no solution)

f'' DNE $\rightarrow x^2-16=0 \rightarrow x=-4,\ x=4$

sign of f''	$- -$ $\frown$	DNE	$+ +$ $\smile$	0	$- -$ $\frown$	DNE	$+ +$ $\smile$
		$x=-4$		$x=0$		$x=4$	

CU: $(-4, 0),\ (4, \infty)$

CD: $(-\infty, -4),\ (0, 4)$

Inflection pts: at $x=0$ only because even though f'' changes sign at $x=-4$, $x=4$, $x=\pm 4$ are not in domain so are NOT points on the graph

inf. pt: $\left(0, \overset{f(0)}{0}\right)$

Asymptotes: vertical : $x = 4$, $x = -4$

horizontal : $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$

$$\left.\begin{array}{l} \lim_{x \to \infty} \dfrac{x}{x^2-16} = 0 \\ \lim_{x \to -\infty} \dfrac{x}{x^2-16} = 0 \end{array}\right\} \quad y = 0$$

~~horiz~~ horiz. asym. on both sides

Intercepts: x-ints @ $y = 0$

$$f(x) = \frac{x}{x^2-16}$$

$$0 = \frac{x}{x^2-16} \rightarrow x = 0$$

y-int~~s~~ @ $x = 0$

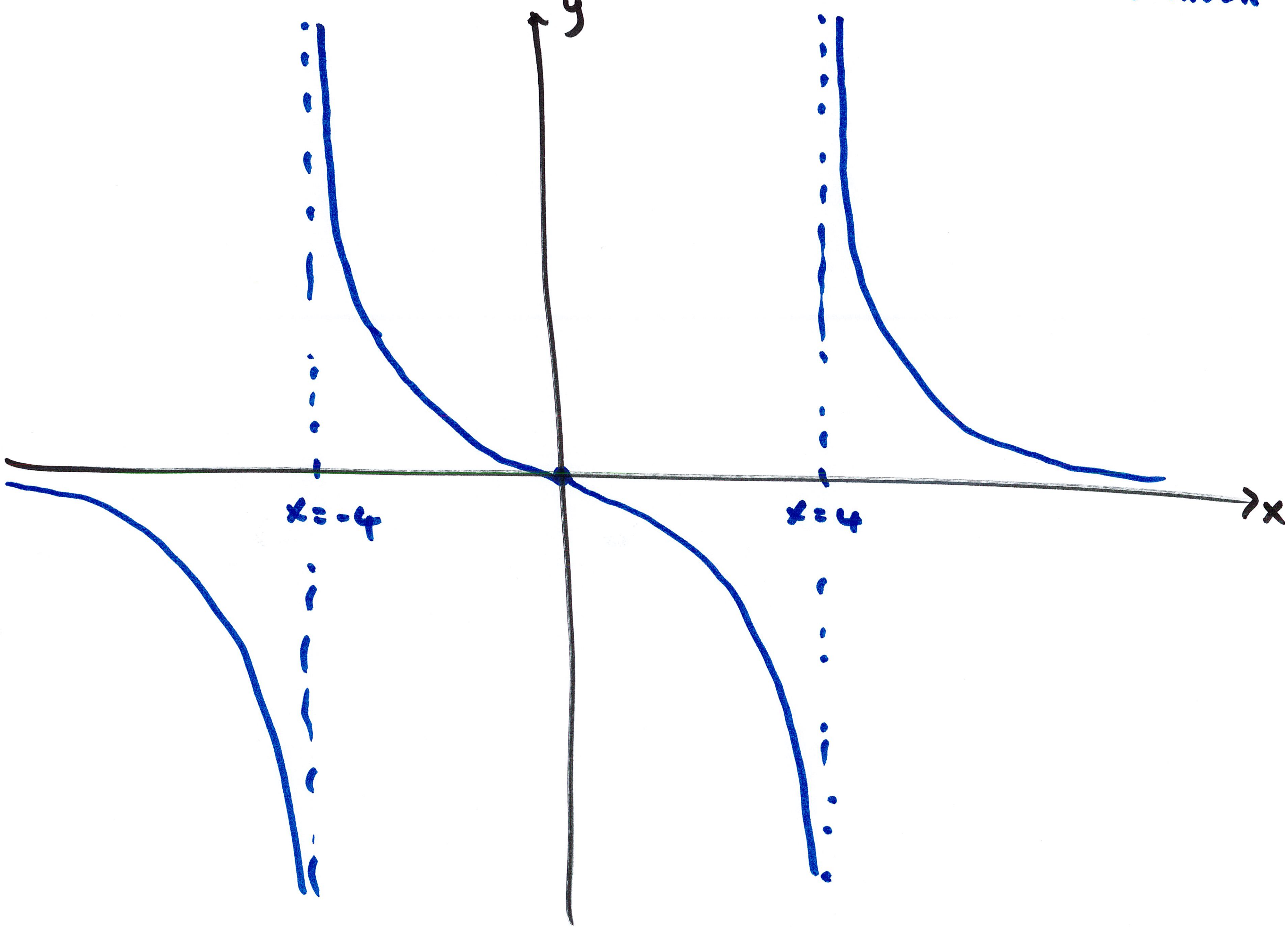
$$f(x) = \frac{x}{x^2-16}$$

$$y = \frac{0}{-16} \rightarrow y = 0$$

points we know: $\underset{\text{x-int}}{x = 0}$, $\underset{\text{y-int}}{y = 0}$, $\underset{\text{inf. pt}}{(0, 0)}$ same point

now we use asymptotes to help us

use CU/CD to fill in
in between

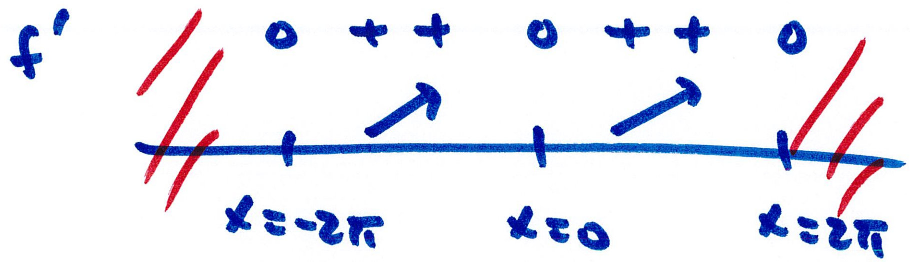


example $f(x) = x - \sin x$ on $[-2\pi, 2\pi]$

Domain: $[-2\pi, 2\pi]$

Inc/dec: $f'(x) = 1 - \cos x$

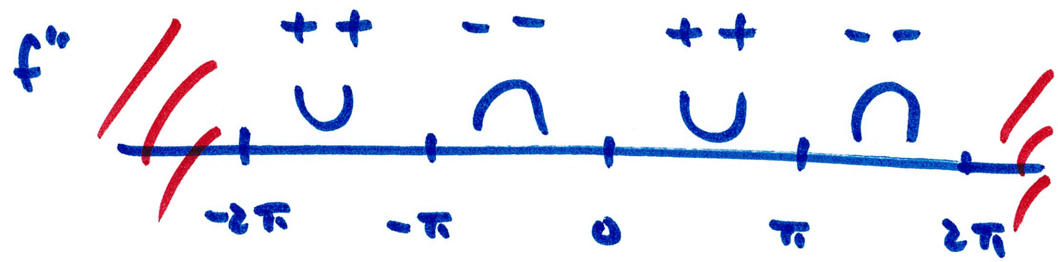
$f' = 0 \rightarrow \cos x = 1 \rightarrow x = -2\pi,\ x = 0,\ x = 2\pi$



Rel. max/min: none (no f' sign change)

CU/CD: $f''(x) = \sin x$

$f'' = 0 \rightarrow \sin x = 0 \rightarrow x = -2\pi, -\pi, 0, \pi, 2\pi$



Inf. pts: $x = -\pi,\ x = 0,\ x = \pi \rightarrow (-\pi, -\pi),\ (0, 0),\ (\pi, \pi)$

Asymptotes : none

Intercepts: x-int $(y=0) \rightarrow x=0$

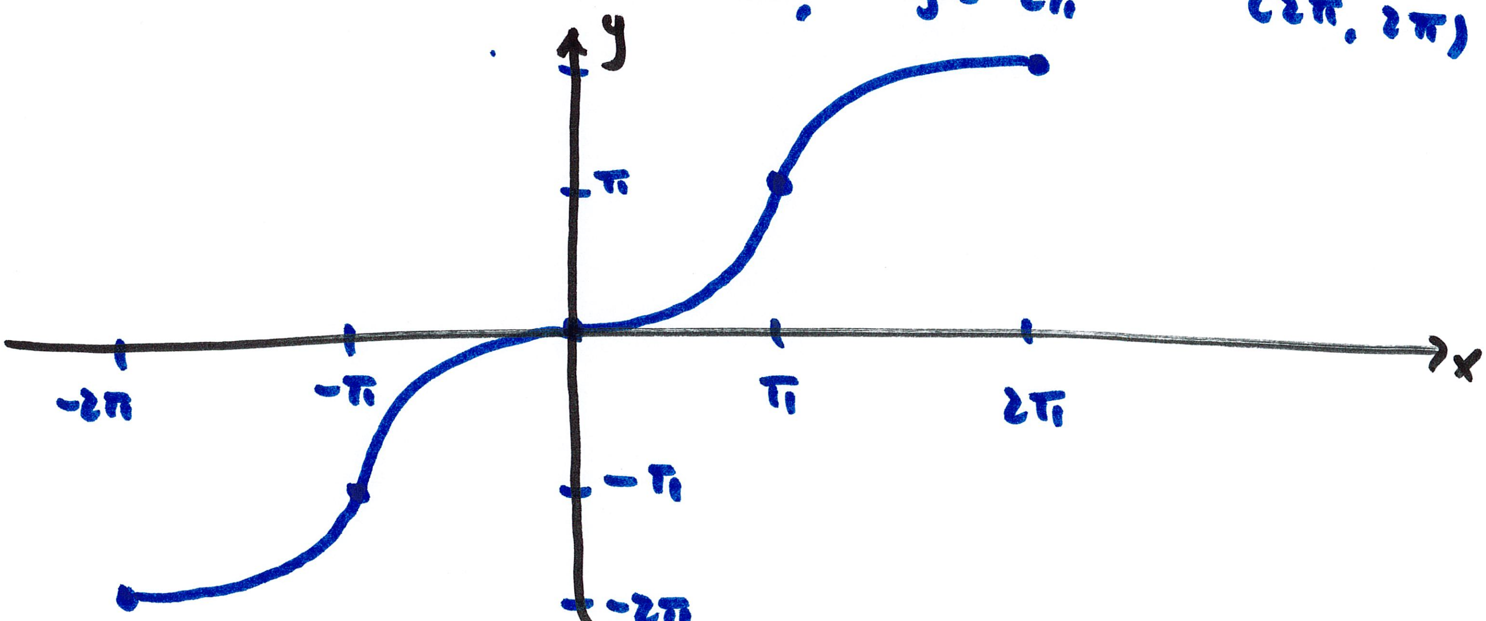
y-int $(x=0) \rightarrow y=0$

points we known: inf. pts $(-\pi, -\pi)$, $(0, 0)$, (π, π)

intercepts $(0, 0)$

end points $x=-2\pi$, $y=-2\pi$ $(-2\pi, -2\pi)$

$x=2\pi$, $y=2\pi$ $(2\pi, 2\pi)$



Some functions have slant asymptotes

$$f(x) = \frac{x^2+15}{5x+1}$$

notice as $x \to \infty$, $x^2+15 \approx x^2$, $5x+1 \approx 5x$

so, $\frac{x^2+15}{5x+1} \to \frac{x^2}{5x} \to \frac{1}{5}x$

this means the graph will approach $y = \frac{1}{5}x$ as $x \to \pm\infty$

vertical : $x = -\frac{1}{5}$

go through CU/CD step

CU: $(-\frac{1}{5}, \infty)$

CD: $(-\infty, -\frac{1}{5})$