4.4 Graphing Functions (part 2)

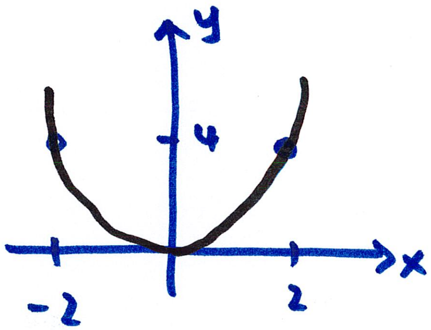
Symmetry: y-axis, origin

y-axis symmetry: when f(x) is <u>even</u> → $f(x) = f(-x)$

example: $f(x) = x^2$

$f(-x) = (-x)^2 = x^2 = f(x)$

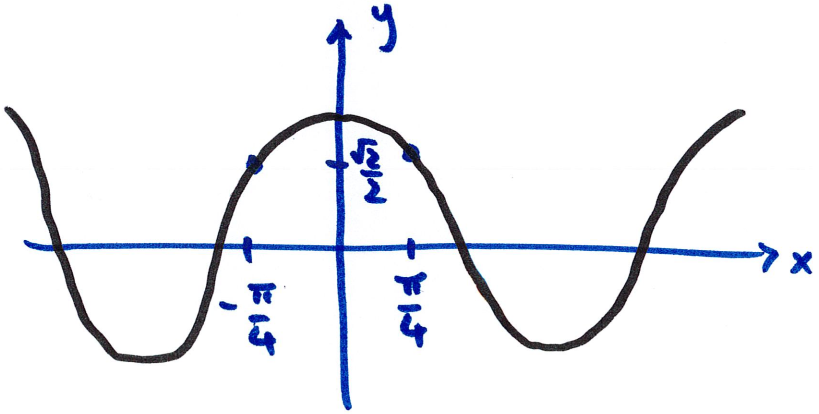
graph:



same thing on either side of y-axis

another example: $f(x) = \cos(x)$

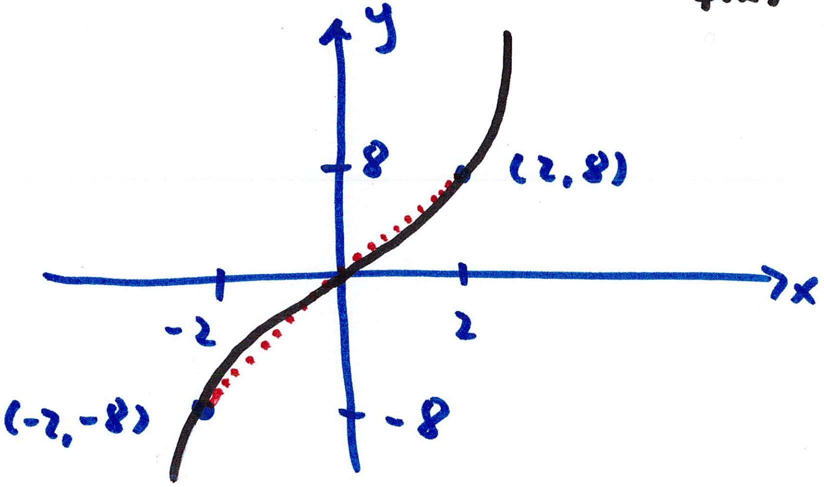
$f(-x) = \cos(-x) = \cos(x) = f(x)$



origin symmetry : f(x) is odd → $f(-x) = -f(x)$

example : $f(x) = x^3$

$$f(-x) = (-x)^3 = -\underbrace{x^3}_{f(x)} = -f(x)$$

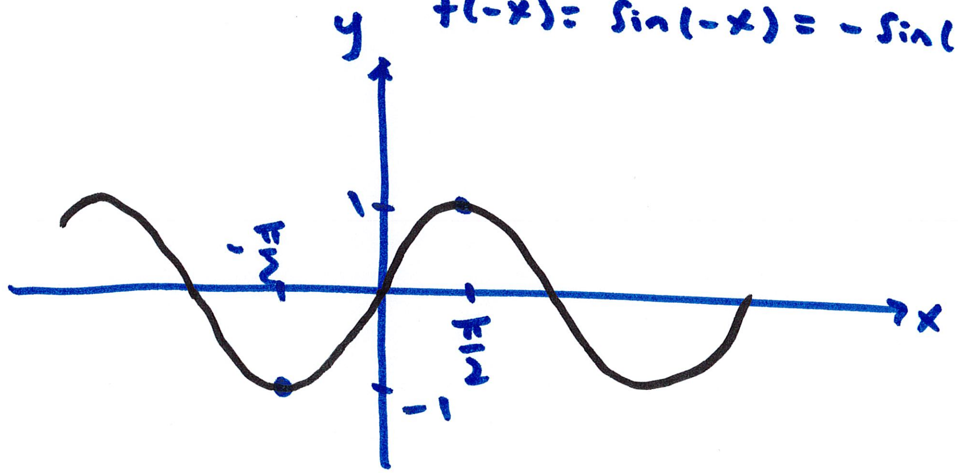


whatever is in one quadrant is duplicated in the diagonally opposite quadrant

(point same distance away from origin on line y = x)

another example : $f(x) = \sin(x)$

$$f(-x) = \sin(-x) = -\sin(x) = -f(x)$$



The first derivative alone is enough to get a reasonable sketch

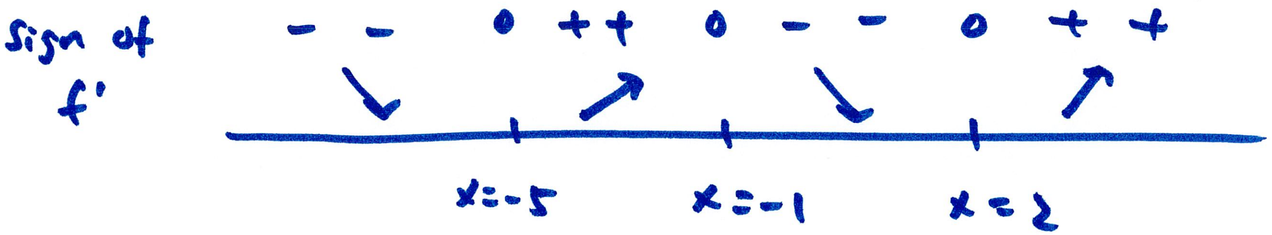
example sketch $f(x)$ if we know $f'(x) = (x-2)(x+1)(x+5)$ only

use the given $f'(x)$ to complete the inc/dec and rel. max/min step

find critical numbers: $f'(x) = 0 \rightarrow (x-2)(x+1)(x+5) = 0$

$x = -5, -1, +2$

f' DNE → never



dec: $(-\infty, -5), (-1, 2)$

~~dec~~ inc: $(-5, -1), (2, \infty)$

$$\left.\begin{array}{ll} \text{rel min:} & \text{at } x = -5 \\ & \text{at } x = 2 \\ \text{rel max:} & \text{at } x = -1 \end{array}\right\} \; y = ?$$

we don't know since we don't have $f(x)$

we can make up y's for these

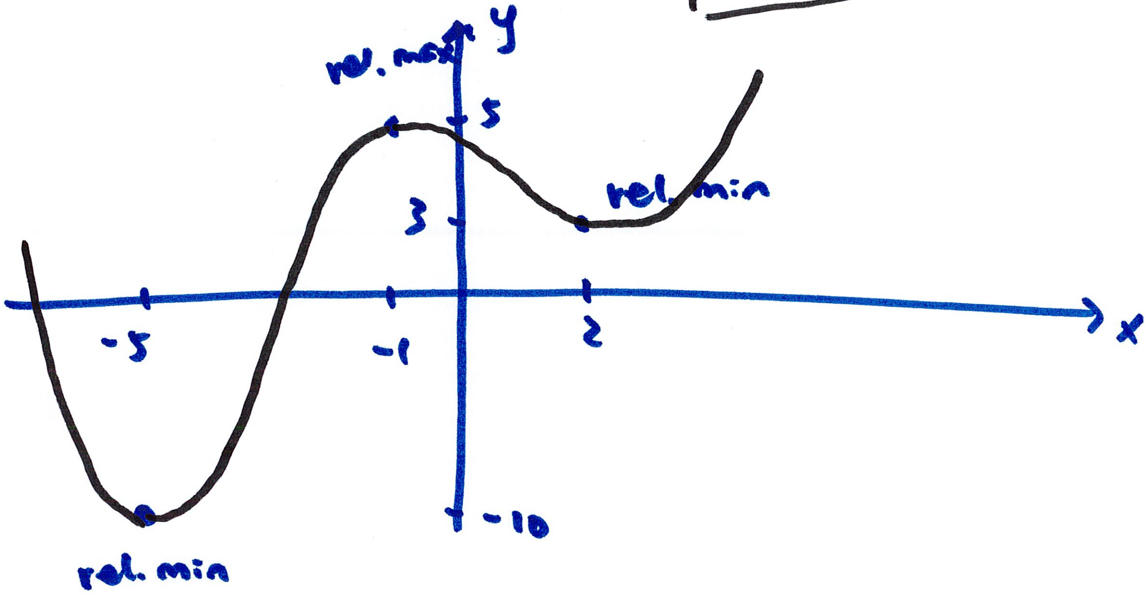
→ we will get the right shape but not the right points

answer is not unique

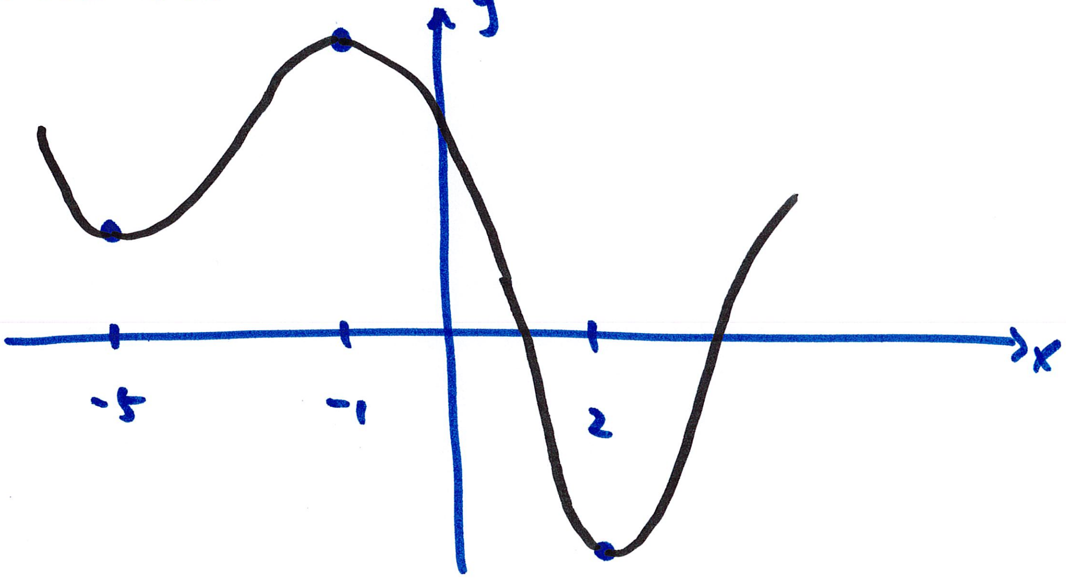
one possible graph: rel. min at $x = -5$, $\boxed{y = -10}$ ← made up numbers

rel. min at $x = 2$, $y = 3$

rel. max at $x = -1$, $y = 5$



equally right alternative



example $f'(x) = \sin(\underline{\underline{3}}x)$ on $\underbrace{\left[-\frac{4\pi}{3}, \frac{4\pi}{3}\right]} \rightarrow -\frac{4\pi}{3} \le x \le \frac{4\pi}{3}$

$-4\pi \le 3x \le 4\pi$

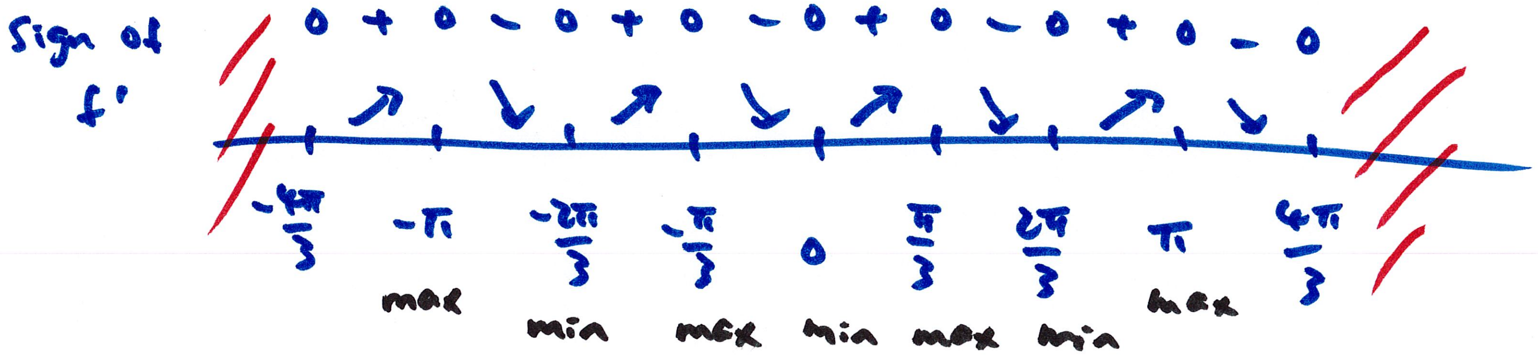
convenient to know this since we are working with $\sin(\underline{\underline{3}}x)$

find critical numbers: $f'(x) = 0$

$\sin(3x) = 0$

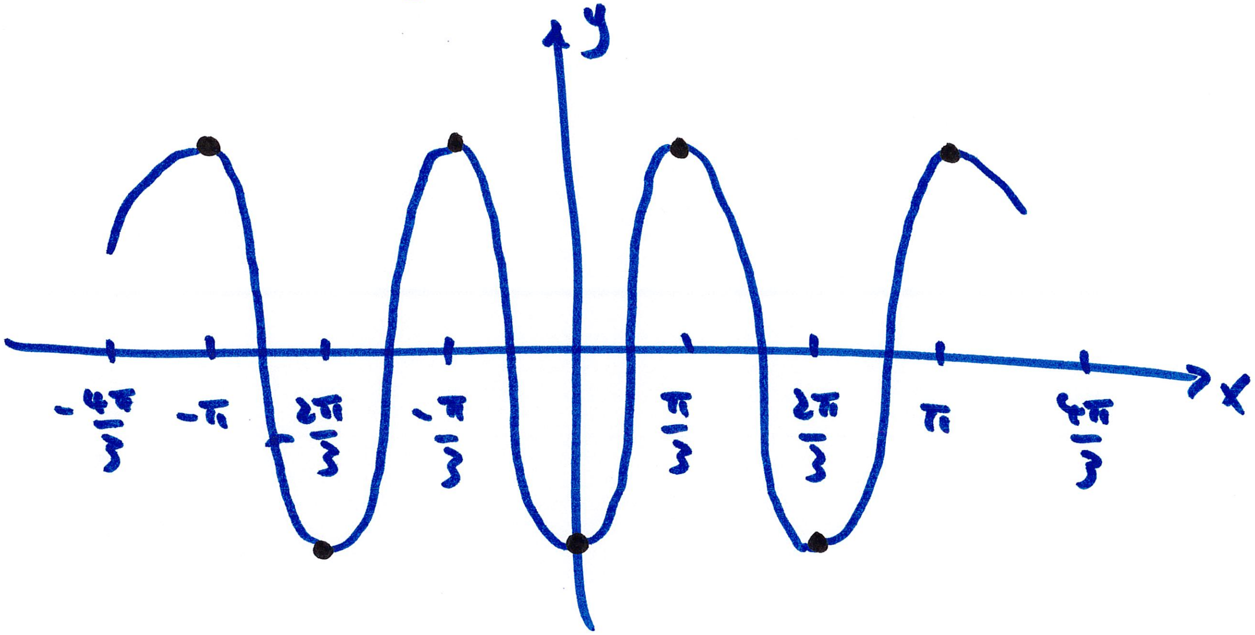
$3x = -4\pi, -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, 4\pi$

$x = -\frac{4\pi}{3}, -\pi, -\frac{2\pi}{3}, -\frac{\pi}{3}, 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$



rel. max at $x = -\pi, -\frac{\pi}{3}, \frac{\pi}{3}, \pi$

rel. min at $x = -\frac{2\pi}{3}, 0, \frac{2\pi}{3}$



example Sketch $f(x) = x^x$ using the first derivative

use logarithmic differentiation to find $f'(x)$

$y = x^x$

$\ln y = \ln x^x$

$\ln y = x \ln x$ now implicitly differentiate to find y'

$\frac{1}{y} y' = x \cdot \frac{1}{x} + \ln x \cdot 1$

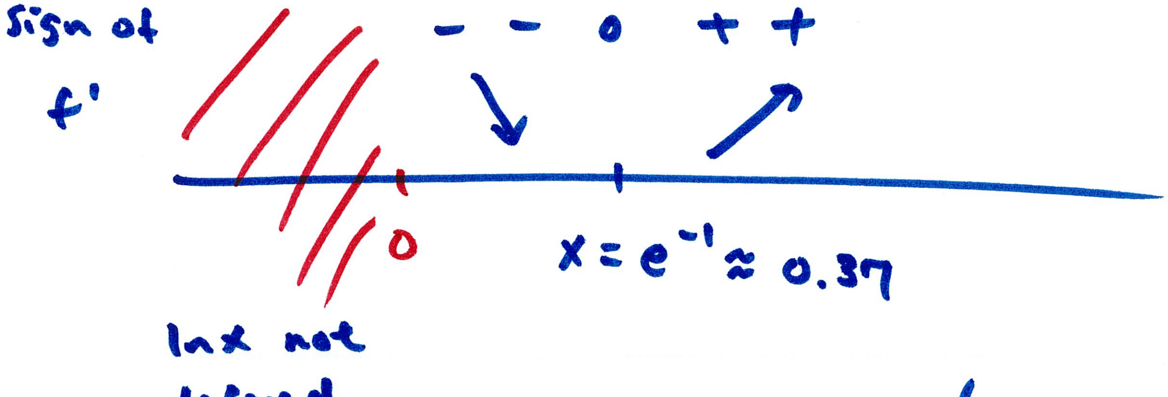
$= 1 + \ln x$

$y' = y(1+\ln x) = x^x (1+\ln x)$

so, $\boxed{f'(x) = x^x (1+\ln x)}$

$f' = 0 \rightarrow \underbrace{x^x = 0}_{\text{never}}$ or $\underbrace{1+\ln x = 0}_{\ln x = -1}$

$x = e^{-1}$ this is the only critical number



rel. min at $x=e^{-1}$

we know $f(x)=x^x$

so we know the real y

$y=f(e^{-1})=(e^{-1})^{e^{-1}}\approx 0.7$