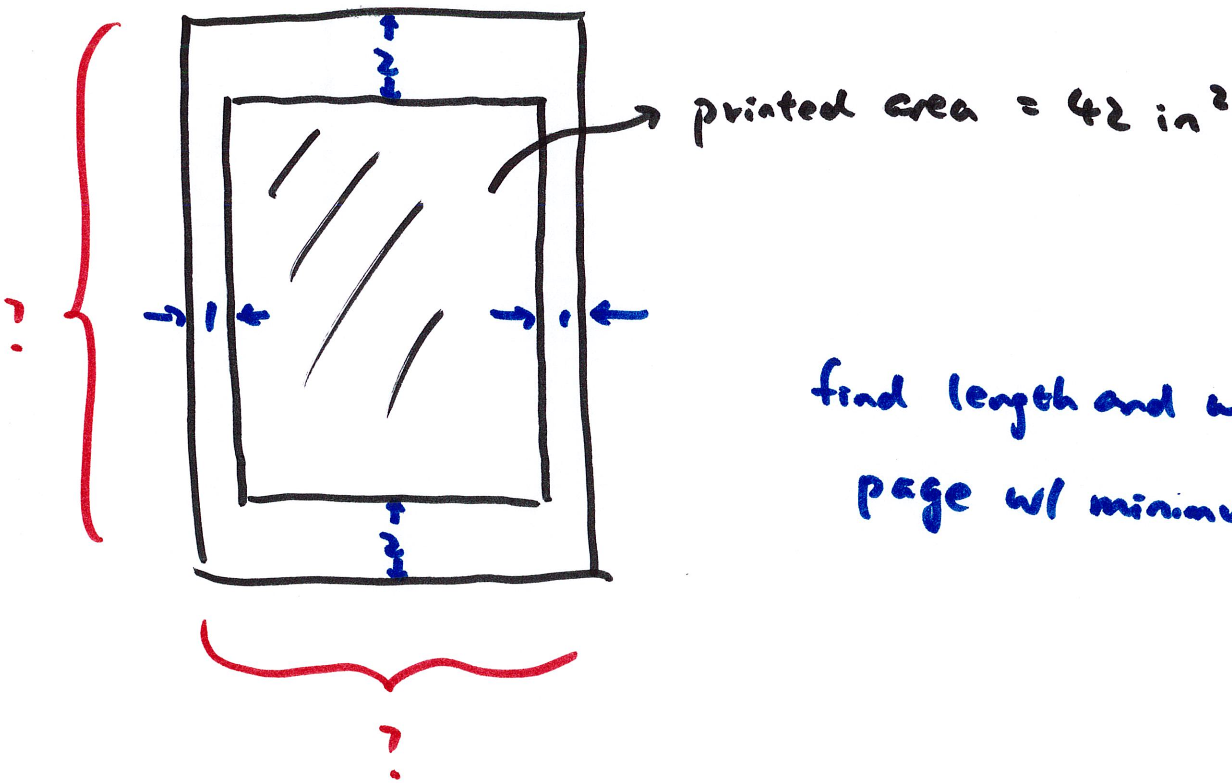
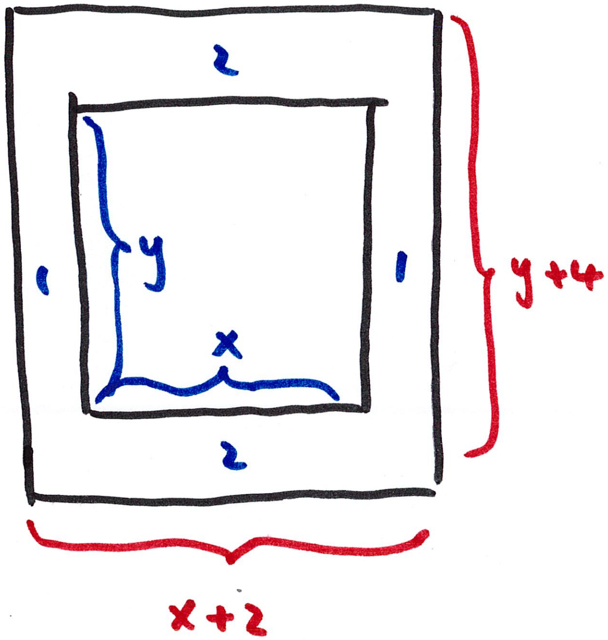
4.5 Optimization Problems (part 2)

example A page from a book is to have a printed area of 42 in^2. The margins at top and bottom are 2 in each and the margins at left and right are 1 in each. Find the dimensions of the page to minimize total area.



find length and width of page w/ minimum total area.



x: width of printed area

y: length " " "

printed area has area 42 in^2

so, $\boxed{xy = 42}$ constraint

width of page : $x+2$

length " " : $y+4$

area of page : $\boxed{A = (x+2)(y+4)}$ objective

goal : minimize A

objective has two variables : use constraint to eliminate one

$xy = 42 \rightarrow y = \frac{42}{x}$ sub into A

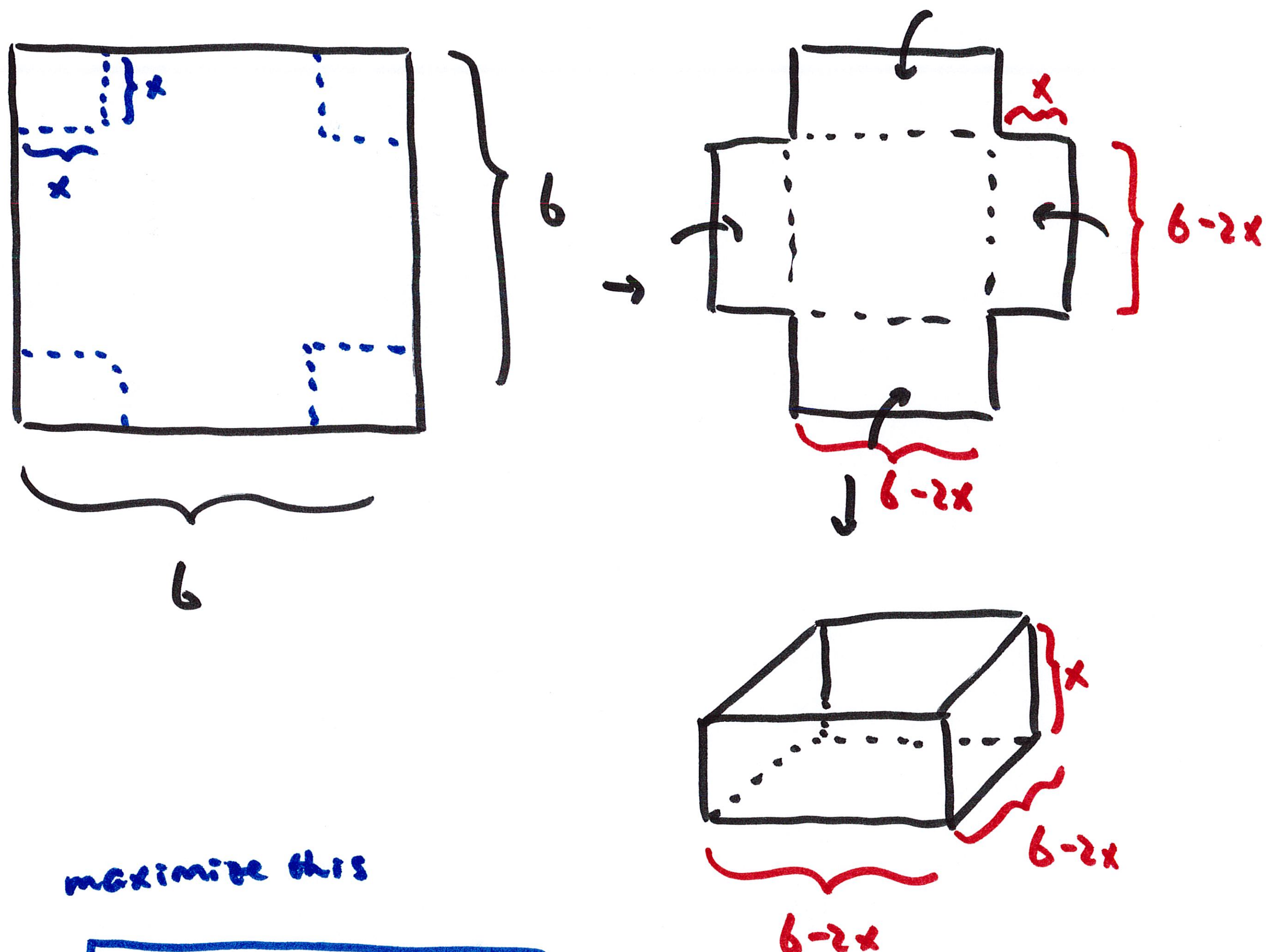
$$\boxed{A = (x+2)\left(\frac{42}{x} + 4\right)}$$

next steps: find critical numbers
check for max/min

example A piece of ~~cardbo~~ cardboard is 6 in by 6 in.

Small squares are removed from the corners and the flaps are folded up to make a box.

Find the volume of the largest possible such box.



maximize this

volume: $\boxed{V(x) = (6-2x)^2(x)}$

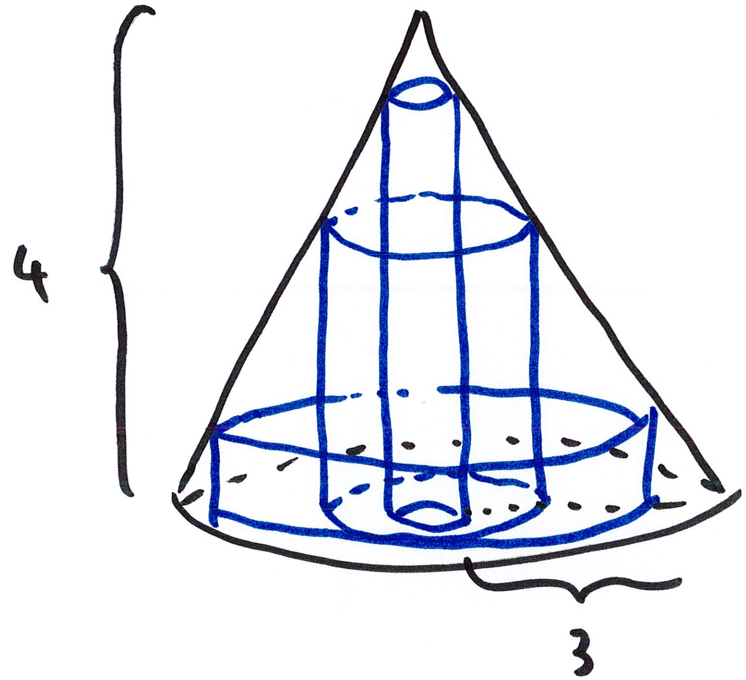
$0 \le x \le 3$

next steps: find V'

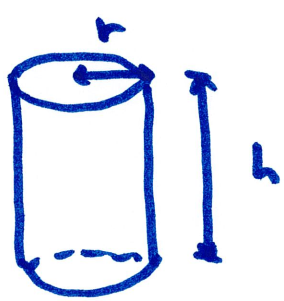
find critical numbers ($V' = 0$)

then compare V at critical numbers and end points.

__example__ Find the volume of the largest possible right circular cylinder that can be inscribed in a cone of height 4 and radius 3.

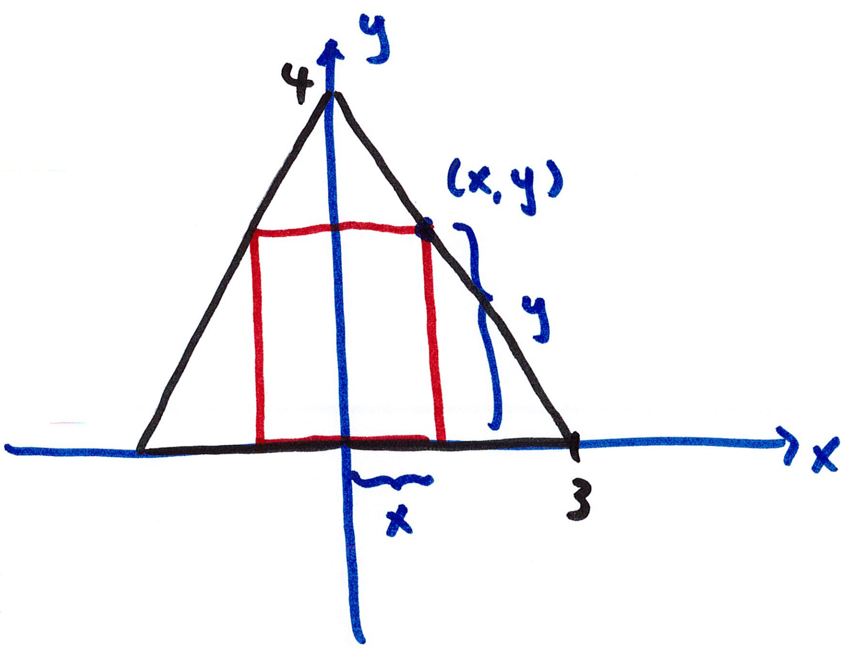


largest possible cylinder → maximize the volume



volume = $V = \pi r^2 h$

cut the cone / cylinder in half and look at cross section on xy axes



x: half width of rectangle
(also radius of cylinder)

y: height of rectangle
(also height of cylinder)

corner of rectangle (x, y) must be on a $\underbrace{\text{line through } (3,0) \text{ and } (0,4)}_{\text{equation}: \ y = -\frac{4}{3}x + 4}$

volume of cylinder: $V = \pi (x)^2 (y)$

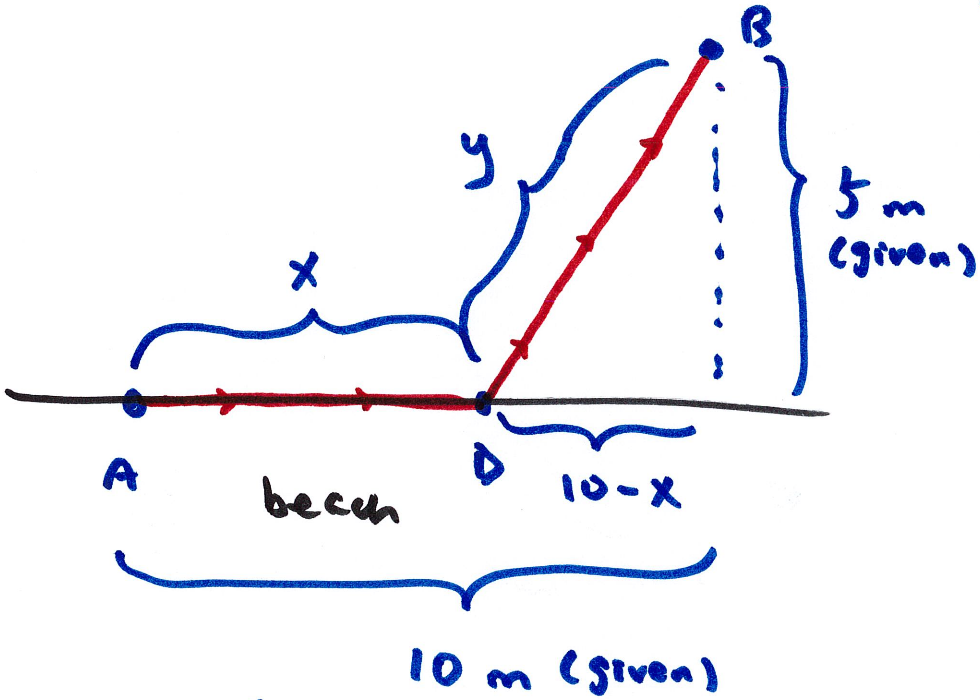
max this → $\boxed{V = \pi x^2 \left(-\frac{4}{3}x + 4\right)}$ $\boxed{0 \leq x \leq 3}$

next steps: find critical numbers
compare V at critical numbers and end points

example A dog can run at 5 m/s and swim at 1 m/s.

A dog runs along the beach and jumps into the water and swim in a straight line to reach a ball floating in the water.

Where should the dog jump into the water to minimize the time to reach the ball?



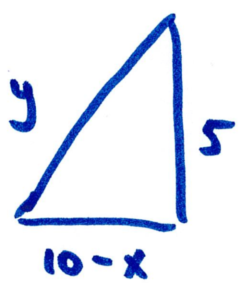
A: start

B: the ball

D: the point where the dog jumps in

x: distance on land

y: distance in water



$$y = \sqrt{5^2 + (10-x)^2}$$

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

time on land : $T_L = \dfrac{x}{5}$

time spent swimming $T_W = \dfrac{\sqrt{5^2 + (10-x)^2}}{1} = \sqrt{25 + (10-x)^2} = (x^2 - 20x + 125)^{1/2}$

total time = time on land + time in water

$$\boxed{T = \frac{x}{5} + (x^2 - 20x + 125)^{1/2}}$$

minimize this

$$\boxed{0 \le x \le 10}$$

swim entire way

run until even w/ ball then jumps in

next: find critical numbers

compare T at critical numbers and at end points