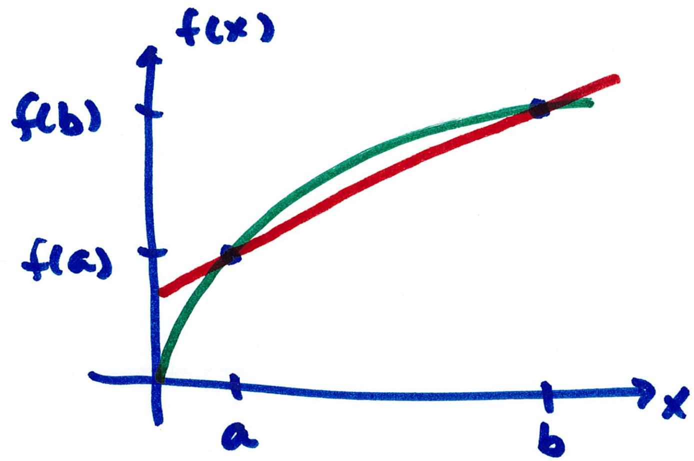
2.1 + 2.2 The Limit

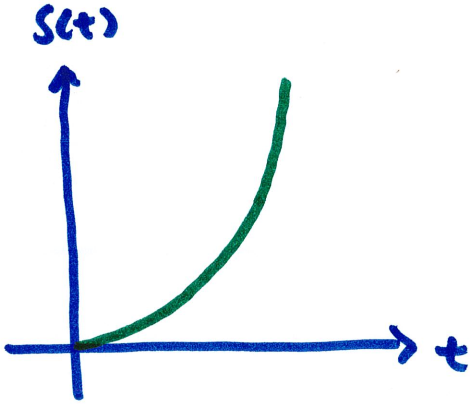
$y = f(x)$

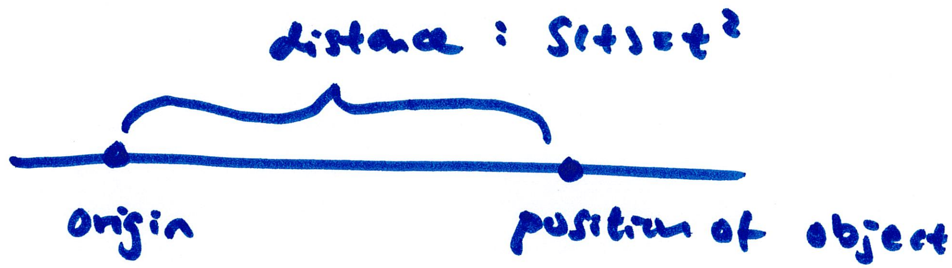


the line that goes the points $(a, f(a))$, $(b, f(b))$ is called the <u>secant line</u> whose slope is $\dfrac{f(b) - f(a)}{b - a}$

if $f(x)$ measures distance and x is time, then the secant line slope is the <u>average velocity</u> over the interval a to b.

if $S(t) = t^2$ gives us the distance of an object from the origin on a straight line





on average, how fast did the object move between $t=2$ and $t=3$?

average velocity : $\dfrac{S(3)-S(2)}{3-2} = \dfrac{9-4}{1} = 5$

on average, the object moved at velocity of 5 during $2 \le t \le 3$

but that does NOT mean it traveled at velocity of 5 at all times

the <u>instantaneous velocity</u> is always changing.

what if we want the instantaneous velocity at a specific time?
for example, at $t=2$

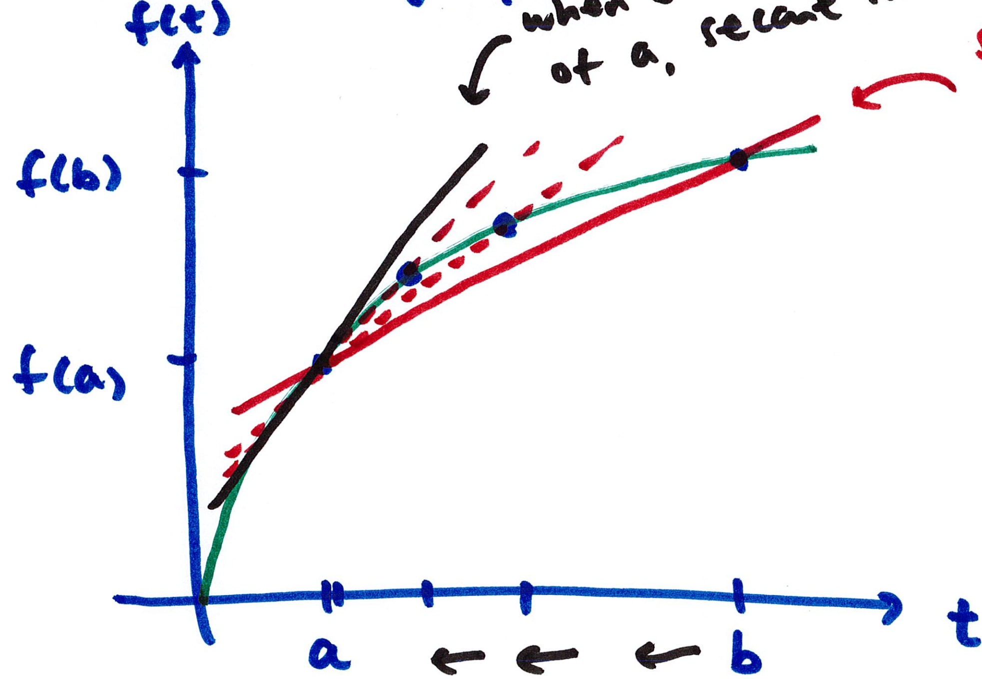
to find the instantaneous velocity at $t=2$, we calculate the average ~~vo~~ velocity using smaller and smaller time interval

for example, $2 \le t \le 2.5$ $\quad [2, 2.5] \to [2, 2.1] \to \underbrace{[2, 2.000001]}_{\text{interval is so short, average velocity is practically the instantaneous velocity}}$

let's see those on a graph

when b is practically on top of a, secant line → tangent line

slope = average velocity on [a, b]



by moving b closer and closer to a, the secant line slope approach the instantaneous velocity at $t=a$

the idea of bringing something closer to another is basically how limit in calculus works.

we write: $\lim_{x \to a} f(x)$ means we want to know what $f(x)$ gets close to as x gets close to a

$x \to a$: make x close to a without equalling a

for example, $f(x) = x^2$

$\lim_{x \to 10} x^2$ means find what x^2 gets close to as x gets close to 10

get close to 10

x	9.9	9.999	10	10.001	10.1
$f(x) = x^2$	98.01	99.98001	X	100.02	102.01

the bottom row suggests $\lim_{x \to 10} x^2 = 100$

the fact that x can be 10 is irrelevant for limit another example: $\lim\limits_{x \to -4} \dfrac{x^2-16}{x+4}$

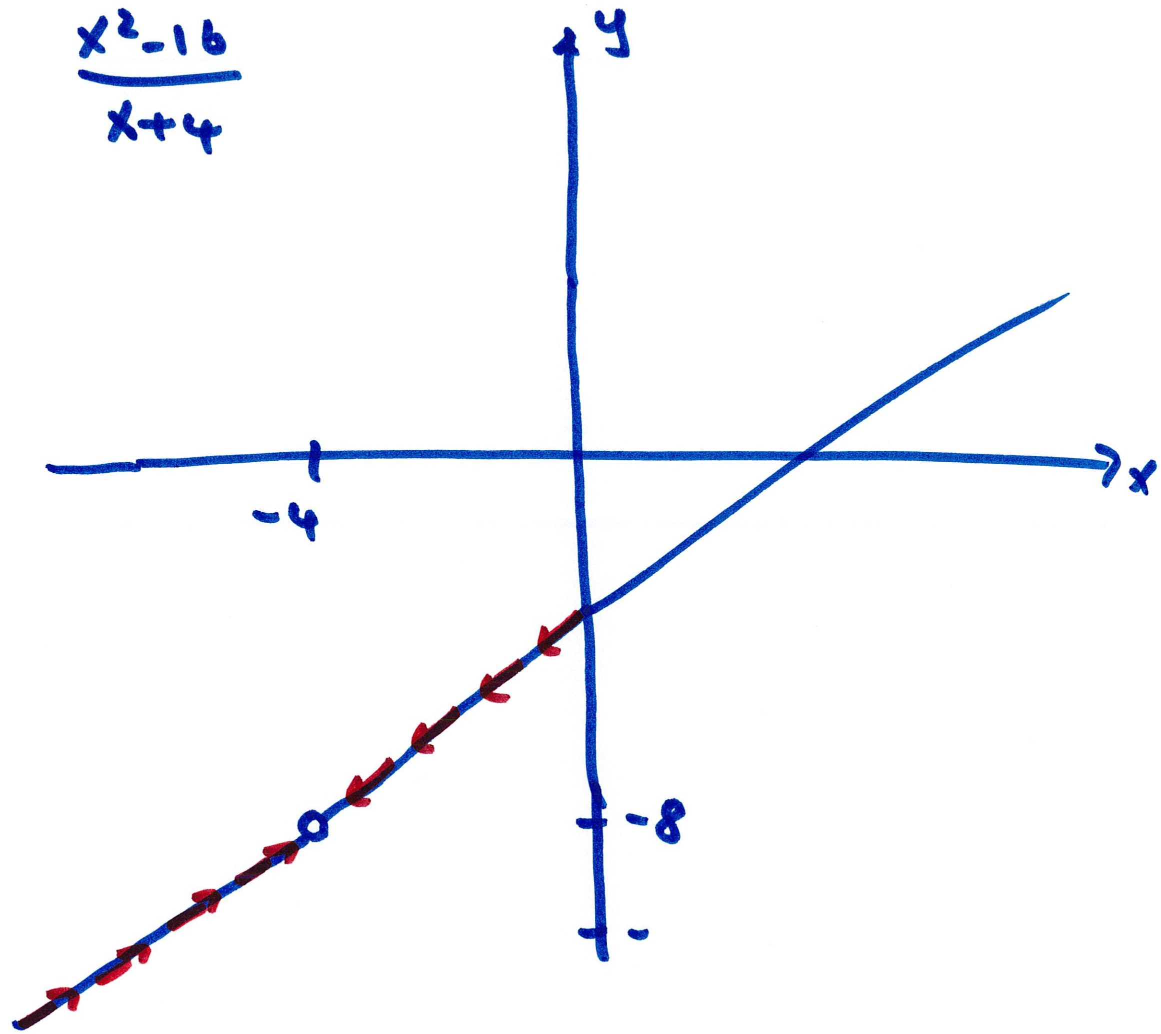
clearly, we cannot go to exactly $x=-4$
but we can get close to $x=-4$
and that is what the limit is about

x	-4.01	-4.001	-4.0001	-4	-3.9999	-3.999	-3.99
$\dfrac{x^2-16}{x+4}$	-8.01	-8.001	-8.0001		-7.9999	-7.999	-7.99

so the bottom row suggests that $\dfrac{x^2-16}{x+4}$ "wants" to be -8 as x gets close to -4

so, $\lim\limits_{x \to -4} \dfrac{x^2-16}{x+4} = -8$

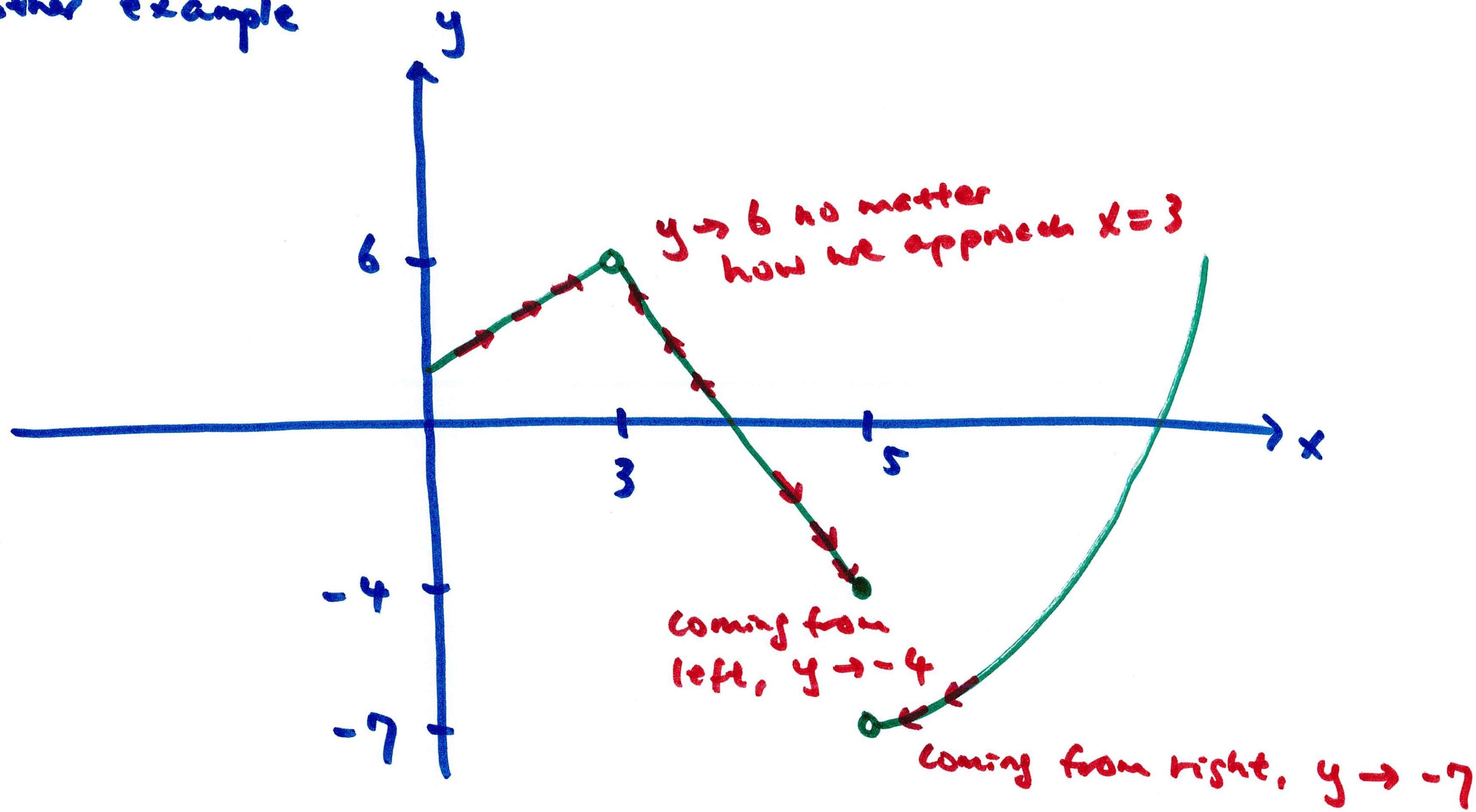
graph of $\frac{x^2-16}{x+4}$



looks like a line with a point removed at $x=-4$

we can make $\frac{x^2-16}{x+4}$ as close to -8 as we want by making x close to -4

another example



f(3) does not exist (DNE) because of the open circle

but $\lim_{x \to 3} f(x) = 6$

f(5) = -4 → we can get to x = 5

but $\lim_{x \to 5} f(x)$ does not exist because the y values approaches different numbers depending on which side we come from even though $\lim_{x \to 5} f(x)$ does not exist, because the value depends on how we approach $x = 5$

if we only look at one side at a time, we can still have <u>one-sided limits</u>

$$\underbrace{\lim_{x \to 5^-}}_{\text{approach } x=5 \text{ from the LEFT}} f(x) = -4$$

$$\underbrace{\lim_{x \to 5^+}}_{\text{approach } x=5 \text{ from the RIGHT}} f(x) = -7$$

if these are different, the the limit at $x = 5$ does not exist