2.4 Infinite Limits

$f(x) = \frac{1}{(x-2)^2}$

$\lim_{x \to 2} \frac{1}{(x-2)^2} = ?$

x	1.99	1.999	1.9999	2	2.0001	2.001	2.01
$\frac{1}{(x-2)^2}$	10,000	1,000,000	100,000,000	X	100,000,000	1,000,000	10,000

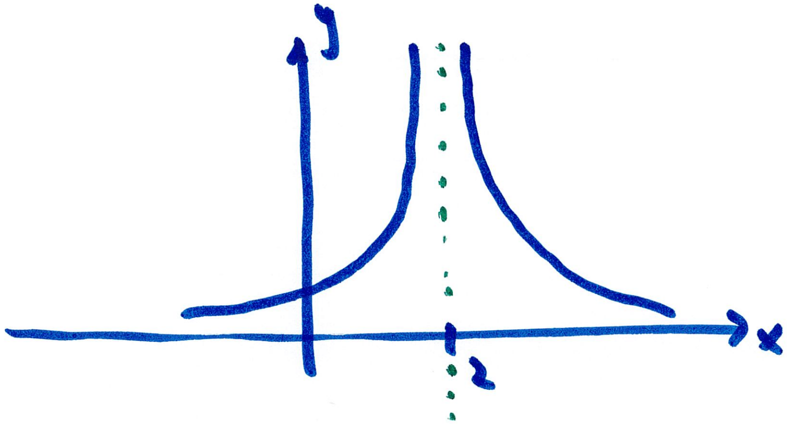
it appears as $x \to 2$ from either side, $\frac{1}{(x-2)^2}$ keeps getting bigger, the closer we get to $x=2$, the bigger $\frac{1}{(x-2)^2}$ is

since $\frac{1}{(x-2)^2}$ as $x \to 2$ grows without bound

we say $\lim_{x \to 2} \frac{1}{(x-2)^2} = \infty$ (infinite limit → when limit is $\pm\infty$)

we also ~~see~~ see: $\lim_{x \to 2^+} \frac{1}{(x-2)^2} = \infty$, $\lim_{x \to 2^-} \frac{1}{(x-2)^2} = \infty$

graph:



vertical asymptote at $x = 2$

$\downarrow$

graph goes to ∞ or $-\infty$ on either side

vertical asymptote: numerator $\neq 0$ while denominator $= 0$

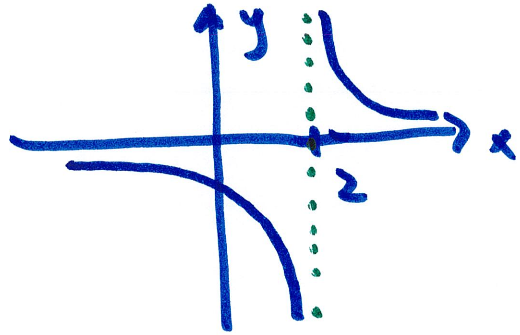
Example: $\displaystyle\lim_{x \to 2} \frac{1}{x-2}$

x	1.99	1.999	1.9999	2	2.0001	2.001	2.01
$\frac{1}{x-2}$	-100	-1000	-10,000	X	10,000	1000	100

we see $\displaystyle\lim_{x \to 2^-} \frac{1}{x-2} = -\infty$ (keeps getting bigger but negative)

$\displaystyle\lim_{x \to 2^+} \frac{1}{x-2} = \infty$ (keeps increasing remains positive)

$\displaystyle\lim_{x \to 2} \frac{1}{x-2}$ DNE because the one-sided limits do not match graph:



in practice, we can just check using numbers close to the target on either side

$$\frac{\text{non zero}}{\text{small number}} \to \infty \text{ or } -\infty$$

depending on the sign of the ratio

example

$$\lim_{x\to 5^+} \frac{x^2-7x+12}{(x-5)^2}$$

$x \to 5^+$ → pick a number slightly bigger than 5

for example, $x = 5.0001$

plug into $\dfrac{x^2-7x+12}{(x-5)^2}$

$$= \frac{(5.0001)^2 - 7(5.0001)+12}{(5.0001-5)^2} \approx \frac{\text{roughly } 2}{\text{very small positive number}}$$

ratio is positive

→ big positive #

so, $\displaystyle\lim_{x\to 5^+} \frac{x^2-7x+12}{(x-5)^2} = \infty$

follow-up: $\displaystyle\lim_{x\to 5^-} \frac{x^2-7x+12}{(x-5)^2}$

pick slightly smaller than 5 : $x = 4.999$

plug into $\dfrac{x^2-7x+12}{(x-5)^2} \approx \dfrac{\text{roughly } 2}{\text{positive small \#}}$ ratio is positive

so, $\lim\limits_{x \to 5^-} \dfrac{x^2-7x+12}{(x-5)^2} = \infty$

Since the one-sided limits match, $\lim\limits_{x \to 5} \dfrac{x^2-7x+12}{(x-5)^2} = \infty$

<u>example</u> $\lim\limits_{x \to 4^+} \dfrac{x-7}{\sqrt{x-4}}$

$x \to 4^+$: x is ~~something~~ something bigger than 4

pick $x = 4.01$

then $\dfrac{x-7}{\sqrt{x-4}} = \dfrac{\text{roughly } -3}{\sqrt{0.01}} \to \dfrac{\text{roughly } -3}{\text{small positive \#}}$ ratio is negative

so, $\lim\limits_{x \to 4^+} \dfrac{x-7}{\sqrt{x-4}} = -\infty$

what about $\lim_{x \to 4^-} \frac{x-7}{\sqrt{x-4}}$?

$x \to 4^-$ means pick something smaller than 4

pick $x = 3.99$

$$\frac{x-7}{\sqrt{x-4}} = \frac{\text{roughly } -3}{\sqrt{-0.01}}$$

↳ undefined

no ratio to talk about

the entire ratio is undefined

all we can say is $\lim_{x \to 4^-} \frac{x-7}{\sqrt{x-4}}$ DNE (does not exist) example

$$\lim_{x \to 4} \frac{x^2 - 9x + 14}{x^2 - 6x + 8}$$

looks like we can factor things and simplify

do that first

$$\frac{x^2 - 9x + 14}{x^2 - 6x + 8} = \frac{(x-7)\cancel{(x-2)}}{(x-4)\cancel{(x-2)}}$$

$$= \lim_{x \to 4} \frac{x-7}{x-4}$$

find $\lim_{x \to 4^+} \frac{x-7}{x-4}$ choose $x = 4.001$

$$\frac{x-7}{x-4} \to \frac{\text{roughly } -3}{\text{small positive \#}} \to -\infty$$

find $\lim_{x \to 4^-} \frac{x-7}{x-4} = \infty$

$\lim_{x \to 4} \frac{x-7}{x-4}$ DNE