MA 161 Exam 1

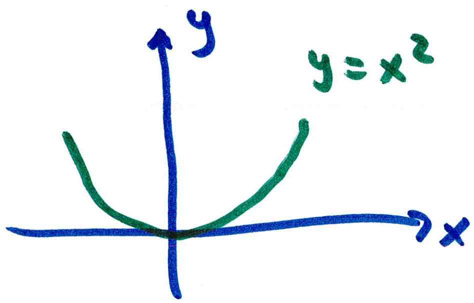
Tuesday, 2/1/2022, 6:30 pm

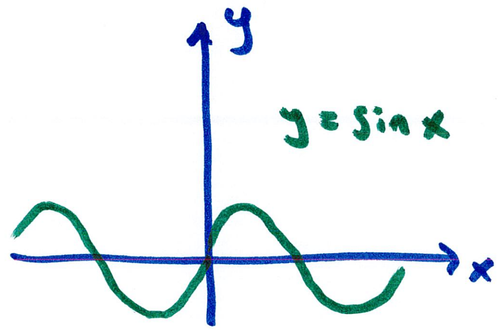
Recitation Instructor	Location
Isaac Chiu	CL50 224
Rowan Desjardins	CL50 224
Christopher Dewey	CL50 224
Hanan Gadi	WALC 1055
Mohit Pathak	WALC 1055
Vittal Srinivasan	PHYS 112
Ezekiel (Seun) Yinka Kehinde	PHYS 112

2.6 Continuity

if a function is continuous, then there are no breaks in its graph

→ if tracing you never need to lift hand to complete the trace





mathematically, if a function is continuous at $x = a$ then

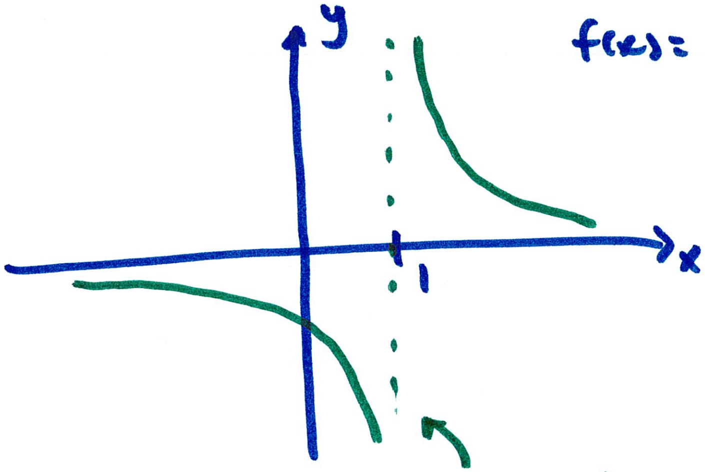
1) $f(a)$ is defined

2) $\lim_{x \to a} f(x)$ exist

3) $f(a) = \lim_{x \to a} f(x)$

ALL 3 need to happen for $f(x)$ to be continuous at $x = a$

$f(x) = \dfrac{-1}{1-x}$



infinite discontinuity

$f(1)$ is not defined because of the asymptote

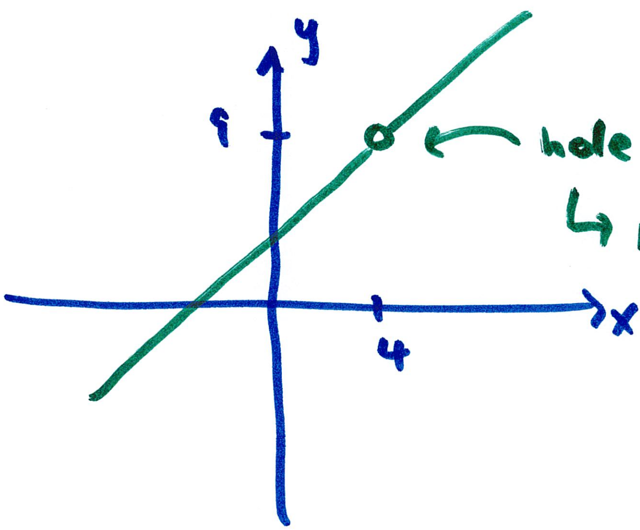
and $\lim\limits_{x \to 1} f(x)$ DNE

so $f(x) = \dfrac{-1}{1-x}$ is <u>discontinuous at $x=1$</u>

because the first two requirements are met

but is continuous at all $x \neq 1$

$$f(x) = \frac{x^2+x-20}{x-4} = \frac{(x+5)(x-4)}{x-4} = x+5, \quad x \neq 4$$



hole at $x=4$

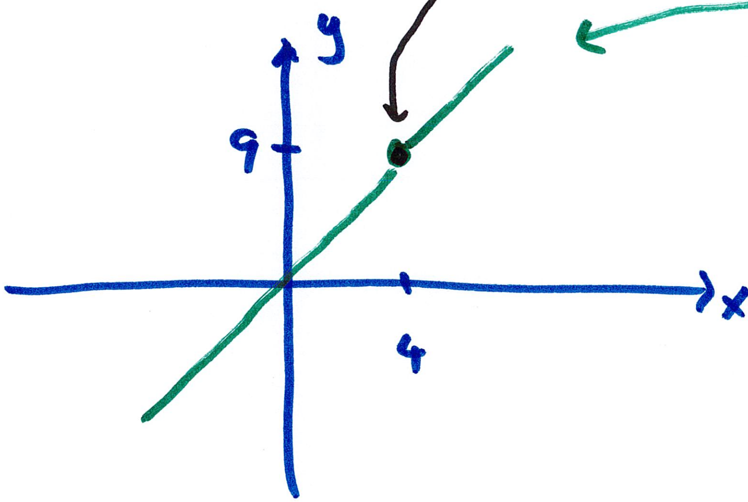
↳ removable discontinuity

$f(4)$ is not defined

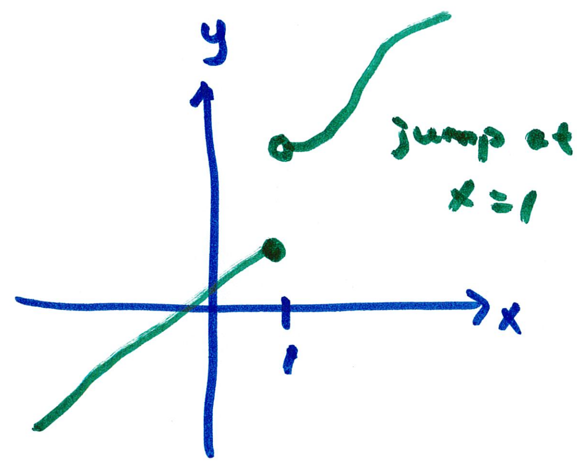
so $f(x)$ is discontinuous at $x=4$

the hole is called a <u>removable discontinuity</u> because we can define f(x) there to "patch" the hole

$$f(x) = \begin{cases} \dfrac{x^2+x-20}{x-4} & \text{if } x \neq 4 \\ \boxed{9} & \text{if } x = 4 \end{cases}$$

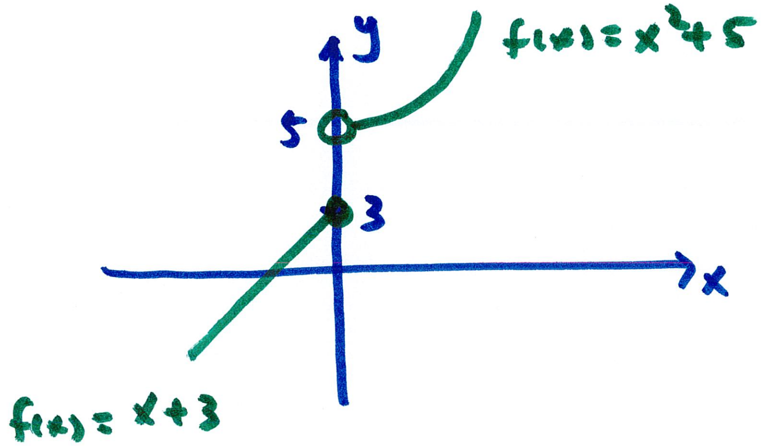


another type of discontinuity: jump discontinuity



Just like with limits we can talk about <u>one-sided continuities</u>

$$f(x) = \begin{cases} x+3 & x \le 0 \\ x^2+5 & x > 0 \end{cases}$$



discontinuous at $x=0$

but is <u>continuous from the LEFT</u> at $x=0$

because $\lim_{x\to 0^-} f(x) = f(0)$

but <u>not</u> <u>continuous from the RIGHT</u> at $x=0$

because $\underbrace{\lim_{x\to 0^+} f(x)}_{5} \neq \underbrace{f(0)}_{3}$

many common functions are continuous <u>on their domains</u>

polynomials
rationals
root
trig
exponential
log

$\Bigg\}$ wherever defined. $f(a) = \lim\limits_{x \to a} f(x)$

$f(a) = \lim\limits_{x \to a} f(x)$ due to continuity is the reason why we can plug in $x = a$ into $f(x)$ to find $\lim\limits_{x \to a} f(x)$

$$\lim_{x \to 1} (\underbrace{x^2 + 2x - 1}) = (1)^2 + (2)(1) - 1 = 2$$

polynomial
continuous everywhere
so $\lim\limits_{x \to 1} f(x) = f(1)$

$$\lim_{x\to 3} \underbrace{\ln(5x)}_{\text{log function}} = \ln(5\cdot 3) = \ln 15$$

defined $x > 0$

so continuous $x > 0$

so, $\lim_{x\to 3} f(x) = f(3)$

$$\lim_{x\to 0} \left(\underbrace{5e^x}_{\text{exponential}} + \underbrace{\cos(\pi x)}_{\text{trig}} + \underbrace{\sqrt{5x^2+10}}_{\text{root function}} \right) = 5e^0 + \cos(\pi\cdot 0) + \sqrt{5\cdot 0 + 10}$$

$$= 6 + \sqrt{10}$$

continuous	continuous	continuous
on $(-\infty, \infty)$	$(-\infty, \infty)$	on $5x^2+10 \geq 0$

$x \to 0$ 0 is in interval of continuity for all parts

so, $\lim_{x\to 0} f(x) = f(0)$

example

$$f(x) = \begin{cases} cx^2 + 2x & \text{if} \quad x < 2 \\ x^3 - cx & \text{if} \quad x \geq 2 \end{cases}$$

find c so $f(x)$ is continuous everywhere

both pieces are polynomial so continuous on their own domains

so $f(x)$ is continuous everywhere except possibly at $x = 2$

for $f(x)$ to be continuous at $x = 2$

need:

1) $f(2)$ defined — yes, $f(2) = 8 - 2c$

2) $\lim_{x \to 2} f(x)$ exist $\rightarrow$ $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

3) $\lim_{x \to 2} f(x) = f(2)$

$$\underbrace{\lim_{x \to 2^+}}_{x > 2} f(x) = \lim_{x \to 2^+} x^3 - cx = 8 - 2c$$

$$\underbrace{\lim_{x \to 2^-}}_{x < 2} f(x) = \lim_{x \to 2^-} cx^2 + 2x = 4c + 4$$

for there to be equal, $8-2c = 4c+4 \rightarrow 6c = +4 \quad c = 2/3$

if $c = 2/3$, then 2) $\lim_{x \to 2} f(x)$ exists. is true

now check 3) $\underbrace{\lim_{x \to 2} f(x) = f(2)}$

use $c = 2/3$

in $8-2c$ or $4c+4$

$$\frac{20}{3} = 8 - 2c \leftarrow 2/3$$

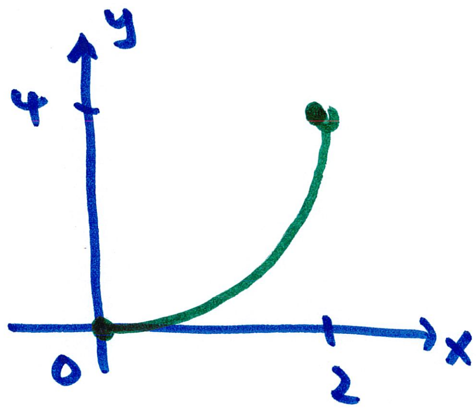
$$= \frac{20}{3}$$

so, if $c = 2/3$ then $f(x)$ is continuous everywhere

Indetermediate Value Theorem (IVT)

If $f(x)$ is continuous on $[a, b]$, then $f(x)$ takes on every value between $f(a)$ and $f(b)$

for example, $f(x) = x^2$ on $[0, 2]$



IVT says the function takes on every value between 0 and 4

this is because a continuous function has no breaks so $f(x) = x^2$ cannot skip any value between 0 and 4.