The equation

$$x^2 + y^2 + z^2 + 2x - 4y + 6z = C$$

is a sphere of radius 2. What is C?

A. $C = -12$

B. $C = 4$

C. $C = 14$

D. $C = 2$

E. $C = -10$

sphere: $(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$

center: (h, k, l)

radius: r

$x^2 + 2x + y^2 - 4y + z^2 + 6z = C$

$x^2 + 2x + 1 + y^2 - 4y + 4 + z^2 + 6z + 9 = C + 1 + 4 + 9$

$(x+1)^2 + (y-2)^2 + (z+3)^2 = \underbrace{C + 14}_{r^2 = 2^2 = 4}$

so, $C + 14 = r^2 = 2^2 = 4$

$C = -10$

The angle between $\vec{a} = \langle 2, -1, 2\rangle$ and $\vec{b} = \langle 1, -1, 0\rangle$ is

A. $\pi/4$

B. $\pi/6$

C. $\pi/3$

D. $2\pi/3$

E. $3\pi/4$

dot product: $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$\vec{a}\cdot\vec{b} = \langle 2,-1,2\rangle \cdot \langle 1,-1,0\rangle$

$= (2)(1) + (-1)(-1) + (2)(0) = 3$

$|\vec{a}| = \sqrt{(2)^2 + (-1)^2 + (2)^2} = \sqrt{9} = 3$

$|\vec{b}| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$3 = (3)(\sqrt{2})\cos\theta$

$\cos\theta = \frac{1}{\sqrt{2}}$ $\quad \theta = \pi/4$

If $\vec{a} =< 2, -1, 4 >$ and $\vec{b} =< 2, 2, 1 >$, find the $\vec{j}$-component of $\vec{a} \times \vec{b}$.

A. 6

B. -6

C. 9

D. -9

E. -2

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 4 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} -1 & 4 \\ 2 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & 4 \\ 2 & 1 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & -1 \\ 2 & 2 \end{vmatrix}$$

$$= \vec{i}(-1 \cdot 1 - 4 \cdot 2) - \vec{j}(2 \cdot 1 - 2 \cdot 4) + \vec{k}(2 \cdot 2 - 2 \cdot -1)$$

$$= -9\vec{i} + 6\vec{j} + 6\vec{k}$$

Find the area enclosed by the curves $x = \underbrace{2y - y^2}$ and $x = y^2$

parabola opening LEFT

$0 = 2y - y^2$

$= y(2-y) = 0$

$y = 0,\ y = 2$

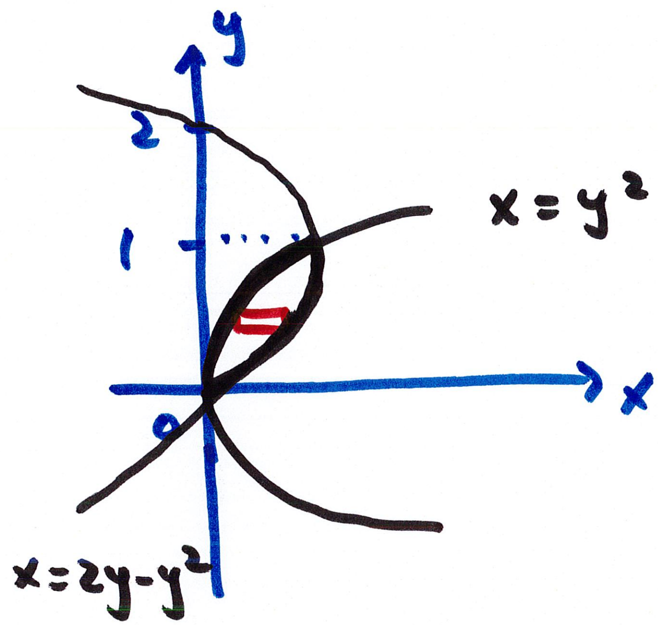
A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. 1

E. $\frac{5}{3}$



Since equations are x in terms of y, probably easier to integrate in terms of y

interfection: $2y - y^2 = y^2$

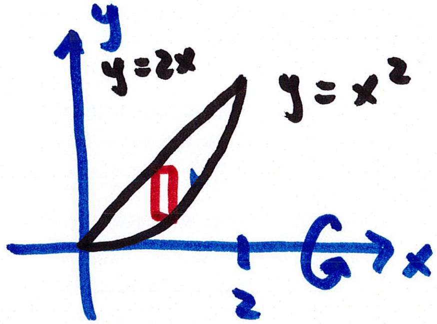
$2y^2 = 2y \qquad y^2 - \cancel{2}y = 0$

$y(y - \cancel{2}) = 0 \qquad y = 0,\ y = 1$

$$\int_0^1 (\underbrace{2y - y^2}_{\text{right}} - \underbrace{y^2}_{\text{left}})\,dy = \int_0^1 2y - 2y^2\,dy = y^2 - \frac{2}{3}y^3 \Big|_0^1$$

$$= 1 - \frac{2}{3} = \frac{1}{3}$$

The area between the graphs of $y = x^2$ and $y = 2x$ is revolved around the x-axis. If the disk/washer method is used, the integral representing the volume of the resulting solid is



disk/washer : rectangle perpendicular to axis of revolution

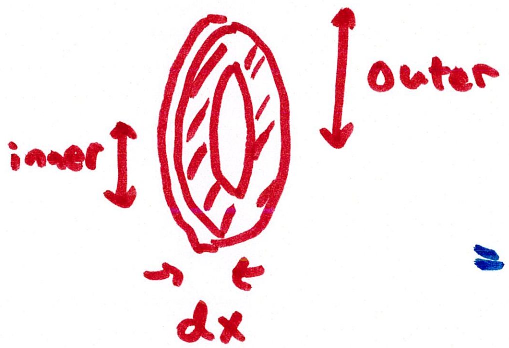
A. $\int_0^2 \pi(2x - x^2)\, dx$

B. $\int_0^2 \pi(2x - x^2)^2\, dx$

C. $\int_0^4 2\pi x(2x - x^2)\, dx$

D. $\int_0^4 2\pi x(4x^2 - x^4)\, dx$

E. $\int_0^2 \pi(4x^2 - x^4)\, dx$



$[\pi(\text{outer})^2 - \pi(\text{inner})^2] \cdot \text{thickness}$

$= [\pi(2x)^2 - \pi(x^2)^2]dx$

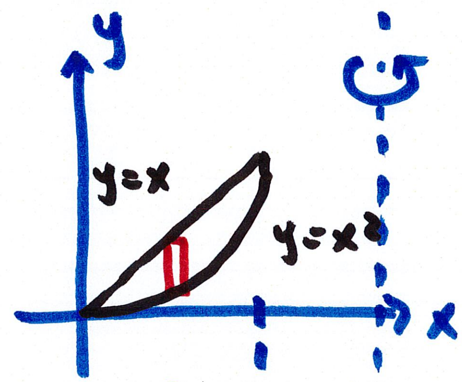
total volume

$$\int_0^2 [\pi(4x^2) - \pi(x^4)]dx$$

$$= \int_0^2 \pi(4x^2 - x^4)dx$$

Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = x$ about the line $x = 3$.

no method specified but answers have 2π so use shell



Shell: rectangle parallel to axis of revolution

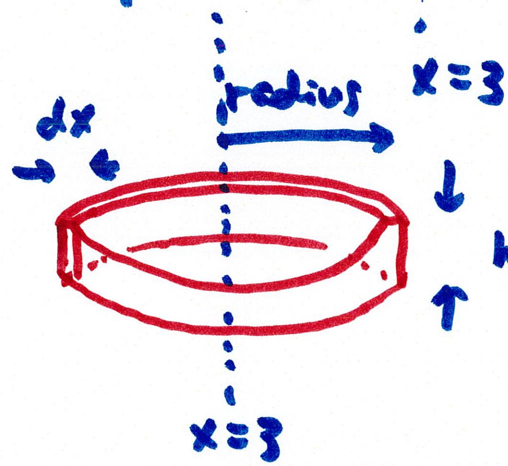
A. $\int_0^1 2\pi(1-x)^2(x-x^2)\,dx$

B. $\int_0^1 2\pi(3-x)(x-x^2)\,dx$

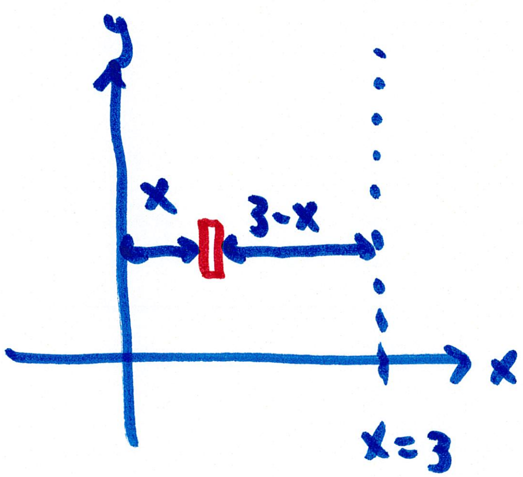
C. $\int_0^1 2\pi(1-x)(x-x^2)^2\,dx$

D. $\int_0^1 2\pi(1-x)^2(x-x^2)^2\,dx$

E. $\int_0^1 2\pi(3-x)^2(x-x^2)\,dx$



height = top - bottom



radius = 3 - x

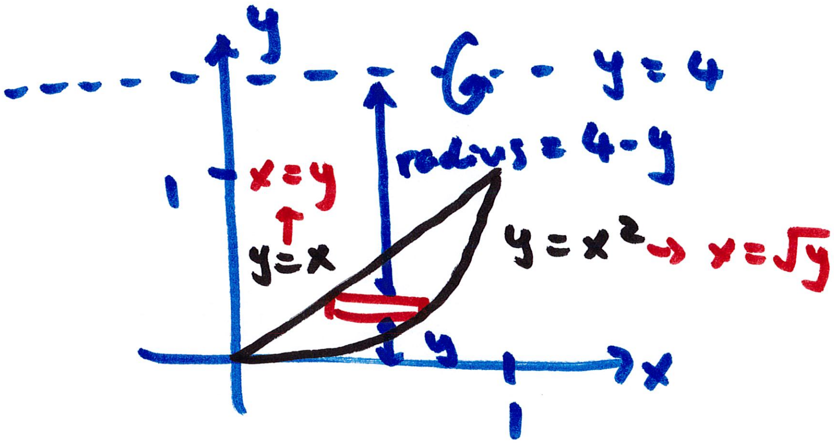
volume of one shell

2π (radius) (height) (thickness)

$= 2\pi\,(3-x)(x-x^2)\,dx$

total: $\int_0^1 2\pi(3-x)(x-x^2)\,dx$

Same region, shell, revolved around $y=4$



shell so horizontal rectangle

radius $= 4-y$

thickness $= dy$

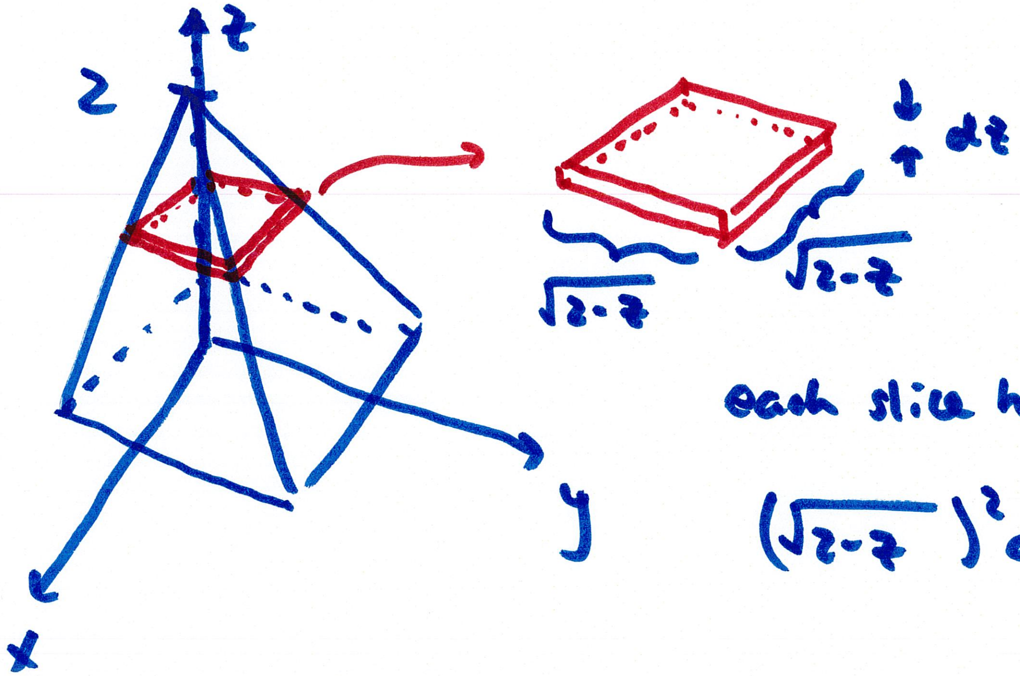
"height" = right - left

$= \sqrt{y} - y$

one shell $\quad 2\pi(4-y)(\sqrt{y}-y)\,dy$

total $\quad \int_0^1 2\pi(4-y)(\sqrt{y}-y)\,dy$

An unusual pyramid of height 2 is sitting on the xy-plane. If the cross-section at a level $z \geq 0$ is a square of side $\sqrt{2-z}$, what is the volume of this strange pyramid?



A. 2

B. $\frac{8}{3}$

C. $\frac{2^{5/2}}{3}$

D. 4

E. 3π

each slice has volume

$$(\sqrt{2-z})^2 dz = (2-z)dz$$

accumulate from $z=0$ to $z=2$

$$\int_0^2 (2-z)\,dz = 2z - \frac{1}{2}z^2 \Big|_0^2$$

$$= 4 - \frac{1}{2}(4) = 2$$

A force of 50 N is required to hold a spring that has been stretched from its natural length of 1 m to a length of 1.5 m. How much work is done (in joules) by stretching the spring from a length of 2 m to a length of 3 m?

1 beyond natural

2 beyond natural

A. 75

B. 125

C. 150

D. 250

E. 300

Spring : $F = kx$ change from natural length

$50 = k \cdot (0.5)$

NOT 1.5 !
this is the change from natural
so is $1.5 - 1.0 = 0.5$

$k = 100$

work : $\int_a^b kx\,dx$

a: start measured with respect to natural

b: end measured with respect to natural

$$W = \int_1^2 100x\,dx = 50x^2 \Big|_1^2 = 50(4-1) = 150$$