Determine whether the series converges or diverges. If it is convergent, find its sum.

$$-4 + 3 - \frac{9}{4} + \frac{27}{16} - \frac{81}{64} + - \cdots$$

$-\frac{3}{4}$ $-\frac{3}{4}$ $-\frac{3}{4}$

A. converges to $-\frac{3}{4}$

B. converges to 0

[✓ C.] converges to $-\frac{16}{7}$

D. converges to 4

E. diverges

geometric series with ratio $-\frac{3}{4} = r$

↳ converges if $|r| < 1$

so, this series converges

Sum of geometric series

$$S = \frac{a}{1-r}$$

a: first term

r: ratio

here, $S = \frac{-4}{1-(-\frac{3}{4})} = \frac{-4}{\frac{7}{4}} = -\frac{16}{7}$

Determine if the series converges or diverges. If it converges, find its sum.

$$\sum_{n=2}^{\infty} \frac{6}{(n)(n+3)}$$

A. Converges to 0.

B. Converges to 13/2.

C. Converges to 13/3.

[✓ D.] Converges to 13/6.

E. Diverges.

clearly not a geometric series ($\sum ar^k$)

can we break it up ?

$$\frac{6}{(n)(n+3)} = \frac{A}{n} + \frac{B}{n+3}$$

$$6 = A(n+3) + B(n)$$

$$6 = (A+B)n + 3A$$

$$A+B = 0$$

$$3A = 6 \rightarrow A = 2 \rightarrow B = -2$$

Series: $\sum_{n=2}^{\infty} \left(\frac{2}{n} - \frac{2}{n+3} \right)$ telescoping series

$$= \underbrace{\left(\frac{2}{2}-\frac{2}{5}\right)}_{n=2} + \underbrace{\left(\frac{2}{3}-\frac{2}{6}\right)}_{n=3} + \underbrace{\left(\frac{2}{4}-\frac{2}{7}\right)}_{n=4} + \underbrace{\left(\frac{2}{5}-\frac{2}{8}\right)}_{n=5} + \underbrace{\left(\frac{2}{6}-\frac{2}{9}\right)}_{n=6}$$

$$+ \underbrace{\left(\frac{2}{7}-\frac{2}{10}\right)}_{n=7} + \underbrace{\left(\frac{2}{8}-\frac{2}{11}\right)}_{n=8} + \underbrace{\left(\frac{2}{9}-\frac{2}{12}\right)}_{n=9}$$

seventh-partial sum (sum from $n=2$ to $n=8$)

$$S_7 = \left(\frac{2}{2}-\cancel{\frac{2}{5}}\right) + \left(\frac{2}{3}-\cancel{\frac{2}{6}}\right) + \left(\frac{2}{4}-\cancel{\frac{2}{7}}\right) + \left(\cancel{\frac{2}{5}}-\cancel{\frac{2}{8}}\right) + \left(\cancel{\frac{2}{6}}-\frac{2}{9}\right) + \left(\cancel{\frac{2}{7}}-\frac{2}{10}\right) + \left(\cancel{\frac{2}{8}}-\frac{2}{11}\right)$$

$$= \frac{2}{2}+\frac{2}{3}+\frac{2}{4}-\underset{\substack{\uparrow \\ n+2}}{\frac{2}{9}}-\underset{\substack{\uparrow \\ n+3}}{\frac{2}{10}}-\underset{\substack{\uparrow \\ n+4}}{\frac{2}{11}}$$

$$S_8 = \frac{2}{2}+\frac{2}{3}+\frac{2}{4}-\underset{\substack{\uparrow \\ n+2}}{\frac{2}{10}}-\underset{\substack{\uparrow \\ n+3}}{\frac{2}{11}}-\underset{\substack{\uparrow \\ n+4}}{\frac{2}{12}}$$

$$S_n = \frac{2}{2}+\frac{2}{3}+\frac{2}{4}-\frac{2}{n+2}-\frac{2}{n+3}-\frac{2}{n+4}$$

$$\lim_{n\to\infty} S_n = S = \frac{2}{2}+\frac{2}{3}+\frac{2}{4} = \frac{13}{6}$$

Which comparison could be used to determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^4+1}}{2n^3+n^2}$ converges or diverges?

A. converges by comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

B. converges by limit comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

C. converges by comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

D. diverges by comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

[✓ E.] diverges by limit comparison with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

look at the tail behavior

$n \to \infty$ (large n)

large n: $\sqrt{n^4+1} \approx \sqrt{n^4} \approx n^2$

$2n^3+n^2 \approx 2n^3$

so $\dfrac{\sqrt{n^4+1}}{2n^3+n^2} \approx \dfrac{n^2}{2n^3} \approx \dfrac{1}{2n}$

compare to $\dfrac{1}{2n}$ or $\dfrac{1}{n}$

try limit comparison: $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = c$

← unknown

← known

$0 < c < \infty$ then both converge or both diverge

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n^4+1}}{2n^3+n^2}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\sqrt{n^4+1}}{2n^2+n} = \frac{1}{2}$$

so both converge or diverge

we know $\sum \frac{1}{n}$ diverges

so our series also diverges

Given the following table of values for e^{-n}, which partial sum of the series $s = \sum_{n=1}^{\infty}(-1)^n e^{-n}$ is the first to be within 0.01 of s?

which partial sum is within 0.01 of ~~true~~ sum
true

n	e^{-n}
1	0.3679
2	0.1353
3	0.0498
4	0.0183
5	0.0067

A. s_1

B. s_2

C. s_3

[✓ D. s_4]

E. s_5

alternating series: partial sum is no more than the absolute value of the first term we throw out away from the true sum

$$\sum_{n=1}^{\infty}(-1)^n e^{-n} = \underbrace{\underbrace{\underbrace{-0.3679+0.1353}_{S_2 \text{ is } \le |-0.0498| \text{ from } S}-0.0498}_{S_3 \text{ is } \le |0.0183| \text{ away}}+0.0183}_{S_4 \text{ is } \le |\underbrace{-0.0067}_{<0.01}| \text{ away}}-0.0067+\cdots$$

given $\sum a_k$, $\sum |a_k|$ converges

Find all of the series below that are convergent but **not** absolutely convergent.

↳ conditionally convergent

(i.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ ✓

(ii.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ ✓

(iii.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ X

(iv.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$ X

i) is, because $\sum \frac{(-1)^n}{n}$ converges but $\sum \left|\frac{(-1)^n}{n}\right| = \sum \frac{1}{n}$ does not

ii) is by the same reasoning ($\sum \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{1/2}}$ p-series w/ $p < 1$)

iii) is absolutely convergent $\sum \frac{1}{n^2}$ p-series w/ $p > 1$

iv) same as iii)

[✓ A.] (i) and (ii) only.

B. (iii) and (iv) only.

C. All of these series.

D. None of these series.

E. Another combination not listed above.

In the Taylor series for e^{2x} centered at $a = 3$, what is the term that contains $\underline{\underline{(x-3)^3}}$?

A. $\frac{e^6}{3}(x-3)^3$

B. $\frac{2e^6}{3}(x-3)^3$

[✓ C.] $\frac{4e^6}{3}(x-3)^3$

D. $\frac{5e^6}{3}(x-3)^3$

E. $\frac{7e^6}{3}(x-3)^3$

Taylor series: $\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

$$= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \boxed{\frac{f'''(a)}{3!}(x-a)^3} + \dots$$

$f(x) = e^{2x}$

$f'(x) = 2e^{2x}$

$f''(x) = 4e^{2x}$

$f'''(x) = 8e^{2x}$

$f'''(3) = 8e^6$

$$\frac{f'''(3)}{3!}(x-3)^3 = \frac{8e^6}{6}(x-3)^3$$

$$= \frac{4e^6}{3}(x-3)^3$$

5) Which of the following follows from the remainder formula in a Taylor series?

(a) The equation $e^2 = 3 + \frac{4}{2!} + \frac{8}{3!} + \frac{16e^z}{4!}$ has a solution for $1 < z < 2$

[✓ (b)] The equation $e^2 = 3 + \frac{4}{2!} + \frac{8}{3!} + \frac{16e^z}{4!}$ has a solution for $0 < z < 2$

(c)The equation $e^2 = 3 + \frac{4}{2!} + \frac{8}{3!} + \frac{16e^z}{4!}$ has a solution for $z > 2$

(d) The equation $e^2 = 3 + \frac{4}{2!} + \frac{8}{3!} + \frac{e^z}{4!}$ has a solution for $0 < z < 2$

(e)The equation $e^2 = 3 + \frac{4}{2!} + \frac{8}{3!} + \frac{e^z}{4!}$ has a solution for $z > 2$

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^3$$

n^{th} partial sum has remainder $\frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$ $\quad a < c < x$

choices above: 3rd partial sum (up to 3rd deriv.) with remainder function is $f(x) = e^x$, $a = 0$

$f'(x) = e^x$

$\vdots$

$f^{(4)}(x) = e^x$

remainder: $\dfrac{f^{(4)}(z)}{4!}(x-a)^4$

at $x = 2$ (how we estimate e^2)

remainder: $\dfrac{e^z}{4!}(2)^4 = \dfrac{16 e^z}{4!}$

the remainder gives us the error in partial sum