8.2 Integration by Parts

HW includes problems from 8.1 (basic integration)

integration by substitution: the chain rule in reverse

integration by parts: the product rule in reverse

u, v both functions of x

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

the differential:

$$d(uv) = u\,dv + v\,du$$

$$u\,dv = d(uv) - v\,du$$

integrate

$$\int u\,dv = \int d(uv) - \int v\,du$$

$$= uv - \int v\,du$$

we get:

$$\boxed{\int u\,dv = uv - \int v\,du}$$

this is integration by parts

given an integral, identify u, v then use the formula above

<u>example</u>

$$\int x \cdot \ln x \, dx$$

not something we can integrate directly

not something we can integrate by subs

so consider integration by parts

we need to identify u, v

$$\int \underline{x} \cdot \underline{\ln x} \, dx$$

→ pick u ~~to be~~ such that its derivative is simpler than itself

pick dv such that it is easy to integrate

focus on u

rule of thumb for picking u: in this order

<u>L</u>ogarithmic

<u>I</u>nverse trig

<u>A</u>lgebraic (x^n)

<u>T</u>rig

<u>E</u>xponential

LIATE

here, we have x and $\ln x$

Algebraic (A) Logarithmic (L)

LIATE ~~A before~~ L before A so choose the "L" to be u

$\int x \cdot \ln x \, dx$ let $u = \ln x$ $dv = x\,dx$ ← any leftover after choosing u

$du = \frac{1}{x} dx$ differentiate u

$v = \int dv$

$= \int x \, dx$

$v = \frac{1}{2} x^2$ don't need $+C$ here

now use $uv - \int v \, du$

$= (\ln x)(\frac{1}{2}x^2) - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx = \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x \, dx$

$= \frac{1}{2}x^2 \ln x - \frac{1}{2}(\frac{1}{2}x^2) + C = \boxed{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C}$

what if I chose ~~ln~~ $u = x$ instead?

$$\int x \cdot \ln x \, dx \qquad u = x \qquad dv = \ln x \, dx$$

$$du = dx \qquad v = \int dv = \underbrace{\int \ln x \, dx}_{}$$

actually needs integration by parts itself!

we can still do this but it's harder

dv being hard to integrate is a common sign of wrong u

example

$$\int \underbrace{x}_{A} \underbrace{\cos x}_{T}\, dx$$

LIATE

A before T so u is the "A" part

$u = x \qquad dv = \cos x \, dx$

$du = dx \qquad v = \sin x$

$$uv - \int v \, du$$

$$= x \sin x - \int \sin x \, dx$$

$$= \boxed{x \sin x + \cos x + C}$$

definite integral : $\displaystyle\int_0^{\pi/2} x \cos x \, dx$ pick u, dv as usual

then $\displaystyle uv\Big|_0^{\pi/2} - \int_0^{\pi/2} v \, du$

$$= x \sin x \Big|_0^{\pi/2} - \int_0^{\pi/2} \sin x \, dx = \ldots = \boxed{\frac{\pi}{2} - 1}$$

example $\int \underset{A}{x^2} \underset{T}{\cos x}\, dx$

LIATE

so $u = x^2$ $dv = \cos x\, dx$

$du = 2x\, dx$ $v = \sin x$

$uv - \int v\, du$

$= x^2 \sin x - \int \sin x \cdot 2x\, dx$

$= x^2 \sin x - 2 \underbrace{\int \overset{A}{x} \cdot \overset{T}{\sin x}\, dx}_{\text{needs int. by parts}}$

$U = x$ $dV = \sin x\, dx$

$dU = dx$ $V = -\cos x$

$= x^2 \sin x - 2\left(UV - \int V\, dU\right) = x^2 \sin x - 2\left(-x\cos x - \int -\cos x\, dx\right)$

$= x^2 \sin x + 2x\cos x - 2\int \cos x\, dx$

$= \boxed{x^2 \sin x + 2x \cos x - 2\sin x + C}$

example

$$\int \underbrace{e^x}_{E} \underbrace{\cos x}_{T}\, dx$$

LIATE

$u = \cos x \qquad dv = e^x\,dx$

$du = -\sin x\,dx \qquad v = e^x$

$$\int e^x \cos x\,dx = uv - \int v\,du$$

$$= e^x \cos x - \int e^x \cdot -\sin x\,dx$$

$$= e^x \cos x + \underbrace{\int e^x \sin x\,dx}_{\text{by parts}}$$

$U = \sin x \qquad dV = e^x\,dx$

$dU = \cos x\,dx \qquad V = e^x$

$$\underbrace{\int e^x \cos x\,dx}_{\text{this is what I want}} = e^x \cos x + e^x \sin x - \underbrace{\int e^x \cos x\,dx}_{\text{this is what I want}}$$

STOP! integration by parts again results in endless loop

$$2\int e^x \cos x\,dx = e^x \cos x + e^x \sin x$$

$$\boxed{\int e^x \cos x\,dx = \frac{1}{2}\left(e^x \cos x + e^x \sin x\right) + C}$$

example $\int \ln x \, dx$ this we can do by parts

$$= \int \underset{L}{\underline{\ln x}} \cdot \underset{A\ (x^0)}{\underline{1}} \cdot dx$$

LIATE

$u = \ln x$ $\quad dv = dx$

$du = \frac{1}{x} dx$ $\quad v = x$

$$uv - \int v \, du$$

$$= x \ln x - \int x \cdot \frac{1}{x} dx$$

$$= x \ln x - \int dx = \boxed{x \ln x - x + C}$$

Alex & Phoebe Present:

EXAM 1 REVIEW: Sept 18th 4:30 pm - LILY 2102