13.1 Vectors

scalars : numbers such as $5, -12, \pi$, etc
sense of magnitude but no direction

vectors: magnitude and direction
e.g. wind from north at 15 mph

vectors tell us the position of one point relative to another

two points: $P(x_1, y_1)$ $Q(x_2, y_2)$

vector from P to Q $\vec{PQ} = \langle x_2 - x_1, y_2 - y_1 \rangle$

for example, $P(1, 2)$ $Q(3, -4)$

six units DOWN

$\vec{PQ} = \langle 3-1, -4-2 \rangle = \langle 2, -6 \rangle$

two steps to RIGHT

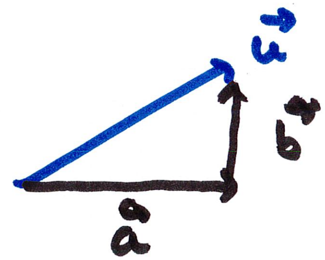
$\vec{QP} = \langle 1-3, 2--4 \rangle = \langle -2, 6 \rangle$

note $\vec{PQ} = -\vec{QP}$

the negative reverses direction

magnitude (or length) of vectors

$$\vec{u} = \langle a, b \rangle$$



right angle triangle

so $|\vec{u}| = \sqrt{a^2 + b^2}$

$\vec{PQ} = \langle 2, -6 \rangle$ $\quad |\vec{PQ}| = \sqrt{(2)^2 + (-6)^2} = \sqrt{40}$

basic operations with vectors

$\vec{u} = \langle 1, 2 \rangle \quad \vec{v} = \langle 0, 3 \rangle$

$\vec{u} + \vec{v} = \langle 1+0, 2+3 \rangle = \langle 1, 5 \rangle$

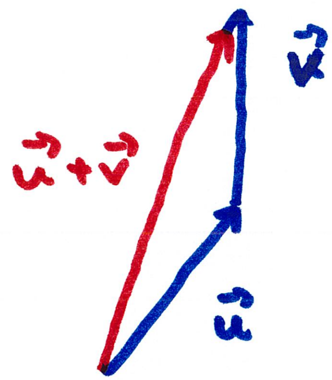
$\vec{u} - \vec{v} = \langle 1-0, 2-3 \rangle = \langle 1, -1 \rangle$

$2\vec{u} - 3\vec{v} = 2\langle 1, 2 \rangle - 3\langle 0, 3 \rangle$

$= \langle 2, 4 \rangle - \langle 0, 9 \rangle = \langle 2, -5 \rangle$

graphically:

$\vec{u}+\vec{v}$ put tail of second ($\vec{v}$) on head of first ($\vec{u}$)

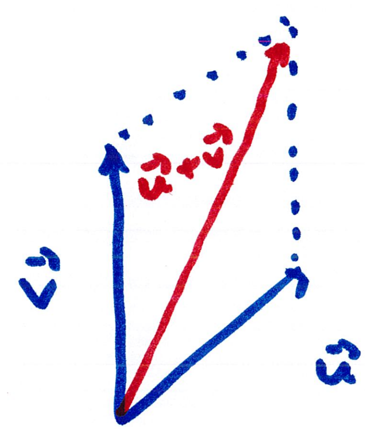


vector from tail of first to head of second is the vector sum

"triangle rule"

"parallelogram rule"

put tail to tail
complete parallelogram
go from tail to far corner



unit vectors: vectors with magnitude of 1

for example, $\vec{u} = \langle 3, 2 \rangle$ is <u>NOT</u> a unit vector

because $|\vec{u}| = \sqrt{9+4} = \sqrt{13} \neq 1$

but $\frac{\vec{u}}{|\vec{u}|}$ is a unit vector $\frac{\vec{u}}{|\vec{u}|} = \left\langle \frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle$

$$\left|\frac{\vec{u}}{|\vec{u}|}\right| = \sqrt{\frac{9}{13} + \frac{4}{13}} = \sqrt{\frac{13}{13}} = 1$$

unit vector in the opposite direction of $\vec{u}$?

$$-\frac{\vec{u}}{|\vec{u}|}$$

a vector with magnitude of 3 in the same direction as $\vec{u}$?

$$3 \cdot \underbrace{\frac{\vec{u}}{|\vec{u}|}}_{\text{unit vector same direction as } \vec{u}} = \frac{3\vec{u}}{|\vec{u}|}$$

Special unit vectors in $\mathbb{R}^2$ (2D space)

$\vec{i} = \langle 1, 0 \rangle$ unit vector in x direction

$\vec{j} = \langle 0, 1 \rangle$ " " " y "

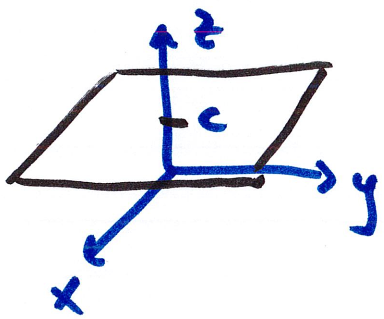
so, $\vec{u} = \langle 3, 2 \rangle = 3\langle 1, 0 \rangle + 2\langle 0, 1 \rangle = 3\vec{i} + 2\vec{j}$

13.2 Vectors in 3D

$P(1,2,3)$ $Q(4,6,8)$

$\vec{PQ} = \langle 4-1, 6-2, 8-3 \rangle = \langle 3, 4, 5 \rangle = 3\vec{i} + 4\vec{j} + 5\vec{k}$

basic shapes in $\mathbb{R}^3$: plane parallel to a coordinate plane

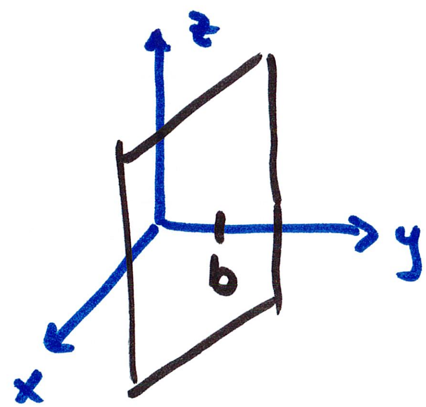


plane parallel to xy-plane, through $(0, 0, c)$

x, y can be any real numbers, but $z = c$

so equation of this plane is $\boxed{z = c}$

Similarly, $y = b$



parallel to xz-plane, thru $(0, b, 0)$

$x = a$?