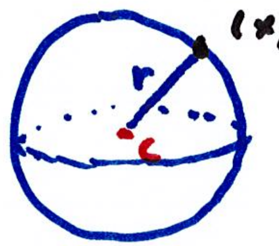
13.2 Vectors in 3D (part 2)

another basic shape: sphere



a collection of points that are all at a distance called radius from a point inside the sphere called center

let the center be the point $c\,(h, k, l)$

(x, y, z) is a point on sphere

vector from center to sphere surface: $\langle x-h,\ y-k,\ z-l \rangle$

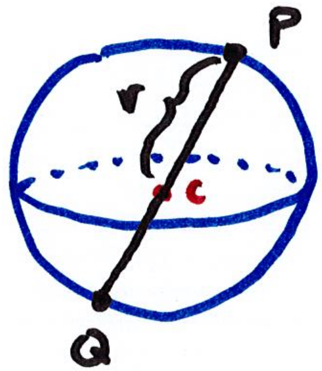
its magnitute is $\sqrt{(x-h)^2+(y-k)^2+(z-l)^2}$ = radius = r

or $$\boxed{(x-h)^2+(y-k)^2+(z-l)^2 = r^2}$$

this is the Standard Form of equation of sphere

all (x, y, z) that satisfies this equation is part of the sphere.

<u>example</u> Find the equation of a sphere whose diameter has endpoints $P(-2, 3, 6)$ and $Q(4, -7, 5)$



center C is midpoint of P, Q

$$C\left(\frac{-2+4}{2}, \frac{3-7}{2}, \frac{6+5}{2}\right)$$

$$= C\left(1, -2, \frac{11}{2}\right)$$

radius is half the diameter which is $|\vec{PQ}|$

$$\vec{PQ} = \langle 4--2, -7-3, 5-6 \rangle = \langle 6, -10, -1 \rangle$$

$$|\vec{PQ}| = \sqrt{36+100+1} = \sqrt{137}$$

$$r = \frac{\sqrt{137}}{2}$$

so, equation is

$$\boxed{(x-1)^2 + (y+2)^2 + \left(z - \frac{11}{2}\right)^2 = \left(\frac{\sqrt{137}}{2}\right)^2 = \frac{137}{4}}$$

example Find center and radius of sphere

$$x^2+y^2+z^2-14x+16y-10z+102=0$$

not in standard form, let's go back to it

$$\cancel{x^2+} \quad x^2-14x+y^2+16y+z^2-10z=-102$$

complete the square for x, y, z

$$x^2-14x+\underline{49}+y^2+16y+\underline{64}+z^2-10z+\underline{25}=-102+49+64+25$$

add square of half of the coefficient of x (-14 here)

add square of half of coeff. of y (16 here)

same idea for z

$$(x-7)^2+(y+8)^2+(z-5)^2=36$$

center: $(7,-8,5)$ radius $=6$

$(x-h)^2+(y-k)^2+(z-l)^2=r^2$ is a sphere radius r centered at (h,k,l)

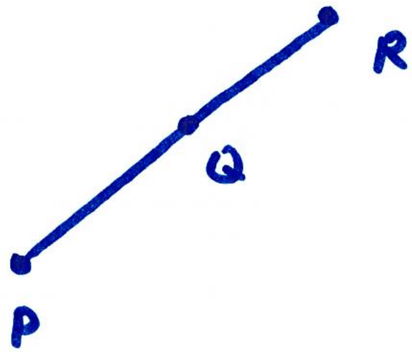
what about $(x-h)^2+(y-k)^2+(z-l)^2 \le r^2$?

(x,y,z) is exactly at distance r from $C(h,k,l)$

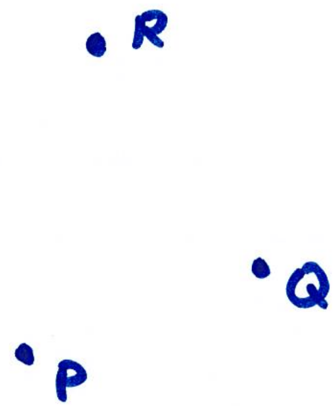
all (x,y,z) at distance equal to or less than r from $C(h,k,l)$ solid ball

what about $(x-h)^2+(y-k)^2+(z-l)^2 < r^2$

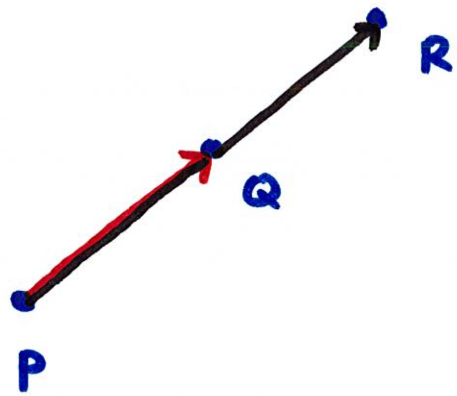
everything less than at distance r from center (ball w/o sphere surface) when given 3 points in $\mathbb{R}^3$, how do we know if they lie along a line (collinear)?



collinear



not collinear



if collinear, $\vec{PQ} \parallel \vec{PR}$

or equivalently,

$$\vec{PQ} = K\,\vec{PR}$$

some constant

<u>example</u> Find x and z such that

$P(1,3,2)$, $Q(5,1,3)$, $R(x,2,z)$ are collinear

$\vec{PQ} = \langle 4, -2, 1 \rangle$ $\quad$ $\vec{PR} = \langle x-1, -1, z-2 \rangle$

$\vec{PQ} \parallel \vec{PR}$ means

$\vec{PQ} = k\,\vec{PR}$ or

$\langle 4, -2, 1 \rangle = k \langle x-1, -1, z-2 \rangle$ vector equation

each component must match up

x: $4 = k(x-1)$

y: $-2 = k(-1)$ $\longrightarrow$ $k = 2$ works in all components

z: $1 = k(z-2)$

$4 = k(x-1) = 2(x-1) \rightarrow \boxed{x = 3}$

$1 = k(z-2) = 2(z-2) \rightarrow \boxed{z = \frac{5}{2}}$