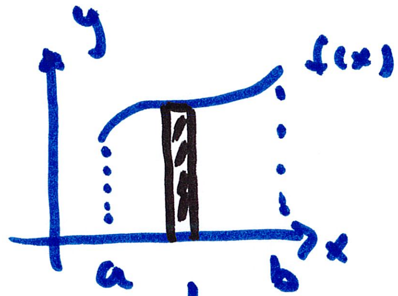
6.3 Volumes by Slicing

Area:



$$\text{area} = \int_a^b f(x)\,dx$$

↳ area = sum of infinitely many of these whose area is $\underbrace{f(x)}_{\text{height}}\,\underbrace{dx}_{dx}$

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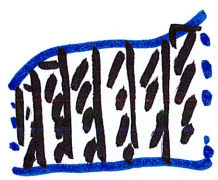
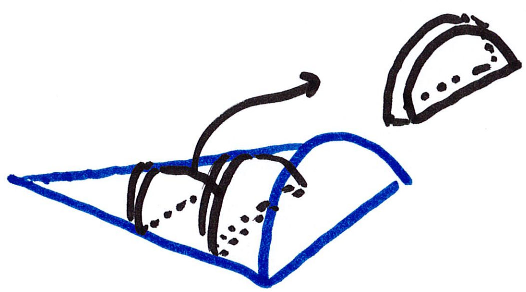


we can do similar things for volumes

for example: volume of half a cone whose cross sections are semicircles

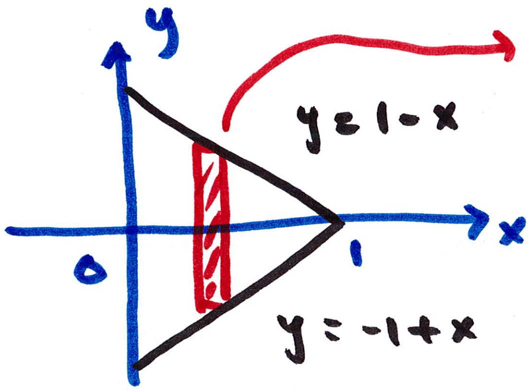


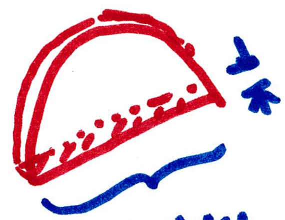
semicircle

volume of this cone = ?

the base of this cone is the region bounded by $y=1-x$ and ~~y=1~~+

$y=-1+x$ from $x=0$ to $x=1$





thickness $= dx$

~~width~~

length = length of rectangle in picture to the left

$= (1-x)-(-1+x) = 2-2x = \text{diameter}$

volume of each semicircle slice

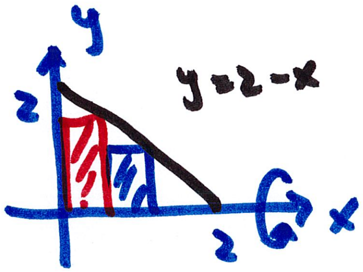
$$= \underbrace{\tfrac{1}{2}}_{\text{half of a circle}} \cdot \underbrace{\pi\left(\frac{2-2x}{2}\right)^2}_{\pi\cdot(\text{radius})^2} \cdot \underbrace{dx}_{\text{thickness}} = \tfrac{1}{2}\pi(1-x)^2\,dx$$

we want to accumulate these starting at $x=0$ ending at $x=1$

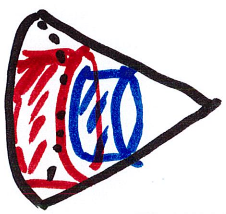
$$\text{volume of this cone} = \int_0^1 \tfrac{1}{2}\pi(1-x)^2\,dx = \cdots = \boxed{\frac{\pi}{6}}$$

volumes of revolutions

volume of an object obtained by revolving a planar region about a certain axis

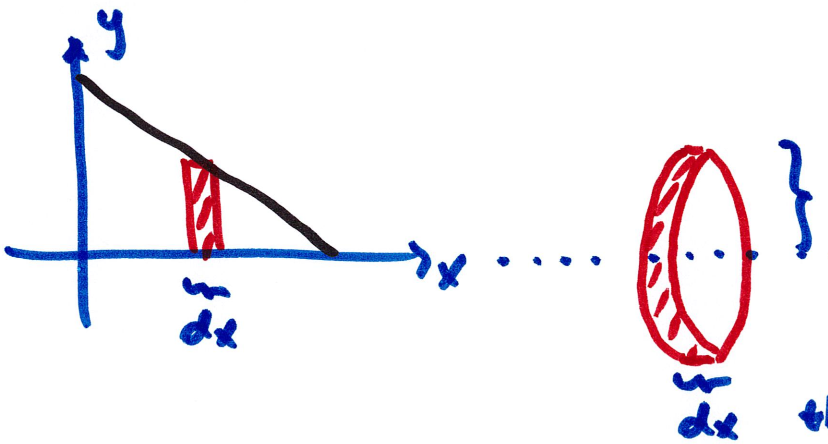


revolve this triangle about x-axis



volume = accumulation of slices

volume of each disk = $\pi \cdot (\text{radius})^2 \cdot \text{thickness}$



radius = height of rectangle = $2-x$

thickness

volume of slice = $\pi(2-x)^2 dx$

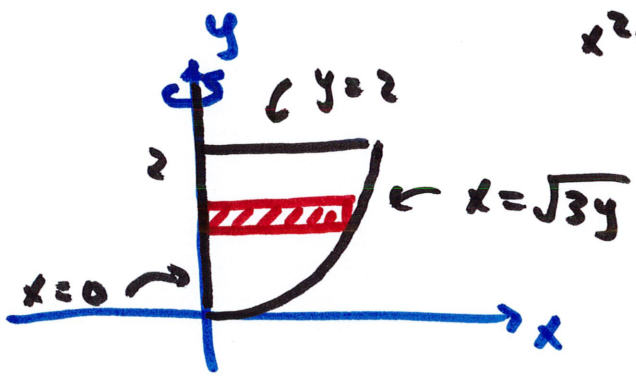
accumulate all, start at $x=0$
end at $x=2$

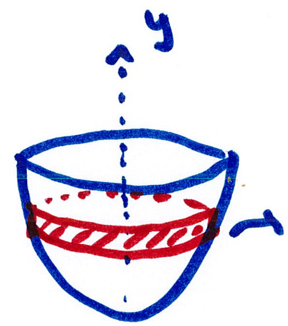
volume of entire cone $= \displaystyle\int_0^2 \pi(2-x)^2\,dx = \cdots = \boxed{\frac{8\pi}{3}}$

this method is called the <u>Disk/Washer Method</u>

<u>example</u> Volume of solid obtained by revolve the region bounded by $\underbrace{x=\sqrt{3y}}_{x^2=3y}$, $x=0$, $y=2$ about the y-axis

$y = \frac{1}{3}x^2$





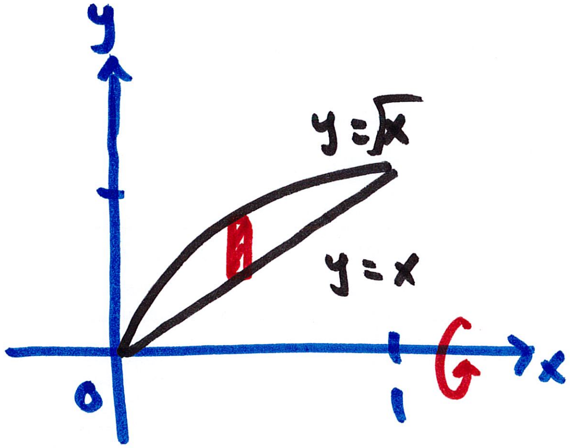
volume $= \pi(\text{radius})^2(\text{thickness})$

each ~~slice of~~ disk has volume $= \pi\left(\sqrt{3y}\right)^2 dy$

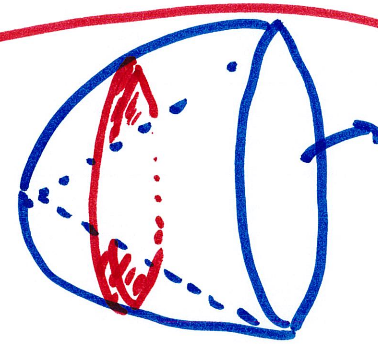
accumulate
start at $y=0$
end at $y=2$

$$V = \int_0^2 \pi\left(\sqrt{3y}\right)^2 dy = \int_0^2 3\pi y\,dy = \cdots = \boxed{6\pi}$$

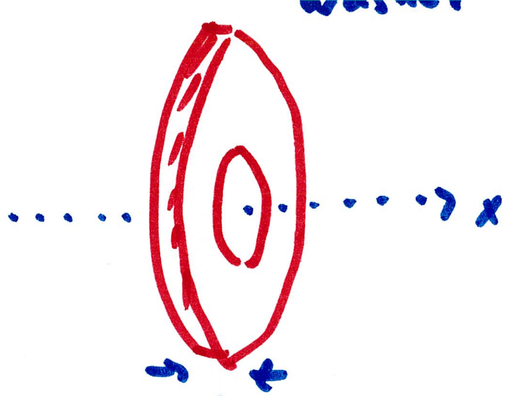
example Solid ~~revolved~~ obtained by revolving region bounded by $y=\sqrt{x}$, $y=x$, about x-axis

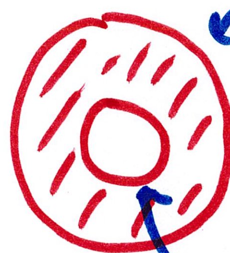


volume by disk/washer: set up rectangle $\perp$ to axis of ~~rotat~~ revolution



hollow shape of a cone





area = big circle − small circle

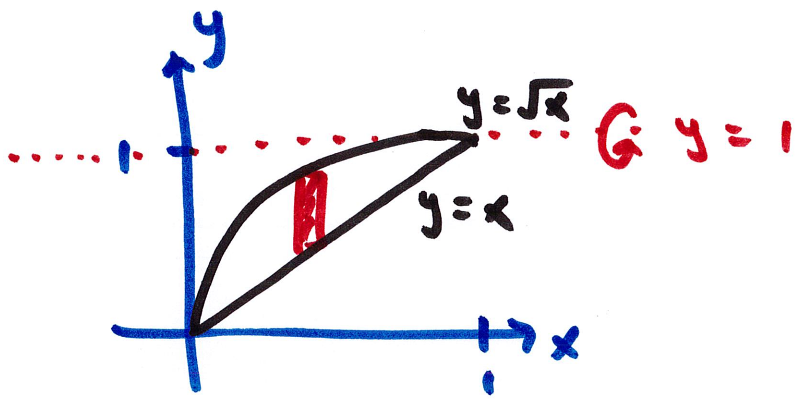
$$\text{volume of washer} = \left[\pi(\text{outer radius})^2 - \pi(\text{inner radius})^2\right]\cdot \text{thickness}$$

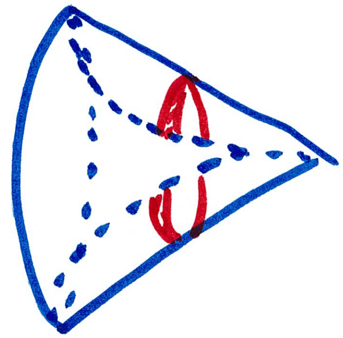
$$= \left[\pi(\sqrt{x})^2 - \pi(x)^2\right]dx$$

accumulate from $x=0$ to $x=1$

$$\int_0^1 \pi (x - x^2)\,dx = \cdots = \boxed{\frac{\pi}{6}}$$

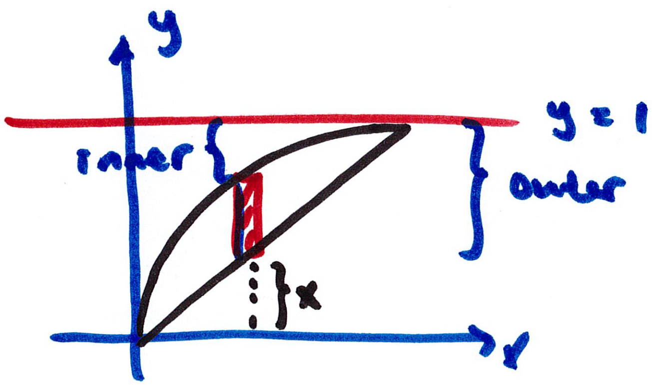
<u>example</u> Same region but revolve around $y = 1$





disk/washer: rectangle $\perp$ axis of revolution

radius: measured from line of revolution



outer radius $= \cancel{y=x}\ 1 - x$

inner radius $= 1 - \sqrt{x}$

volume of slice =

$[\pi\,(\text{outer})^2 - \pi\,(\text{inner})^2] \cdot \text{thickness}$

$= [\pi\,(1-x)^2 - \pi\,(1-\sqrt{x})^2]\,dx$

volume of whole thing $= \int_0^1 [\pi(1-x)^2 - \pi(1-\sqrt{x})^2]\,dx = \cdots = \boxed{\frac{\pi}{6}}$