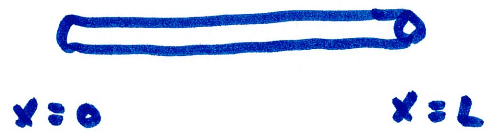
6.7 Physical Applications (part 1)

exam 1 covers up to this ~~section~~ lesson (inclusive)

long thin wire/bar



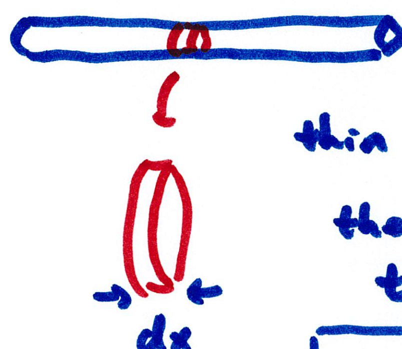
wire is L m long

if density is constant, then the mass is easy

$m = \rho \cdot L$

"rho"

if density is not constant kg but depend on the location within the wire : $\rho = \rho(x)$



thin section : mass $= \rho(x)\,dx$

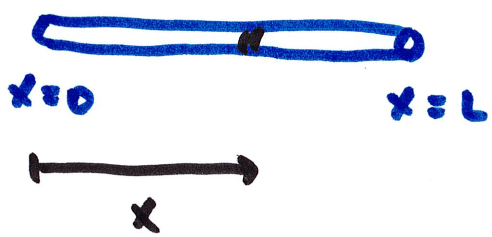
then accumulate all slices from $x=0$ to $x=L$

$$\boxed{m = \int_0^L \rho(x)\,dx}$$

<u>example</u> Wire length 6 m

density is twice the distance from left end

mass = ?



x: dist from left end

$\rho(x) = 2x$

$$m = \int_0^6 2x\,dx = \cdots = 36$$

follow-up question: mass of the right half of wire?

accumulate from middle $(x=3)$ to right end $(x=6)$

$$m = \int_3^6 2x\,dx = \cdots = 27$$

work = force · distance ~~the~~ this assumes force is constant

in many situations force is not constant

for example, a spring

if no force is acting on the spring, its length is called the natural length and we say it's at equilibrium position

one simple way to model the force on a linear spring is Hooke's Law

$$F = k \cdot x$$

change in length from equilibrium

force acting to restore to equilibrium

spring constant

total work ~~on~~ done by spring: $W = \int_a^b \text{force}\, dx = \int_a^b kx\, dx$

a: starting length measured with respect to equilibrium

b: ending " " " " "

example Spring has natural length of 1 m

A force of 40 N stretches and holds it 0.1 m from its equilibrium

How much work is done in compressing the spring to a length of 0.5 m? Starting from natural

How much work is done in compressing it by another 0.5 m?

force : $F = k \cdot x$ measured from equilibrium

$$40 = k \cdot (0.1) \rightarrow k = 400$$

work done in compressing to 0.5 m

$$W = \int_a^b F\,dx = \int_0^{-0.5} 400x\,dx$$

natural is 1 want to go to 0.5 so it is a change of -0.5 with respect to natural

start w/ natural

$$= \cdots = 50 \text{ N} \cdot \text{m} \text{ or Joules (J)}$$

now having compressed by 0.5 m, let's compress it by another 0.5 m

$$W = \int_a^b F\,dx = \int_{-0.5}^{-1} 400x\,dx = \cdots = 150\text{ J}$$

this is the additional work in compressing it by the additional 0.5 m

work done against gravity

if the change in height is "small" we consider gravity as constant : $g = 9.8\ m/s^2$

work = force · distance

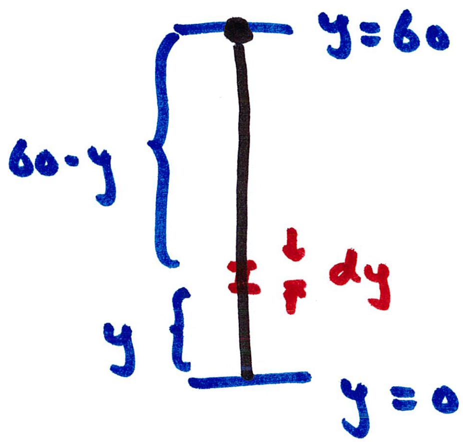
e.g. moving a mass of 45 kg from ground to a height of 60 m

$$W = \underbrace{(45 \cdot g)}_{\text{force on mass}} \cdot (60) = 2700\ J$$

when "distance" is not constant, integration is once again involved

example: winding up a chain

example A 60-m chain with density of 5 kg/m is hanging from a winch. Find the work done in winding up this chain.



each small segment has its own distance to go when winding up chain

small segment at height of y from ground and needs to go 60-y to the top

find work to move one segment, the accumulate all

mass of one small segment : $\underbrace{(dy)}_{\text{length}}\underbrace{(5)}_{\text{density}} = 5\,dy$

work to move that segment : $\underbrace{\underbrace{(5\,dy)}_{\text{mass}}\underbrace{(g)}_{\text{gravity}}}_{\text{force}}\underbrace{(60-y)}_{\text{dist to go}} = 5g(60-y)\,dy$

now accumulate along the chain

$$\int_0^{60} 5g\,(60-y)\,dy = \cdots = 88200 \text{ J} \qquad (g = 9.8)$$