Compute the value of the integral $\int_1^e 2x \ln x \, dx$.

A L

method : direct ✗

Substitution ✗

by parts → different kinds of functions

trig subs

LIATE

A. $\frac{1}{2}$

B. $\frac{1-e^2}{2}$

C. $\frac{1+e^2}{2}$

D. $\frac{3e^2-1}{2}$

E. $\frac{e^2}{2}$

$u = \ln x \quad dv = 2x\,dx$

$du = \frac{1}{x}dx \quad v = x^2$

$uv\Big|_1^e - \int_1^e v\,du$

$= x^2 \ln x \Big|_1^e - \int_1^e x^2 \cdot \frac{1}{x} dx$

$= x^2 \ln x \Big|_1^e - \int_1^e x\,dx$

$= x^2 \ln x \Big|_1^e - \frac{1}{2}x^2 \Big|_1^e = x^2 \ln x - \frac{1}{2}x^2 \Big|_1^e$

$= e^2 \ln e - \frac{1}{2}e^2 - 1 \cdot \ln 1 + \frac{1}{2}$

$= e^2 - \frac{1}{2}e^2 + \frac{1}{2} = \frac{1}{2}e^2 + \frac{1}{2}$

Evaluate $\int_0^{\pi/2} \sin^3 x \cos^4 x dx$

Sinx and cosx : $u = \cos x$ $du = -\sin x dx$

OR

$u = \sin x$ $du = \cos x dx$

then use

$\sin^2 x + \cos^2 x = 1$

A. $\frac{5}{12}$

B. $\frac{1}{7}$

C. $\frac{2}{35}$

D. $\frac{3}{28}$

E. $\frac{5}{16}$

$u = \cos x$, split a factor of sinx

$\sin^2 x$ left : use $\sin^2 x + \cos^2 x = 1$ to change into cosx

$$\int_0^{\pi/2} \sin^2 x \underbrace{\cos^4 x}_{u^4} \underbrace{\sin x\, dx}_{-du}$$

$\sin^2 x = 1 - \cos^2 x$
$= 1 - u^2$

$u = \cos x$
$du = -\sin x\, dx$

$x = \pi/2 \rightarrow u = \cos\frac{\pi}{2} = 0$
$x = 0 \rightarrow u = \cos 0 = 1$

$$= -\int_1^0 (1-u^2)u^4\, du = -\int_1^0 u^4 - u^6\, du = -\left(\frac{u^5}{5} - \frac{u^7}{7}\right)\Big|_1^0$$

$$= -\left[(0) - \left(\frac{1}{5} - \frac{1}{7}\right)\right] = \frac{2}{35}$$

Compute

$$\int 7 \sec^4 x \, dx$$

$u = \sec x \quad du = \sec x \tan x \, dx$

$u = \tan x \quad du = \sec^2 x \, dx$

$\sin^2 x + \cos^2 x = 1$

divide by $\cos^2 x$

$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$\tan^2 x + 1 = \sec^2 x$

$$7 \int \underbrace{\sec^2 x}_{\tan^2 x + 1 \,\to\, u^2+1} \; \underbrace{\sec^2 x \, dx}_{du \text{ if } u = \tan x}$$

$$7 \int (u^2+1) \, du$$

$$= 7\left(\frac{u^3}{3} + u\right) + C$$

$$= \frac{7}{3} u^3 + 7u + C = \frac{7}{3} \tan^3 x + 7 \tan x + C$$

A. $\frac{7}{3} \tan^3 x + C$

B. $-\frac{7}{3} \tan^3 x + C$

C. $7(\sec x + \tan x)^5 + C$

D. $\frac{7}{3} \tan x + 7 \tan^3 x + C$

E. $7 \tan x + \frac{7}{3} \tan^3 x + C$

After a proper trigonometric substitution is used to transform $\int_1^4 \frac{dt}{t^2 - 2t + 10}$ into $\int_a^b f(\theta)\, d\theta$, what is the new upper integration limit b?

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{\pi}{3}$

E. π

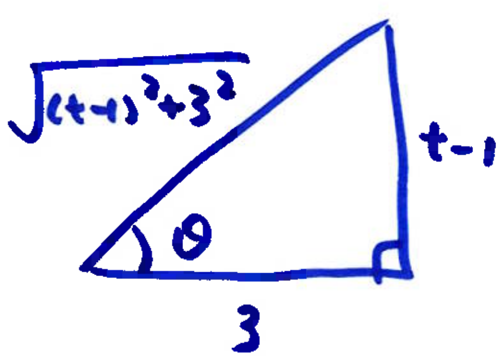
trig subs: difference or sum of squares

$$t^2 - 2t + 10 = t^2 - 2t + 1 + 10 - 1$$
$$= (t-1)^2 + 9$$

$$\int_1^4 \frac{dt}{t^2-2t+10} = \int_1^4 \frac{dt}{\left(\sqrt{(t-1)^2+9}\right)^2}$$

triangle w/ sides: $\sqrt{(t-1)^2+3^2}$, $t-1$, 3

hypotenuse: $\sqrt{(t-1)^2+3^2}$ because it squared is sum of squares of the other two



$$\tan\theta = \frac{t-1}{3}$$
$$3\tan\theta = t-1$$
$$t = 3\tan\theta + 1$$
$$dt = 3\sec^2\theta\, d\theta$$

$$\int_1^4 \frac{dt}{(t-1)^2+9} = \int \frac{3\sec^2\theta \, d\theta}{9\tan^2\theta+9} = \int \frac{\sec^2\theta \, d\theta}{3(\tan^2\theta+1)}$$

$$= \int \frac{\sec^2\theta \, d\theta}{3\sec^2\theta} = \int \frac{1}{3} \, d\theta = \int_{\pi/6}^{\pi/4} \frac{1}{3} d\theta$$

$\cancel{\tan}$ $\quad \tan\theta = \frac{t-1}{3}$

$t = 4 \rightarrow \tan\theta = 1 \rightarrow \theta = \pi/4$

$t = 1 \rightarrow \tan\theta = 0 \rightarrow \theta = \cancel{\pi/6}\ 0$

Use the fact that

$$\int \frac{2x^4 + 15x^3 + 9x^2 + 11x + 3}{x^5 + x^4 - x - 1}\,dx = 5\ln|x-1| - 3\ln|x+1| - \frac{3}{x+1} + 2\tan^{-1}(x) + C$$

to find the partial fraction expansion of $\frac{2x^4 + 15x^3 + 9x^2 + 11x + 3}{x^5 + x^4 - x - 1}$ = expansion ?

if we know the expansion then

$\int$ expansion =

so expansion = deriv. of

$$\frac{5}{x-1} - \frac{3}{x+1} + \frac{3}{(x+1)^2} + \frac{2}{x^2+1}$$

A. $\frac{5}{x-1} + \frac{-3}{x+1} + \frac{3}{(x+1)^2} + \frac{2}{x^2+1}$

B. $\frac{5}{x-1} + \frac{-3}{x+1} + \frac{-3x}{(x+1)^2} + \frac{2}{x^2+1}$

C. $\frac{5}{x-1} + \frac{-3}{x+1} + \frac{-3}{(x+1)^2} + \frac{2x}{x^2+1}$

D. $\frac{-5}{x-1} + \frac{3}{x+1} + \frac{3}{(x+1)^2} + \frac{-2}{x^2+1}$

E. $\frac{-5}{x-1} + \frac{3}{x+1} + \frac{3x}{(x+1)^2} + \frac{2x}{x^2+1}$

Compute $\int_1^2 \frac{dx}{\sqrt{2-x}}$

A. 2

B. $2\sqrt{2}-1$

C. $\sqrt{2}-1$

D. $\sqrt{2}$

E. 1

improper because integrand is undefined somewhere between or at the two bounds

here, $x=2$ is problem

$$\lim_{a\to 2^-} \int_1^a \frac{dx}{\sqrt{2-x}} = \lim_{a\to 2^-} \int_1^a (2-x)^{-1/2}\,dx$$

$u = 2-x$
$du = -dx$

$$= \lim_{a\to 2^-} \int_1^{2-a} -u^{-1/2}\,du$$

$$= \lim_{a\to 2^-} -2u^{1/2}\Big|_1^{2-a} = \lim_{a\to 2^-} \left(-2(2-a)^{1/2}+2\right)$$

$$= 2$$

$$\int_1^3 \frac{dx}{\sqrt{2-x}}$$ $x=2$ is the problem

$$= \int_1^2 \frac{dx}{\sqrt{2-x}} + \int_2^3 \frac{dx}{\sqrt{2-x}}$$

$$= \lim_{a \to 2^-} \int_1^a \frac{dx}{\sqrt{2-x}} + \lim_{b \to 2^+} \int_b^3 \frac{dx}{\sqrt{2-x}}$$

Evaluate $\int_0^\infty x^2 e^{-x^3}\,dx$

(A) $\frac{1}{2}$

(B) 1

(C) the integral diverges

(D) $\frac{1}{3}$

(E) $\frac{1}{6}$

$$\lim_{a\to\infty}\int_0^a x^2 e^{-x^3}\,dx$$

$u = x^3$

$du = 3x^2\,dx$

$$= \lim_{a\to\infty}\int_0^{a^3} \frac{1}{3} e^{-u}\,du$$

$$= \lim_{a\to\infty} -\frac{1}{3}e^{-u}\Big|_0^{a^3} = \lim_{a\to\infty}\left(\underbrace{-\frac{1}{3}e^{-a^3}}_{0} + \frac{1}{3}\right) = \frac{1}{3}$$

Determine whether the following sequences are convergent or divergent.

(1) $\{a_n = 2n/(3n+1)\}$ C

(2) $\{a_n = \cos n\pi\}$ D

(3) $\{a_n = n\sin(1/n)\}$ C

Sequence: convergent if $\lim_{n\to\infty} a_n$ exists

divergent if $\lim_{n\to\infty} a_n$ DNE

A. (1) convergent (2) convergent (3) convergent

B. (1) divergent (2) convergent (3) convergent

C. (1) convergent (2) divergent (3) convergent

D. (1) convergent (2) convergent (3) divergent

E. (1) convergent (2) divergent (3) divergent

(1) $\lim_{n\to\infty} \frac{2n}{3n+1} \to \frac{\infty}{\infty}$ l'Hospital's : $\lim_{n\to\infty} \frac{2}{3} = \frac{2}{3}$ <u>conv</u>

(2) $\{\cos n\pi\}_{n=1}^{\infty} = \{-1, \cancel{0}, -1, 1, -1, 1, -1, 1, -1, \dots\}$

not settling anywhere, so no limit, so <u>div</u>

(3) $\lim_{n\to\infty} n\sin\left(\frac{1}{n}\right) = \lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} \to \frac{0}{0}$ l'Hospital's ok

$= \lim_{n\to\infty} \frac{\cos\left(\frac{1}{n}\right)\cdot -\frac{1}{n^2}}{-\frac{1}{n^2}} = \lim_{n\to\infty} \cos\left(\frac{1}{n}\right) = \cos(0) = 1$

<u>conv</u>