

# 13.1 Vectors

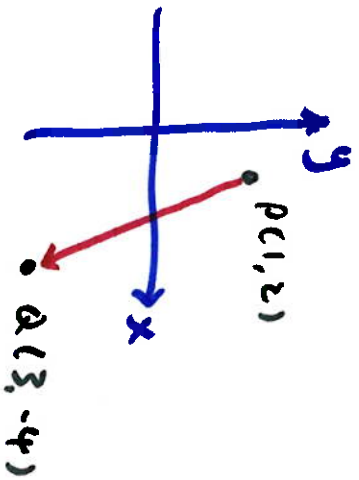
Scalars: numbers such as 5,  $\pi$ , e, 133, etc  
magnitude only, no direction

vectors: magnitude and direction

e.g. wind 25mph from NE

vectors also tell us the relative position from one point to another

for example, P(1, 2), to Q(3, -4)



vector from P to Q

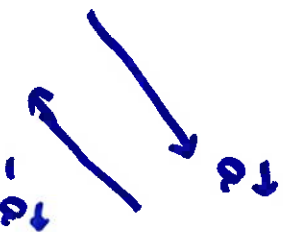
$$\vec{PQ} = \langle \underbrace{3-1}_{\text{end } x \text{ minus start } x}, \underbrace{-4-2}_{\text{end } y \text{ minus start } y} \rangle = \langle 2, -6 \rangle$$

2 unit RIGHT (positive)      6 units down (negative)

Q to P     $\vec{QP} = \langle -2, 6 \rangle$

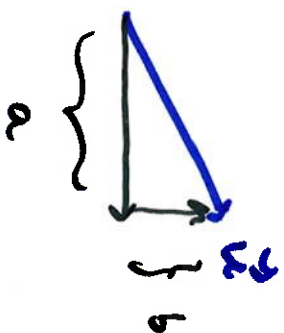
$$\vec{QP} = -\vec{PQ}$$

minus sign reverses direction



magnitude / length

•  $\vec{u} = \langle a, b \rangle$



length :  $|\vec{u}| = \sqrt{a^2 + b^2}$

$\vec{p}_Q = \langle -2, 6 \rangle$      $|\vec{p}_Q| = \sqrt{(-2)^2 + (6)^2} = \sqrt{40}$

addition / subtraction

$\vec{u} = \langle 1, 2 \rangle$      $\vec{v} = \langle 0, 3 \rangle$

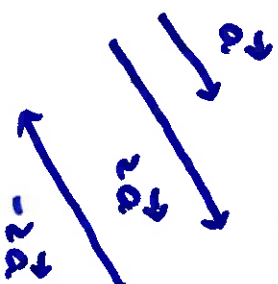
$\vec{u} + \vec{v} = \langle 1+0, 2+3 \rangle = \langle 1, 5 \rangle$

$\vec{u} - \vec{v} = \langle 1-0, 2-3 \rangle = \langle 1, -1 \rangle$

$2\vec{u} - 3\vec{v} = 2\langle 1, 2 \rangle - 3\langle 0, 3 \rangle$

$= \langle 2, 4 \rangle - \langle 0, 9 \rangle$

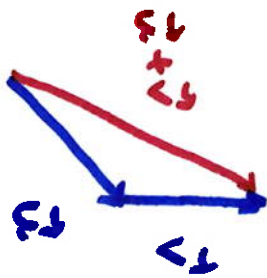
$= \langle 2, -5 \rangle$



Graphically:

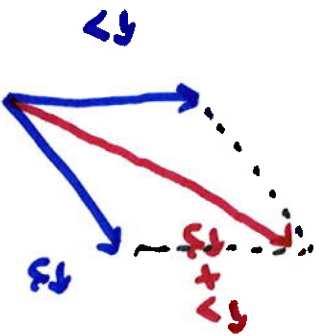


$\vec{u} + \vec{v}$ :



"triangle rule"

$\vec{u} + \vec{v}$ :



"parallelogram rule"

unit vector: vector of length or magnitude of 1

$$\vec{u} = \langle 3, 2 \rangle \quad |\vec{u}| = \sqrt{3^2 + 2^2} = \sqrt{13} \neq 1$$

So,  $\vec{u}$  is NOT a unit vector

$$\text{but } \vec{v} = \frac{\vec{u}}{|\vec{u}|} = \frac{\langle 3, 2 \rangle}{\sqrt{13}} = \left\langle \frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \right\rangle$$

$$|\vec{v}| = \sqrt{\left(\frac{3}{\sqrt{13}}\right)^2 + \left(\frac{2}{\sqrt{13}}\right)^2} = \sqrt{\frac{9}{13} + \frac{4}{13}} = \sqrt{\frac{13}{13}} = 1$$

So, in general,  $\frac{\vec{a}}{|\vec{a}|}$  is a <sup>unit</sup> vector

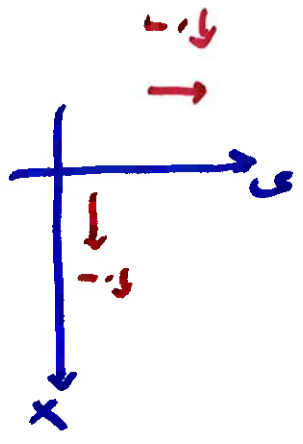
unit vector in opposite direction of  $\vec{b}$ ?

$$-\frac{\vec{b}}{|\vec{b}|}$$

vector length of 3, in opposite direction of  $\vec{b}$ ?  $-3\frac{\vec{b}}{|\vec{b}|}$

Special unit vectors:  $\vec{i} = \langle 1, 0, 0 \rangle$  unit vector in x direction

$\vec{j} = \langle 0, 1, 0 \rangle$  " " " y "

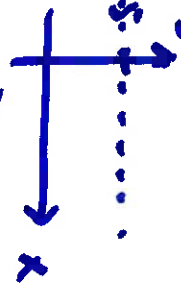


13.2 Vectors in 3D

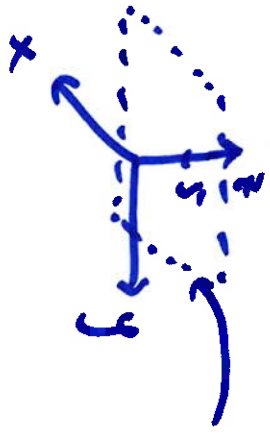
P (1, 2, 3) Q (4, 6, 8)

$\vec{PQ} = \langle 4-1, 6-2, 8-3 \rangle = \langle 3, 4, 5 \rangle = 3\vec{i} + 4\vec{j} + 5\vec{k}$   $\langle 0, 0, 1 \rangle$

shapes: line in 2D:  $y = 5$   $\dots$  all points w/  $y = 5$



in 3D, it's a plane  $z = 5$  plane



all points w/  $z = 5$   $(x, y, 5)$