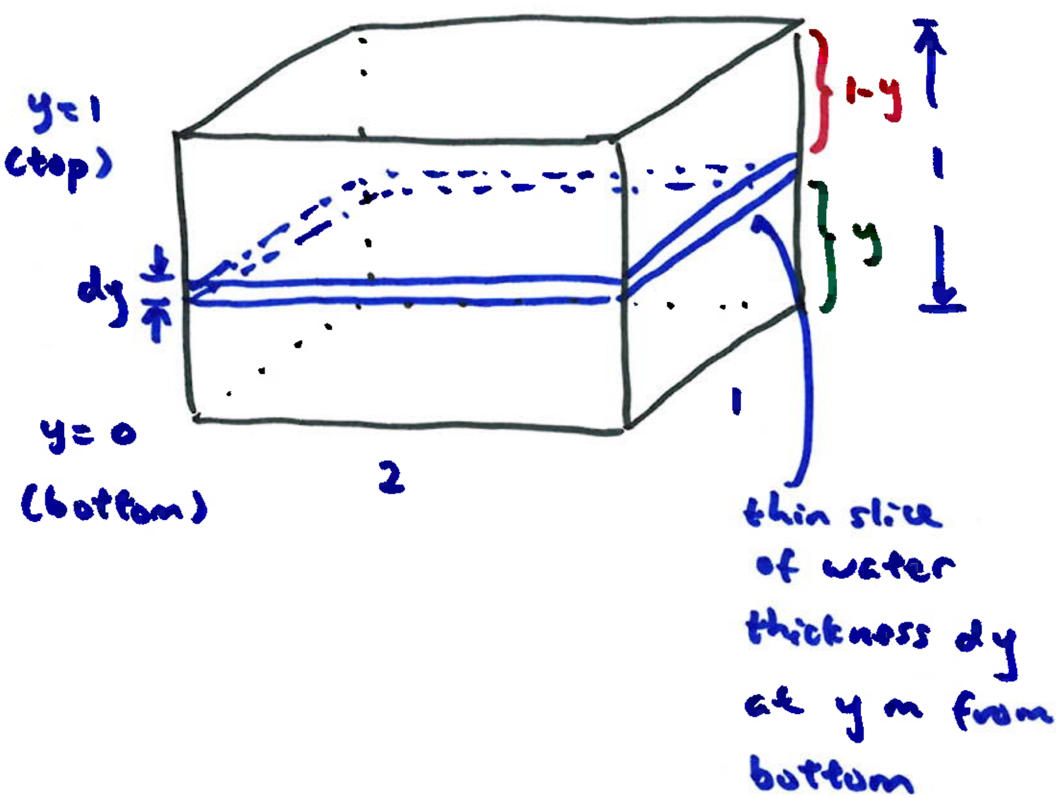
6.7 Physical Applications (part 2)

Exam 1 covers up to this lesson (inclusive)

work examples / applications

<u>example</u> An aquarium length 2 m, width 1 m, height 1 m, is full of water. Find the work done in pumping all the water over the top.



just like with the chain, we will find the work to move one "slice" of water, then integrate to accumulate all slices.

mass of slice : density · volume

density of water : ρ (rho)

volume : $(2)(1)\,dy = 2\rho$

force or weight : mass · gravity

weight of slice : $2\rho\,dy \cdot g$

work to move it to top :

work to move it : (weight)(distance to go)

$$= (2\rho g\, dy)(1-y)$$

$$= 2\rho g (1-y)\, dy$$

we can take infinitely many of these starting at $y=0$ (bottom) to $y=1$ (surface of water)

accumulate all

$$\int_0^1 2\rho g (1-y)\, dy = 2\rho g \int_0^1 (1-y)\, dy = \cdots = \boxed{\rho g} \text{ (J)}$$

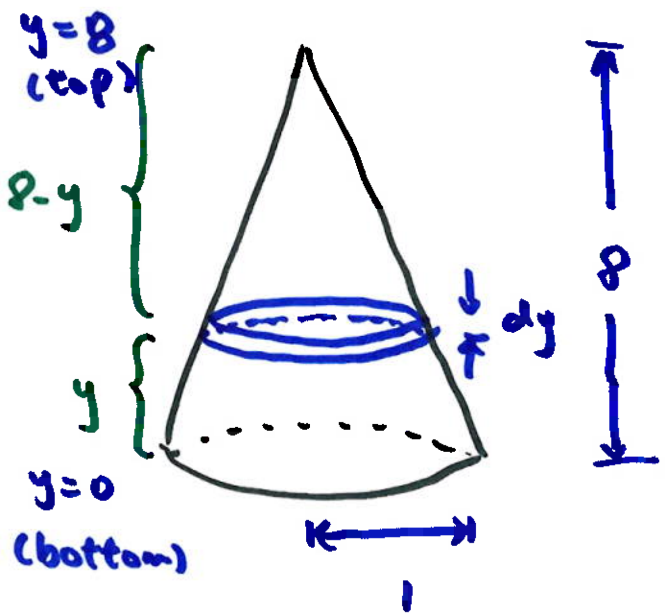
ρ for water : 1000 kg/m³

$g = 9.81$ m/s²

<u>example</u> The tank is in the shape of a cone vertex up.

Height is 8 m and radius at base is 1 m.

Tank is full of water. Find how much work to pump all water out over top?



draw slice at height y

thickness dy, needs to travel up $8-y$ m

mass = volume · density

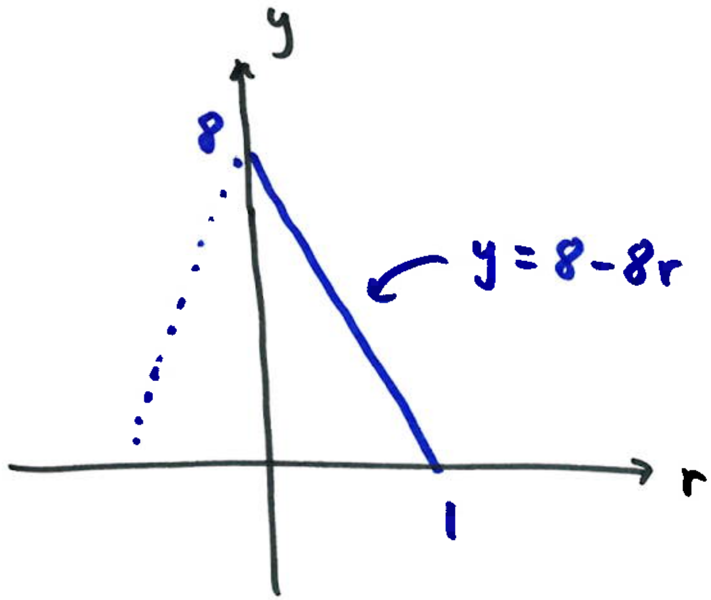
$= \pi\,(\text{radius})^2 \cdot dy \cdot \rho$

weight = mass · gravity

notice radius changes with y

need to find a relationship between radius and height y

Side view



notice outline is a triangle

Side is a line through $(0,8)$, $(1,0)$

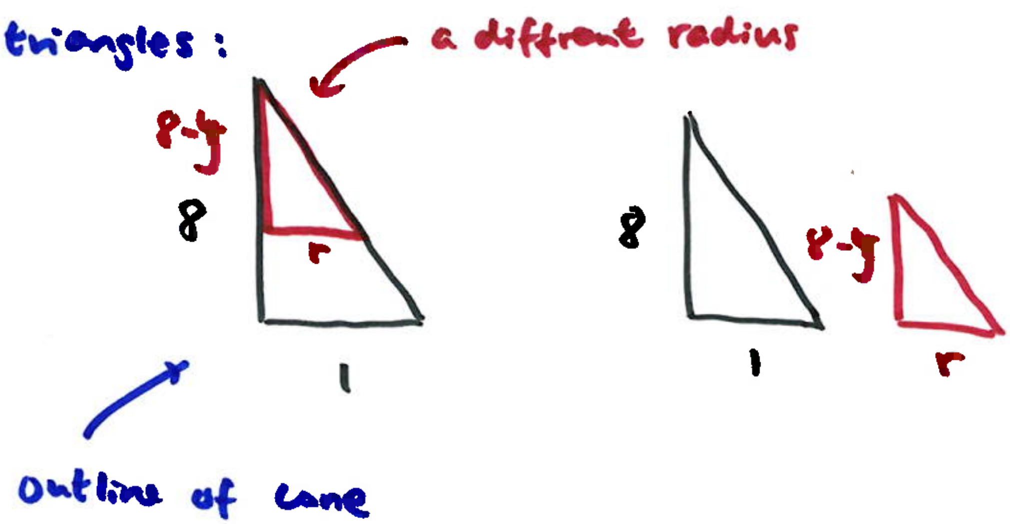
$y = 8-8r$

solve for r: $y-8=-8r$

$8r = 8-y$

$$\boxed{r = 1-\frac{1}{8}y}$$

Similar triangles :



ratios of sides are equal

$$\frac{1}{8} = \frac{r}{8-y}$$

$$r = \frac{1}{8}(8-y)$$

$$r = 1-\frac{1}{8}y$$

mass of one slice : $\pi\,(\text{radius})^2 \cdot dy \cdot \rho$

$$= \pi \left(1 - \tfrac{1}{8} y\right)^2 \cdot dy \cdot \rho$$

weight : $\pi\rho \left(1-\tfrac{1}{8}y\right)^2 dy \cdot g \;=\; \pi\rho g \left(1-\tfrac{1}{8}y\right)^2 dy$

work : $\pi\rho g \left(1-\tfrac{1}{8}y\right)^2 dy \cdot (8-y) \;=\; \pi\rho g \left(1-\tfrac{1}{8}y\right)^2 (8-y)\, dy$

accumulate from bottom $(y=0)$ to water surface $(y=8)$

$$\int_0^8 \pi\rho g \left(1-\tfrac{1}{8}y\right)^2 (8-y)\, dy = \cdots = \boxed{16\pi\rho g} \ (J)$$

$$= 156800\,\pi$$

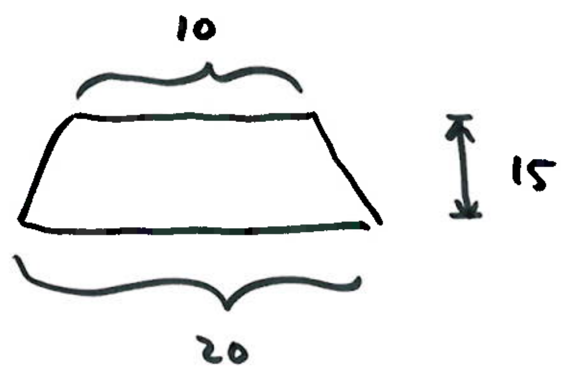
what if tank is half full based on height

same shape, different water surface level (upper lim of integration)

$$\int_0^4 \pi\rho g \left(1-\tfrac{1}{8}y\right)^2 (8-y)\, dy = \cdots = 147{,}000\,\pi$$

__example__ hydrostatic pressure or force

A small dam is in the shape of a trapezoid



all in meters

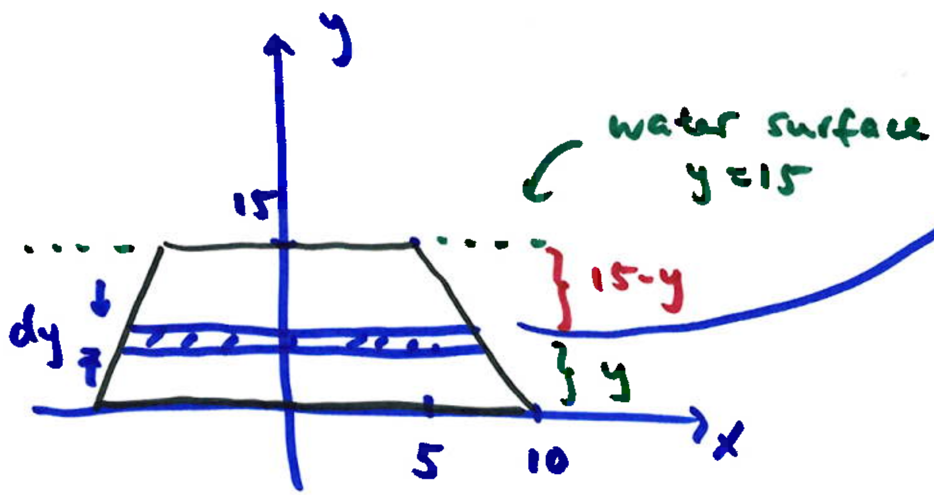
water ~~is~~ level is even with the top

what is the force on this gate?

hydrostatic force = (hydro pressure) · (area)

hydrostatic pressure = (density of water) (gravity) (depth) ~~(area)~~

↳ below water surface



one strip of gate

15-y below surface

need area of ~~slice~~ strip

~~th~~ height : dy