For example, in polar: \((r, \theta)\)

- \(r\): distance from origin to the point.
- \(\theta\): angle formed by a line through origin and the point with the positive x-axis.

In polar, a point is located as \((r, \theta)\).

In rectangular/cartesian coordinates, a point is located as \((x, y)\).

12. 2 Polar Coordinates
\( e^{2/3} (-2, -2^{2/3}) \) can be expressed as \((2, 2^{1/3}) \) \( e^{1/3} \).

Also

\( e^{1/3} (2, 2^{1/3}) \) can be expressed as \((-2, -2^{2/3}) \) \( e^{2/3} \).

\[ \text{Distance is } 2 \]

Get away from origin \( \theta \) 

\[ \text{Distance to } (2, 2^{1/3}) \]

\[ \text{Distance to } (-2, -2^{2/3}) \]
\[ x^2 + y^2 = 2 \]

\[ \theta = \frac{\pi}{4} \]

\[ y = x \]

\[ \theta = \tan^{-1}\left(\frac{y}{x}\right) \]

\[ \sin \theta = \frac{1}{\sqrt{2}} \]

\[ \cos \theta = \frac{1}{\sqrt{2}} \]

Conversion: Polar to Cartesian

\[ (r, \theta) \rightarrow (x, y) \]

\[ (1, \frac{\pi}{4}) \rightarrow (\sqrt{2}, 1) \]

\[ (r, \theta) \rightarrow (x, y) \]

\[ (1, \frac{3\pi}{4}) \rightarrow (-\sqrt{2}, 1) \]

Find two other ways to express location in polar:

Point: \((-1, -\frac{3\pi}{4})\)
In Cartesian, this point is \((-1, -\sqrt{3})\)

\[1 + 3 = 4\quad \text{yes}\]

\[\frac{1}{2} \cdot (\frac{9}{2})^2 = \left(-\frac{9}{2}\right)^2\]

\[x^2 + y^2 = 2^2\]

Check: Sine and cosine

\[\begin{align*}
\theta &= \arcsin\left(-\frac{3}{2}\right) = \left(-2\right) \quad \text{sine} = -\frac{3}{2} \\
x &= \cos\theta = \left(-2\right) \quad \text{cosine} = \frac{\sqrt{3}}{2} \\
y &= \sin\theta = \left(-2\right) \quad \text{sine} = \frac{1}{2}
\end{align*}\]

In \(\theta = \frac{\pi}{3}\), we need \(x < 0, y < 0\)
In Cartesian:

\[ x^2 + y^2 = 9 \]

\[ r = 3 \]

Polar is good with circles and circle-like shapes.

Example: \[ r = 3 \] (polar)

Equations can be transformed to
At each point \((x, y)\), the line \(2x + 4y = 1\) passes:

\[
\begin{align*}
\rho &= \frac{x \cos \theta + y \sin \theta}{\sqrt{x^2 + y^2}} \\
\rho &= \frac{\rho \cos \theta + \rho \sin \theta}{\sqrt{\rho^2}} \\
\rho &= \cos \theta + \sin \theta
\end{align*}
\]

This is a polar equation for a line.
\[ \frac{x}{h} + \frac{y}{k} = \theta \]

\[ y = h \cos \theta \]

\[ x = h \sin \theta \]

\[ \begin{align*}
\theta &= \theta_0 + \theta_0' \\
\sin \theta &= \frac{x}{h} \\
\cos \theta &= \frac{y}{h}
\end{align*} \]

Check solution!
now pair $r = 2$, $z$ and $\theta = \frac{\pi}{3}$.

So,

$$\frac{3}{\sqrt{3}} \cdot \frac{3}{\sqrt{3}} = \frac{3}{\sqrt{3}}$$

$$= \left( \frac{\sqrt{3}}{2}, -\frac{1}{2} \right)$$

$$= \cos \left( \frac{\pi}{3} \right) \sin \left( -\frac{\pi}{3} \right)$$

Then $\theta = 0$, $z = \tan^{-1} \left( \sqrt{3} \right)$.

Let's check $w$, $v$: $r = 1$, $\theta = \frac{\pi}{3}$.

Example: Converse: $\left( -1, -\frac{\sqrt{3}}{2} \right)$.
$r^2 = \sec^2 \theta$

\[
\sin \theta \cos \theta = \frac{1}{1} \Rightarrow \sin \theta \cos \theta = \frac{\sin \theta}{\cos \theta} \cdot \cos \theta = 1
\]

\[
\frac{\sin \theta \cos \theta}{\cos \theta} = 1
\]

\[
\tan \theta = \frac{\sin \theta}{\cos \theta}
\]

\[
\tan \theta = \frac{\sin \theta}{\cos \theta}
\]

\[
x = \frac{1}{1} = 1
\]

\[
y = 1
\]

\[
x = r \cos \theta
\]

\[
y = r \sin \theta
\]

Basic Relations:

\[
x^2 + y^2 = r^2
\]

Example Problem:

Convert to Polar

\[
x = \frac{1}{1}
\]

\[
y = \frac{x}{1}
\]