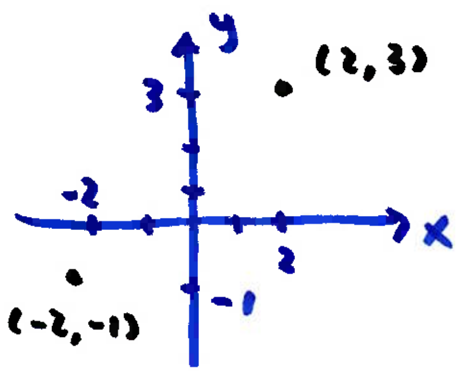
12.2 Polar Coordinates

in Rectangular / Cartesian coordinates, a point is located as (x, y)



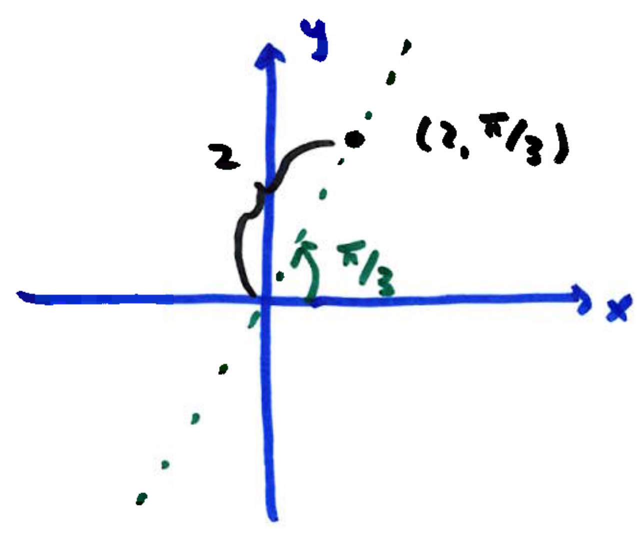
in Polar, a point is located as (r, θ)

r: displacement from origin to the point

θ: angle formed by a line through origin and the point with the positive x-axis

for example, in polar: $(2, \pi/3)$

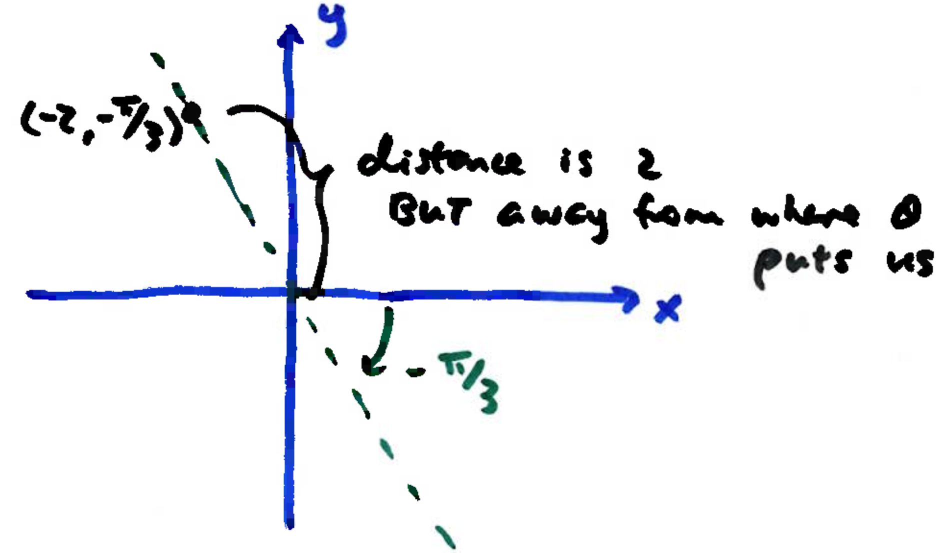
r → 2, θ → $\pi/3$



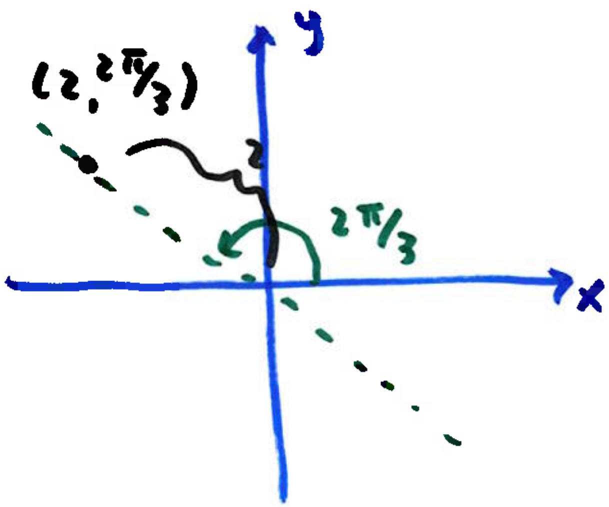
both r and θ can be negative

for example, $(\underbrace{-2}_{r}, \underbrace{-\pi/3}_{\theta})$

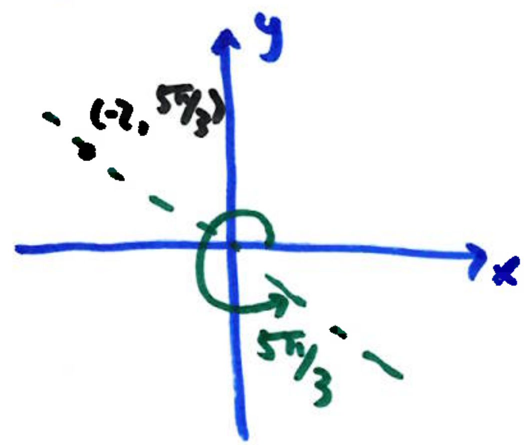
when $r < 0$, go away from the quadrant that θ puts us in



notice $(-2, -\pi/3)$ can be expressed as $(2, 2\pi/3)$ also

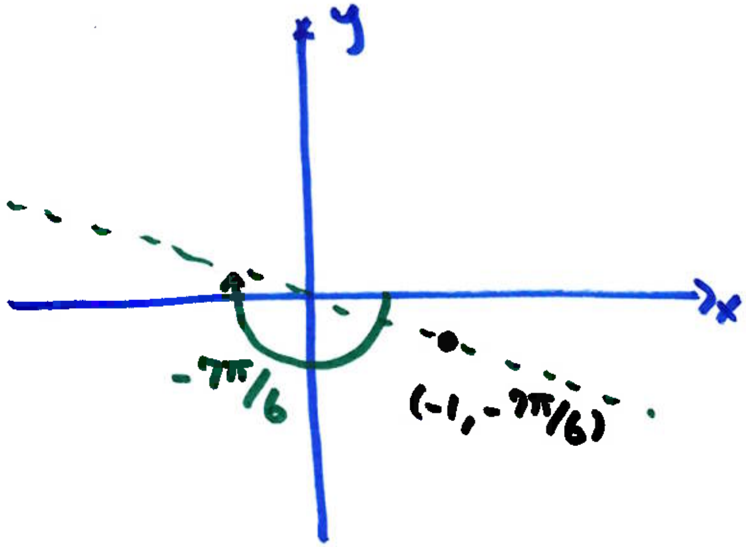


also $(-2, 5\pi/3)$



point : $(-1, -7\pi/6)$

find two other ways to express location in polar

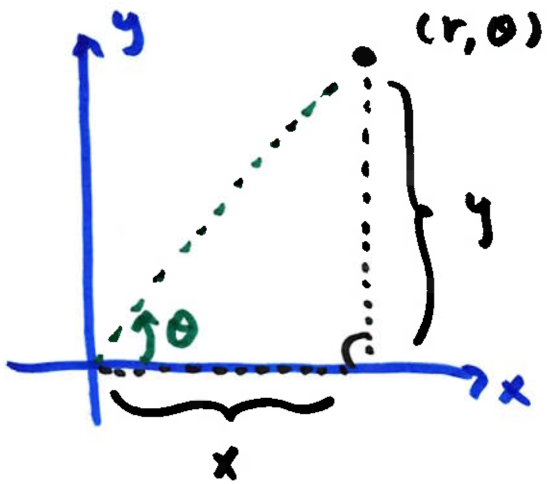


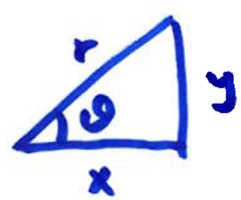
$(1, -\pi/6)$

$(-1, +5\pi/6)$ ← positive

$(1, 11\pi/6)$

Conversion: Polar → Cartesian $(r, \theta) \to (x, y)$





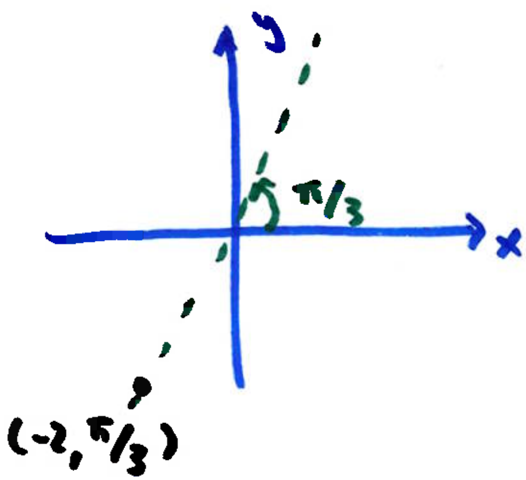
$\cos\theta = \dfrac{x}{r} \to \boxed{x = r\cos\theta}$

$\sin\theta = \dfrac{y}{r} \to \boxed{y = r\sin\theta}$

also, $\boxed{x^2 + y^2 = r^2}$

<u>example</u> In polar: $(-2, \pi/3)$

in Cartesian?



in QIII, we need $x<0$, $y<0$

$x = r\cos\theta = (-2)\cos(\pi/3) = (-2)(\frac{1}{2}) = -1$

$y = r\sin\theta = (-2)\sin(\pi/3) = (-2)(\frac{\sqrt{3}}{2}) = -\sqrt{3}$

check: signs are right

$x^2+y^2 = r^2$?

$(-1)^2 + (-\sqrt{3})^2 = (-2)^2$?

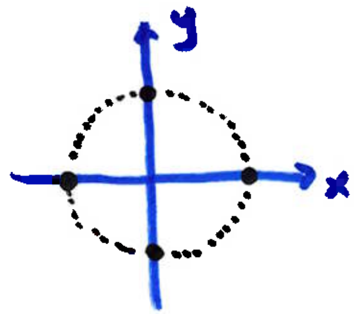
$1 + 3 = 4$ yes

in Cartesian, this point is $(-1, -\sqrt{3})$

equations can be transformed too

<u>example</u> $r = 3$ (polar)

polar is good with circles and circle-like shapes



$r = 3 \rightarrow$ all points with $r = 3$ and $\theta =$ real number

in Cartesian: $r = 3$ from $x^2 + y^2 = r^2$

$r^2 = 9$ $\qquad x^2 + y^2 = 9$

example

$$r = \frac{1}{2\cos\theta + 4\sin\theta}$$

each point (r, θ)

↳ $\frac{1}{2\cos\theta + 4\sin\theta}$

go to Cartesian

$x = r\cos\theta$

$y = r\sin\theta$

$$r(2\cos\theta + 4\sin\theta) = 1$$

$$2\underbrace{r\cos\theta}_{x} + 4\underbrace{r\sin\theta}_{y} = 1$$

$\boxed{2x + 4y = 1}$ line (good in Cartesian, not so good in Polar)

↳ each point (x, y)

$(x, \frac{1}{4} - \frac{1}{2}x)$

Conversion : Cartesian → Polar $(x, y) \to (r, \theta)$

we know $\left.\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}\right\} \longrightarrow \dfrac{y}{x} = \dfrac{r\sin\theta}{r\cos\theta} = \tan\theta$

so, $\tan\theta = \dfrac{y}{x}$ or $\boxed{\theta = \tan^{-1}\left(\dfrac{y}{x}\right)}$

$x^2 + y^2 = r^2$

$$\boxed{\begin{array}{c} r = \sqrt{x^2+y^2} \\ \text{or} \\ r = -\sqrt{x^2+y^2} \end{array}}$$

check quadrants!

__example__ Cartesian : $(-1, -\sqrt{3})$

quadrant check : $\boxed{Q\,\text{III}}$

let's start w/ r : $r^2 = x^2 + y^2$

$r^2 = (-1)^2 + (-\sqrt{3})^2 = 4$

so, $r = 2$ or $r = -2$

then θ : $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = \tan^{-1}\left(\frac{-\sqrt{3}/2}{-1/2}\right)$

$= \frac{\pi}{3}$ or $\frac{4\pi}{3}$

now pair $r = 2, -2$ and $\theta = \pi/3, \frac{4\pi}{3}$ correctly $\rightarrow$ Q III

$(2, 4\pi/3)$, $(-2, \pi/3)$

example ~~xxxx~~ $y = \frac{1}{x}$ convert to Polar

basic relationships: $\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ r^2 &= x^2+y^2 \end{aligned}}$

$$y = \frac{1}{x}$$

$$r\sin\theta = \frac{1}{r\cos\theta}$$

$$(r\sin\theta)(r\cos\theta) = 1$$

$$\boxed{r^2 \sin\theta\cos\theta = 1}$$

or $r^2 = \frac{1}{\sin\theta\cos\theta} = \frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta} = \csc\theta\cdot\sec\theta$

$$\boxed{r^2 = \csc\theta\sec\theta}$$