14.1 Vector-Valued Functions

scalar-valued functions : $f(t) = \underbrace{t^2 + 3}_{\text{scalar output}}$

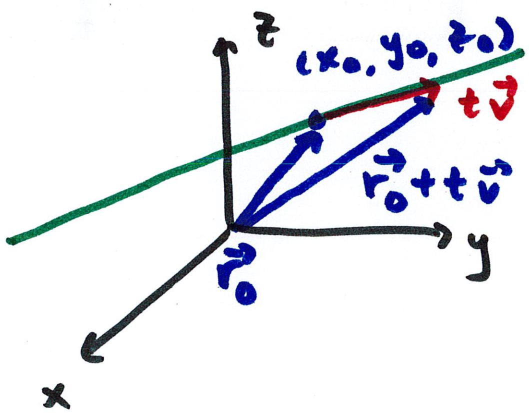
scalar input

vector-valued functions: $\vec{r}(t) = \underbrace{\langle \cos t, \sin t, 0 \rangle}_{\text{vector out}}$

scalar in

you have seen an example already: equation of line

$\vec{r}(t) = \vec{r}_0 + t\vec{v}$

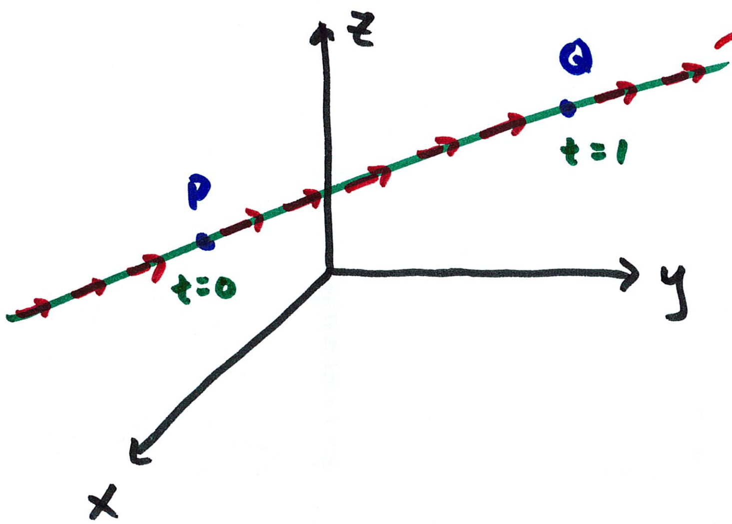


quick example: line through $P(1,2,3)$ and $Q(4,5,6)$

$\vec{r}_0 = \langle 1, 2, 3 \rangle$ $\vec{v} = \vec{PQ} = \langle 3, 3, 3 \rangle$

$\vec{r}(t) = \langle 1, 2, 3 \rangle + t \langle 3, 3, 3 \rangle$

$= \langle 1 + 3t, 2 + 3t, 3 + 3t \rangle$



direction of increasing t

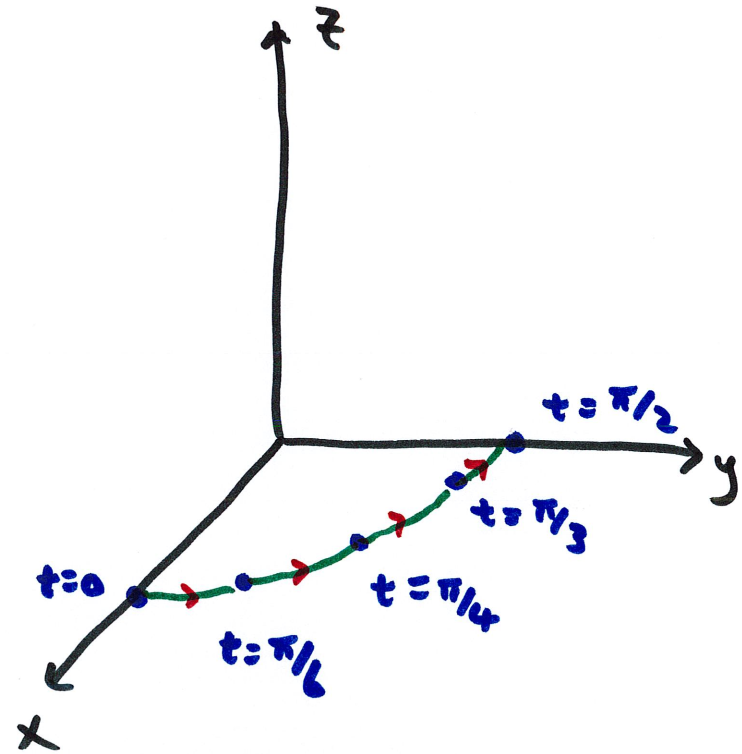
that is called the <u>positive orientation</u>

example $\vec{r}(t) = \langle \cos t, \sin t, 0 \rangle \qquad 0 \le t \le \pi/2$

what is the graph ?

easy thing to do : find points with $x = \cos t$, $y = \sin t$, $z = 0$ by choosing t, then connect them

t	x	y	z
0	1	0	0
$\frac{\pi}{6}$	$\sqrt{3}/2$	$1/2$	0
$\frac{\pi}{4}$	$\sqrt{2}/2$	$\sqrt{2}/2$	0
$\frac{\pi}{3}$	$1/2$	$\sqrt{3}/2$	0
$\frac{\pi}{2}$	0	1	0



positive orientation is counterclockwise when viewed from above another way to visualize: make a connection to surface we know

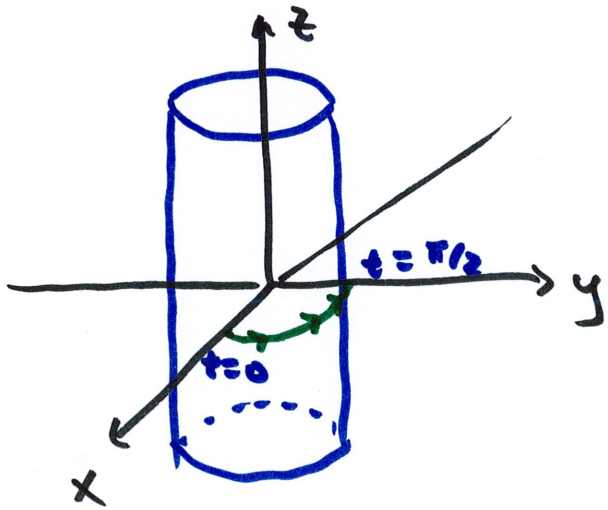
$\vec{r}(t) = \langle \cos t, \sin t, 0 \rangle \qquad 0 \le t \le \pi/2$

$\qquad\qquad x \qquad y \qquad z$

relationship between x, y, z: $\underbrace{x = \cos t, \ y = \sin t}$, $z = 0$

$\cos^2 t + \sin^2 t = 1 \qquad \boxed{x^2 + y^2 = 1}$ in $\mathbb{R}^3$ this is a cylinder

so, $\vec{r}(t) = \langle \cos t, \sin t, 0 \rangle$ is a curve on the cylinder $x^2 + y^2 = 1$ where $z = 0$



example

$\vec{r}(t) = \langle 0, \cos t, 2\sin t \rangle \qquad 0 \le t \le \pi/2$

$x \quad y \quad z$

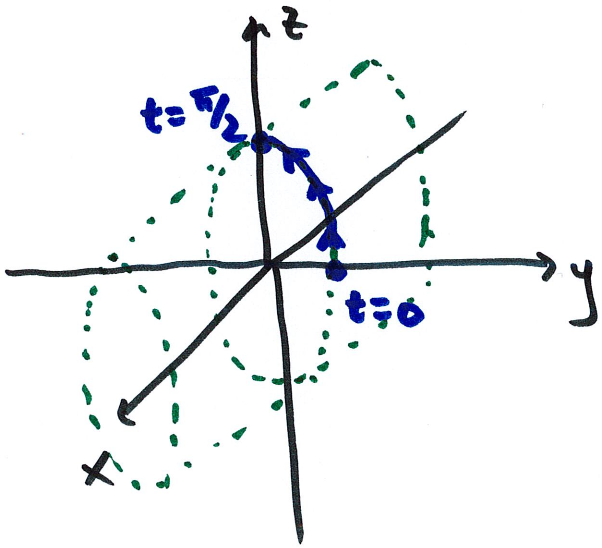
$x = 0, \quad y = \cos t, \quad z = 2\sin t \qquad \cos^2 t + \sin^2 t = 1$

$\cos t = y, \quad \sin t = \frac{1}{2} z \quad \to \quad y^2 + (\frac{1}{2} z^2)^2 = 1$

$$\boxed{y^2 + \frac{z^2}{4} = 1}$$

in $\mathbb{R}^3$ this is a elliptic cylinder

$\vec{r}(t)$ is a portion of that w/ $x = 0$



$t = 0: \quad \vec{r}(0) = \langle 0, 1, 0 \rangle$

$t = \pi/2: \quad \vec{r}(\pi/2) = \langle 0, 0, 2 \rangle$

example

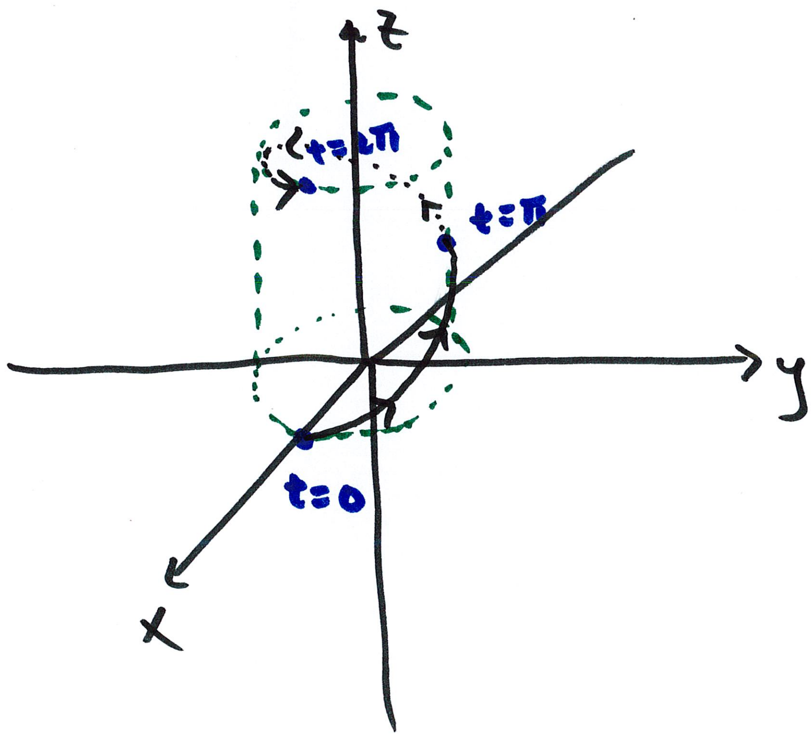
$\vec{r}(t) = \langle \cos t, \sin t, t \rangle \quad 0 \le t \le 2\pi$

$x \quad y \quad z$

$x = \cos t \quad y = \sin t, \quad z = t$

overall shape is still $x^2 + y^2 = 1$ ($z = t$ really doesn't directly fit in)

this curve is still on the cylinder $x^2 + y^2 = 1$ but w/ varying z



$\vec{r}(0) = \langle 1, 0, 0 \rangle$

$\vec{r}(\pi) = \langle -1, 0, \pi \rangle$

$\vec{r}(2\pi) = \langle 1, 0, 2\pi)$

like a spiraling staircase

<u>example</u> $\vec{r}(t) = \langle t\cos t,\ t,\ t\sin t \rangle \qquad 0 \le t \le 2\pi$

$\quad\quad x \quad\quad y \quad\quad z$

$x = t\cos t$

$y = t$

$z = t\sin t$

remember, $\cos^2 t + \sin^2 t = 1$

notice $x^2 + z^2 = t^2\cos^2 t + t^2 \sin^2 t$

$= t^2(\cos^2 t + \sin^2 t)$

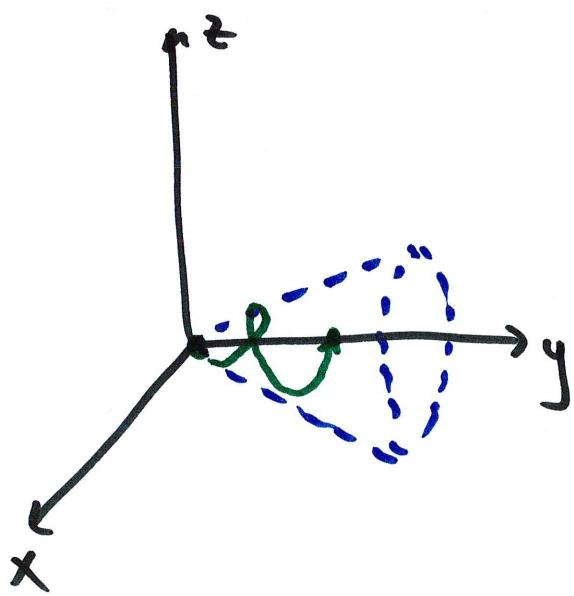
$x^2 + z^2 = t^2$

but $y = t$, so

$$\boxed{x^2 + z^2 = y^2}$$

in $\mathbb{R}^3$ this is a cone

curve is on the cone going toward positive y



Does $\vec{r}(t) = \langle \overset{x}{t\cos t}, \overset{y}{t}, \overset{z}{t\sin t} \rangle$ $\quad 0 \le t \le 2\pi$

intersect the plane $x - z = 0$?

intersection: same x, y, z

so, if they intersect, then at some t, $\quad x = t\cos t,\ y = t,\ z = t\sin t$

must satisfy $x - z = 0$

$x = t\cos t, \quad z = t\sin t$

$x - z = 0 \rightarrow t\cos t - t\sin t = 0$

$t(\cos t - \sin t) = 0 \rightarrow t = 0, \quad \cos t = \sin t$

$\hookrightarrow t = ?$

$t = \pi/4, \ 5\pi/4$

they intersect at $t = 0, \pi/4, 5\pi/4$

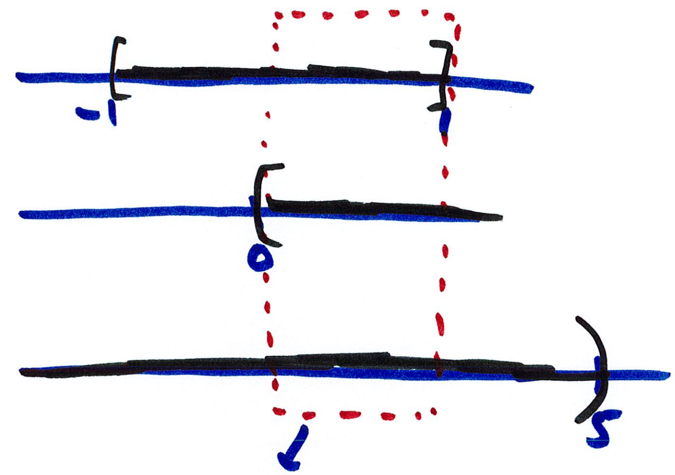
domain of a vector-valued function is the intersection of the domains of all components → where ALL components are defined

<u>example</u> $\vec{r}(t) = \left\langle \sqrt{1-t^2}, \sqrt{t}, \frac{1}{\sqrt{5-t}} \right\rangle$

$\sqrt{1-t^2}$ defined on $[-1, 1]$

$\sqrt{t}$ " " $[0, \infty)$

$\frac{1}{\sqrt{5-t}}$ " " $(-\infty, 5)$



intersection is

$[0, 1]$

this is the domain of $\vec{r}(t)$

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