For the vectors **u** = ⟨ − 1,3⟩ and **v** = ⟨ − 3,2⟩, calculate $\text{proj}_{\mathbf{v}}\mathbf{u}$ and $\text{scal}_{\mathbf{v}}\mathbf{u}$.

$\text{proj}_{\mathbf{v}}\mathbf{u} = \langle \square, \square \rangle$

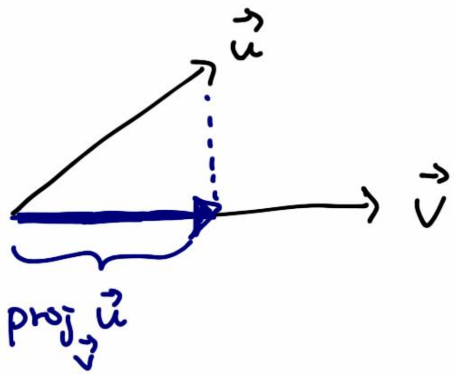
$\text{scal}_{\mathbf{v}}\mathbf{u} = \square$

(Type an exact answer, using radicals as needed.)

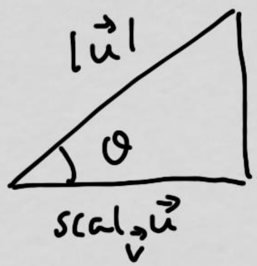
$\text{proj}_{\vec{v}}\vec{u} \rightarrow$ vector projection

$\text{scal}_{\vec{v}}\vec{u} \rightarrow$ scalar projection

$= |\text{proj}_{\vec{v}}\vec{u}|$



it's easier to find the scalar projection first



from trig, $\cos\theta = \dfrac{\text{scal}_{\vec{v}}\vec{u}}{|\vec{u}|}$ but we also know

$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$$

$$\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{\text{scal}_{\vec{v}}\vec{u}}{|\vec{u}|}$$

$$\text{scal}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|} = \frac{\langle -1, 3\rangle \cdot \langle -3, 2\rangle}{|\langle -3, 2\rangle|} = \boxed{\frac{9}{\sqrt{13}}}$$

the vector projection $\mathrm{proj}_{\vec{v}}\vec{u}$ is the scalcer projection $\mathrm{scal}_{\vec{v}}\vec{u}$ times a unit vector in the direction of $\vec{v}$

so, $\mathrm{proj}_{\vec{v}}\vec{u} = \mathrm{scal}_{\vec{v}}\vec{u}\,\dfrac{\vec{v}}{|\vec{v}|} = \dfrac{9}{\sqrt{13}}\,\dfrac{\langle -3, 2\rangle}{\sqrt{13}} = \boxed{\dfrac{9}{13}\langle -3, 2\rangle}$

A line l passes through the point $(1, 1, -2)$ and is perpendicular to the plane $x + y - 2z = 8$. At what point does this line intersect with the yz-plane?

A. (0, 1, 1)

B. (0, −1, 0)

C. (0, 1, −1)

D. (0, 0, 0)

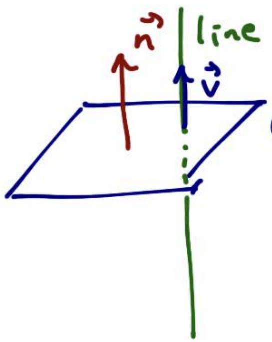
E. (0, 1, 0)

equation of line: need point, need direction vector

point: $(1,1,-2)$

direction vector = normal vector of plane $x+y-2z=8$

because the line is perpendicular to the plane



$\hookrightarrow$ $\underbrace{x+y-2z=8}$

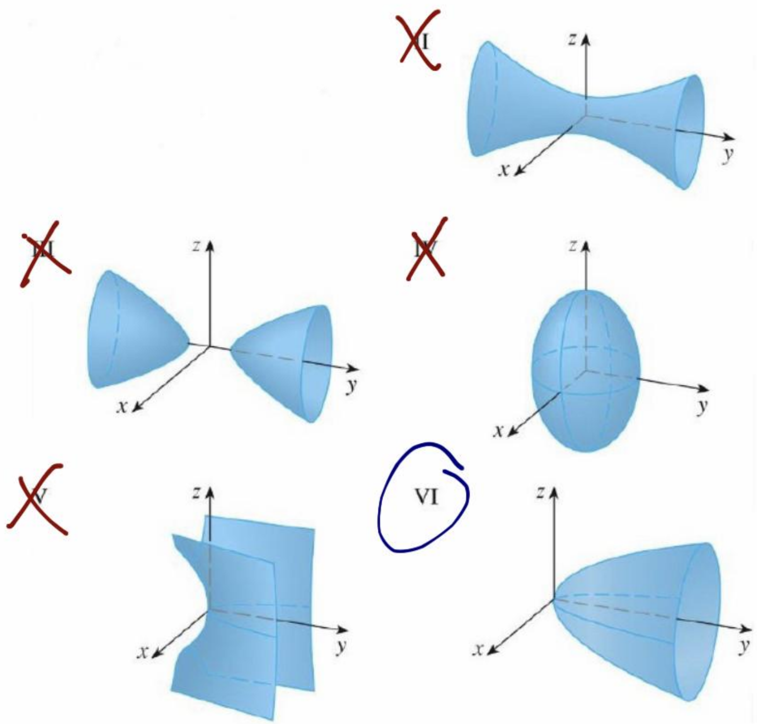
normal vector: $\langle 1, 1, -2\rangle$ (coefficients of x, y, z)

equation of line: $\vec{r}(t) = \underbrace{\langle 1,1,-2\rangle}_{\text{vector to point on the line}} + t\underbrace{\langle 1,1,-2\rangle}_{\text{direction vector}} = \langle 1+t, 1+t, -2-2t\rangle$

it intersects with yz-plane at $x=0 \longrightarrow 1+t=0 \rightarrow t=-1$

$\vec{r}(-1) = \langle 0, 0, 0\rangle \rightarrow$ intersects yz-plane at $\boxed{(0,0,0)}$

3. Which of the following surfaces is NOT represented by any of the equations shown?



$$x^2 - y^2 + z^2 = 1$$
$$-x^2 + y^2 - z^2 = 1$$
$$9x^2 + 4y^2 + z^2 = 1$$
$$y = x^2 - z^2$$

A. II
B. III
C. IV
D. V
E. VI

equation 1: $x^2 - y^2 + z^2 = 1$

xy-trace: $x^2 - y^2 = 1$ hyperbolas with vertices on x-axis
yz-trace: $-y^2 + z^2 = 1$ hyperbolas with vertices on z-axis
xz-trace: $x^2 + z^2 = 1$ circle radius 1
this must be II

equation 2: $-x^2 + y^2 - z^2 = 1$

xy-trace: $-x^2 + y^2 = 1$ hyperbolas with vertices on y-axis
yz-trace: $y^2 - z^2 = 1$ hyperbolas with vertices on y-axis
xz-trace: $-x^2 - z^2 = 1$ none, no intersection w/ xz-plane
this must be III

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A particle is moving with acceleration

$$\vec{\mathbf{a}}(t) = \langle 6,\ 6t,\ 0\rangle.$$

If at time $t = 1$, the particle has position $\vec{\mathbf{r}}(1) = \langle 2, 1, 2\rangle$, and, at time $t = 0$ it has velocity $\vec{\mathbf{v}}(0) = \langle 0, 0, 1\rangle$, compute $|\vec{\mathbf{r}}(2)|$, the magnitude of the position vector at $t = 2$.

A. $2\sqrt{53}$

B. $3\sqrt{21}$

C. $\sqrt{194}$

D. $\sqrt{293}$

E. $\sqrt{57}$

velocity: $\vec{v}(t) = \int \vec{a}(t)\,dt = \int \langle 6, 6t, 0\rangle\,dt = \langle 6t + C_1,\ 3t^2 + C_2,\ C_3\rangle$

to find C_1, C_2, C_3, use the given $\vec{v}(0) = \langle 0, 0, 1\rangle$

must match

from $\vec{v}(t) = \langle 6t + C_1, 3t^2 + C_2, C_3\rangle$, we get $\vec{v}(0) = \langle C_1, C_2, C_3\rangle$

so, $C_1 = 0$, $C_2 = 0$, $C_3 = 1$

so, $\boxed{\vec{v}(t) = \langle 6t,\ 3t^2,\ 1\rangle}$

position: $\vec{r}(t) = \int \vec{v}(t)\,dt = \langle 3t^2 + d_1,\ t^3 + d_2,\ t + d_3\rangle$

at $t = 1$, $\vec{r}(1) = \langle 3 + d_1,\ 1 + d_2,\ 1 + d_3\rangle$ which must match the given $\vec{r}(1) = \langle 2, 1, 2\rangle$ so, $d_1 = -1$, $d_2 = 0$, $d_3 = 1$

$\vec{r}(t) = \langle 3t^2 - 1,\ t^3,\ t + 1\rangle$ $\quad \vec{r}(2) = \langle 11, 8, 3\rangle$ $\quad |\vec{r}(2)| = \sqrt{121 + 64 + 9}$

$= \sqrt{194}$

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Question Help

Determine whether the following curve uses arc length as a parameter. If not, find a description that uses arc length as a parameter.

$\mathbf{r}(t) = \langle 8\cos t, 8\sin t\rangle$, for $0 \le t \le \pi$

Choose the correct answer below.

A. $\mathbf{r}_1(s) = \left\langle 8\cos\frac{s}{8}, 8\sin\frac{s}{8}\right\rangle$, for $0 \le s \le 8\pi$

B. $\mathbf{r}_1(s) = \left\langle \cos\frac{s}{8}, \sin\frac{s}{8}\right\rangle$, for $0 \le s \le \pi$

C. $\mathbf{r}_1(s) = \langle \cos s, \sin s\rangle$, for $0 \le s \le \frac{\pi}{8}$

D. $\mathbf{r}_1(s) = \langle 8\cos s, 8\sin s\rangle$, for $0 \le s \le 8\pi$

E. The given curve uses arc length as a parameter.

$\boxed{\text{arc length: } s(t) = \int_a^t |\vec{r}\,'(u)|\,du \qquad t \text{ starts at } a}$

if t is arc length, then $\frac{ds}{dt} = 1$ (since $s = t - a$)

$$\frac{d}{dt}s(t) = \frac{d}{dt}\int_a^t |\vec{r}\,'(u)|\,du = |\vec{r}\,'(t)| = 1$$

first, check if $|\vec{r}\,'|$ is 1 here

$\vec{r}(t) = \langle 8\cos t, 8\sin t\rangle \qquad \vec{r}\,'(t) = \langle -8\sin t, 8\cos t\rangle$

$|\vec{r}\,'(t)| = \sqrt{64\sin^2 t + 64\cos^2 t} = \sqrt{64} = 8 \neq 1$ so t is not arc length

from $s(t) = \int_a^t |\vec{r}\,'(u)|\,du \rightarrow s = \int_0^t 8\,du = 8t \rightarrow t = \frac{s}{8}$

equation 3: $9x^2+4y^2+z^2=1$ ellipsoid this must be $\overline{\underline{\text{IV}}}$

equation 4: $y = x^2 - z^2$

xy-trace: $y = x^2$ parabola opening toward positive y

yz-trace: $y = -z^2$ " " " negative y

xz-trace: $x^2 - z^2 = 0$ asymptotes of hyperbolas

(at $y \neq 0$, xz slices must be hyperbolas)

so this must be $\overline{\underline{\text{V}}}$

(can't be $\overline{\underline{\text{VI}}}$ because $\overline{\underline{\text{VI}}}$ has circles at $y = \text{constant}$)

So, $\overline{\underline{\text{IV}}}$ must be the surface whose equation is missing

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$\vec{r}(t) = \langle 8\cos t,\ 8\sin t \rangle \qquad 0 \le t \le \pi$

replace with $s = 8t$ or $t = \frac{s}{8}$

$$\boxed{\vec{r}(s) = \left\langle 8\cos\left(\frac{s}{8}\right),\ 8\sin\left(\frac{s}{8}\right) \right\rangle \qquad 0 \le s \le 8\pi}$$

at $t=0$

$s = 8 \cdot 0 = 0$

at $t = \pi$

$s = 8 \cdot \pi = 8\pi$

6. The curvature of the curve $\vec{r}(t) = < 9\cos t, 9\sin t >$ at $t = \pi$ is

A. 9

B. 3

C. $\frac{1}{9}$

D. $\frac{1}{3}$

E. 1

curvature : $\kappa = \dfrac{|\vec{T}'|}{|\vec{r}'|}$ → measures the change in the unit tangent vector T

T is a unit vector, its magnitude cannot change, so $|\vec{T}'|$ must due to change in direction

a line has no change in $\vec{T}$, so $\kappa = 0$

a very curve $\vec{r}$ has frequent change in $\vec{T}$ so high κ

$$\vec{T} = \frac{\vec{r}'}{|\vec{r}'|} = \frac{\langle -9\sin t, 9\cos t \rangle}{\sqrt{81\sin^2 t + 81\cos^2 t}}$$

$$= \frac{\langle -9\sin t, 9\cos t \rangle}{9} = \langle -\sin t, \cos t \rangle$$

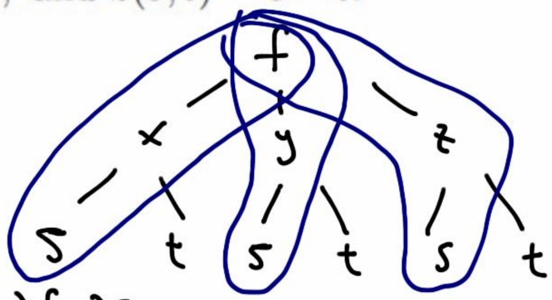
$\vec{T}' = \langle -\cos t, -\sin t \rangle \qquad |\vec{T}'| = 1$

so, $\kappa = \dfrac{|\vec{T}'|}{|\vec{r}'|} = \dfrac{1}{9}$ (at any t)

8. Let $f(x, y, z) = e^{x+y-z}$ and suppose that

$$x(s,t) = ts, \quad y(s,t) = 2s - 2t, \text{ and } z(s,t) = s - t.$$

Compute $\dfrac{\partial f}{\partial s} - 2\dfrac{\partial f}{\partial t}$ when $s = 0$ and $t = -1$.



A. $-3e$

B. $3e$

C. $5e$

D. $2e$

E. $-2e$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial s}$$

$$= (e^{x+y-z})(t) + (e^{x+y-z})(2) - (e^{x+y-z})(1)$$

at $s=0,\ t=-1,\quad x=0,\ y=2,\ z=1$ so $e^{x+y-z} = e$

and $\dfrac{\partial f}{\partial s} = -e + 2e - e = 0$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial t}$$

$$= (e^{x+y-z})(s) + (e^{x+y-z})(-2) - (e^{x+y-z})(-1)$$

at $s=0,\ t=-1$

$$\frac{\partial f}{\partial t} = (e)(0) + (e)(-2) - (e)(-1) = -e$$

so, $\dfrac{\partial f}{\partial s} - 2\dfrac{\partial f}{\partial t} = 0 - 2(-e) = 2e$

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7. Evaluate

$$\lim_{(x,y)\to(0,0)} \frac{y^4 - x^4}{x^2 + y^2}\, e^{x^2+y^2}$$

A. 0

B. 1

C. -1

D. e^2

E. The limit does not exist

limit exists only if ALL possible paths lead to same limit

if possible, try to evaluate limit w/o assuming path to avoid evaluating from many paths

$$\lim_{(x,y)\to(0,0)} \frac{y^4 - x^4}{x^2+y^2} e^{x^2+y^2}$$

$$= \lim_{(x,y)\to(0,0)} \frac{(y^2-x^2)\cancel{(y^2+x^2)}}{\cancel{x^2+y^2}} e^{x^2+y^2}$$

$$= \lim_{(x,y)\to(0,0)} (y^2-x^2)e^{x^2+y^2} = 0$$ because $y^2-x^2 \to 0$ as $(x,y)\to(0,0)$

this is evaluated w/o assuming a path

limit is 0

this also means if we define $f(0,0)=0$, then $f(x,y)$ will be continuous for all (x,y) in $\mathbb{R}^2$

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12. Find the directional derivative of the function $f(x,y,z) = \ln(1 + x^2 + y^2 + e^z)$ in the direction of the vector $\mathbf{u} = \langle 1, 2, 3 \rangle$ at the point $P(1,1,0)$.

A. $\dfrac{9}{4\sqrt{14}}$

B. $\dfrac{15}{2\sqrt{14}}$

C. $\dfrac{16}{7\sqrt{14}}$

D. $\dfrac{9}{2\sqrt{14}}$

E. $\dfrac{18}{7\sqrt{14}}$

directional derivative $= \underbrace{\vec{\nabla} f}_{\text{gradient}} \cdot \vec{u}$ ← unit vector giving direction

here, $\vec{u} = \langle 1, 2, 3 \rangle$ is NOT unit vector

let $\vec{v} = \dfrac{\vec{u}}{|\vec{u}|} = \dfrac{\langle 1, 2, 3 \rangle}{\sqrt{14}} = \left\langle \dfrac{1}{\sqrt{14}}, \dfrac{2}{\sqrt{14}}, \dfrac{3}{\sqrt{14}} \right\rangle$

$$\vec{\nabla} f = \langle f_x, f_y, f_z \rangle = \left\langle \frac{2x}{1+x^2+y^2+e^z}, \frac{2y}{1+x^2+y^2+e^z}, \frac{e^z}{1+x^2+y^2+e^z} \right\rangle$$

$$\vec{\nabla} f(1,1,0) = \left\langle \frac{2}{4}, \frac{2}{4}, \frac{1}{4} \right\rangle = \left\langle \frac{1}{2}, \frac{1}{2}, \frac{1}{4} \right\rangle$$

$$D_{\vec{v}} f = \vec{\nabla} f \cdot \vec{v} = \left\langle \frac{1}{2}, \frac{1}{2}, \frac{1}{4} \right\rangle \cdot \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$$

$$= \frac{1}{2\sqrt{14}} + \frac{2}{2\sqrt{14}} + \frac{3}{4\sqrt{14}} = \frac{2+4+3}{4\sqrt{14}} = \frac{9}{4\sqrt{14}}$$

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Question Help

a. Find the linear approximation to the function f at the point (a,b).
b. Use part (a) to estimate the given function value.

$f(x,y) = (x+y)e^{xy}$; $(a,b) = (4,0)$; estimate $f(3.97,0.06)$.

a. $L(x,y) =$ x + 17y

b. $L(3.97,0.06) =$ 5.0 (Type an integer or decimal rounded to the nearest tenth as needed.)

evaluate at (x_0, y_0): starting values of x, y

linear approximation / tangent plane: $z - z_0 = f_x(x - x_0) + f_y(y - y_0)$

$f_x = (x+y)e^{xy}y + e^{xy}(1) = e^{xy}(y(x+y)+1)$

$f_y = (x+y)e^{xy}x + e^{xy}(1) = e^{xy}(x(x+y)+1)$

at $(4,0)$ $\quad f_x = 1 \qquad f_y = 16+1 = 17$

$f(4,0) = (4+0)e^{4\cdot 0} = 4$

so, linear approx. is: $z = z_0 + f_x(x-x_0) + f_y(y-y_0)$

$z = 4 + (1)(x-4) + 17(y-0)$

$= \boxed{x + 17y \approx f(x,y) = L(x,y)}$

$L(3.97, 0.06) = 3.97 + 17(0.06) = 4.99 = 5$ (to the nearest tenth)