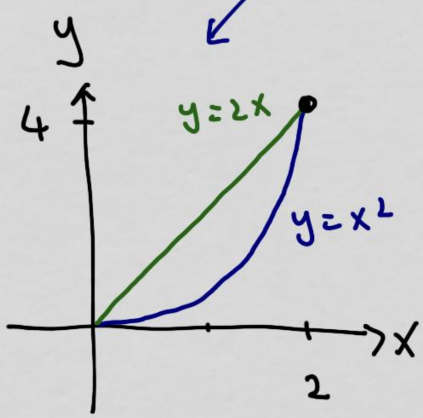
Find a and b for the correct interchange of order of integration:

$$\int_0^2 \int_{x^2}^{2x} f(x,y)dydx = \int_0^4 \int_a^b f(x,y)dxdy.$$

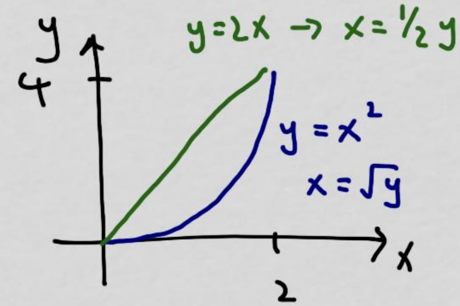
A. $a = y^2, b = 2y$ B. $a = \frac{y}{2}, b = \sqrt{y}$ C. $a = \frac{y}{2}, b = y$

D. $a = \sqrt{y}, b = \frac{y}{2}$ E. cannot be done without explicit knowledge of $f(x,y)$.

$$\underbrace{\int_0^2 \int_{x^2}^{2x} f(x,y)\,dy\,dx}_{\substack{0 \le x \le 2 \\ x^2 \le y \le 2x}} = \int_0^4 \int_a^b f(x,y)\,dx\,dy$$



Type I region
(x bounded
by constants)



Type II

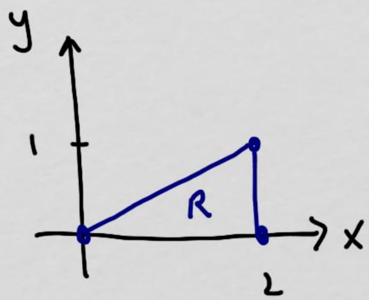
$$0 \le y \le 4$$

$$\underbrace{\tfrac{1}{2}y}_{\text{left curve}} \le x \le \underbrace{\sqrt{y}}_{\text{right curve}}$$

$$\int_0^4 \int_{\frac{y}{2}}^{\sqrt{y}} f(x,y)\,dx\,dy$$

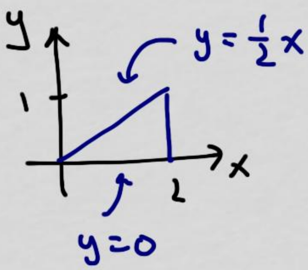
Evaluate the double integral $\iint_R y dA$, where R is the region of the (x, y)-plane inside the triangle with vertices $(0, 0)$, $(2, 0)$ and $(2, 1)$.

A. 2 B. $\frac{8}{3}$ C. $\frac{2}{3}$ D. 1 E. $\frac{1}{3}$.



choices : Type I or Type II

Type I :



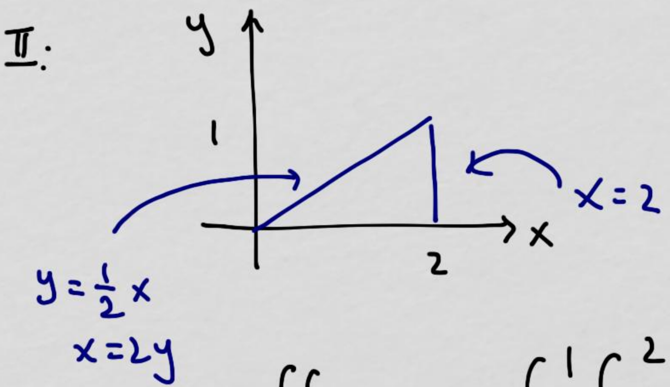
$0 \le x \le 2$

$0 \le y \le \frac{1}{2}x$

$$\iint_R y\,dA = \int_0^2 \int_0^{\frac{1}{2}x} y\,dy\,dx = \int_0^2 \frac{1}{2}y^2 \Big|_0^{\frac{1}{2}x} dx = \int_0^2 \frac{1}{2}\left(\frac{1}{2}x\right)^2 dx$$

$$= \frac{1}{8}\int_0^2 x^2\,dx = \frac{1}{24}x^3\Big|_0^2 = \frac{1}{3}$$

Type II:



$$0 \le y \le 1$$

$$2y \le x \le 2$$

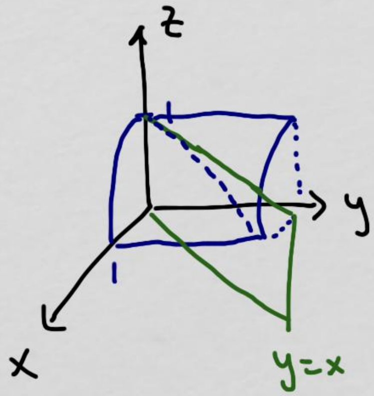
$$\iint_R y\, dA = \int_0^1 \int_{2y}^2 y\, dx\, dy = \int_0^1 yx \Big|_{x=2y}^{x=2} dy$$

$$= \int_0^1 (2y - 2y^2)\, dy = y^2 - \frac{2}{3}y^3 \Big|_0^1 = 1 - \frac{2}{3} = \frac{1}{3}$$

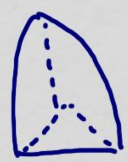
The volume of the solid region in the first octant bounded above by the parabolic sheet $z = 1 - x^2$, below by the xy plane, and on the sides by the planes $y = 0$ and $y = x$ is given by the double integral

A. $\int_0^1 \int_0^x (1 - x^2)\,dy\,dx$ B. $\int_0^1 \int_0^{1-x^2} x\,dy\,dx$ C. $\int_{-1}^1 \int_{-x}^x (1 - x^2)\,dy\,dx$

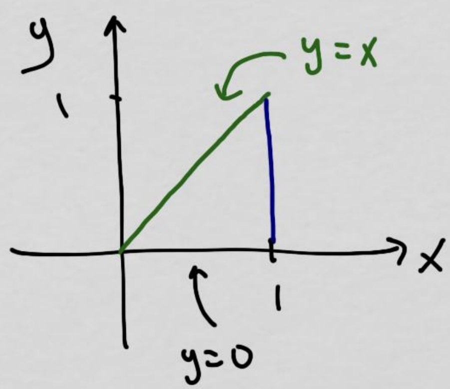
D. $\int_0^1 \int_x^0 (1 - x^2)\,dy\,dx$ E. $\int_0^1 \int_x^{1-x^2} dy\,dx$.



$z = 1 - x^2 \rightarrow$ parabolas for xz slices



all responses are $dy\,dx \rightarrow$ xy-plane as the "floor"



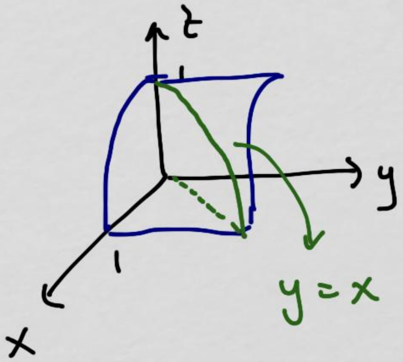
$0 \leq x \leq 1$

$0 \leq y \leq x$

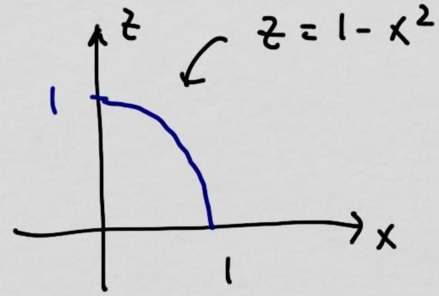
integrate the height ("ceiling") $z = 1 - x^2$

$$\int_0^1 \int_0^x (1 - x^2)\,dy\,dx$$

what if we were asked to use the order $dz\,dx$?



projection onto xz-plane



$0 \le x \le 1$

$0 \le z \le 1 - x^2$

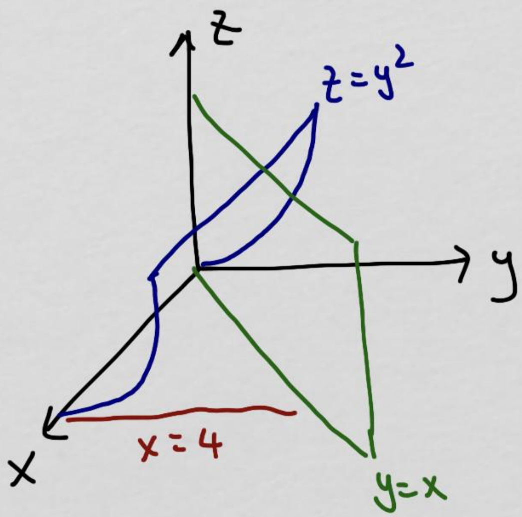
integrate the "ceiling" above:

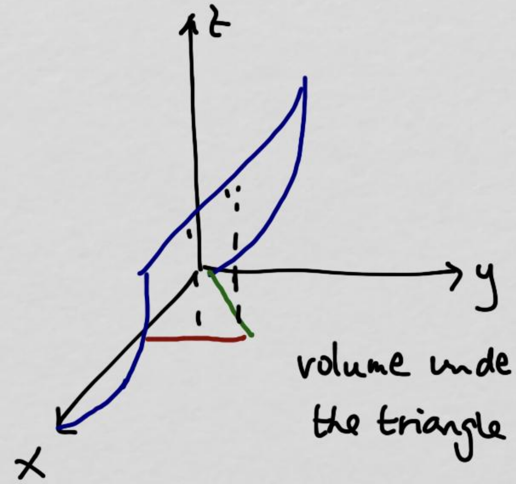
$y = x$

$$\int_0^1 \int_0^{1-x^2} x \, dz \, dx$$

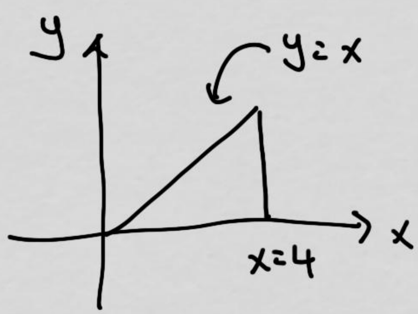
A solid region in the first octant is bounded by the surfaces $z = y^2$, $y = x$, $y = 0$, $z = 0$ and $x = 4$. The volume of the region is

A. 64 B. $\frac{64}{3}$ C. $\frac{32}{3}$ D. 32 E. $\frac{16}{3}$.





volume under $z=y^2$ above the triangle on the xy-plane



$$\left.\begin{array}{l} 0 \le x \le 4 \\ 0 \le y \le x \end{array}\right\} \text{"floor"}$$

$0 \le z \le y^2$ "ceiling"

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$$\text{volume} = \int_0^4 \int_0^x \int_0^{y^2} \underbrace{dz\,dy\,dx}_{dV} = \underbrace{\int_0^4 \int_0^x y^2\,dy\,dx}$$

could have started with this double integral instead of the triple integral

$$= \int_0^4 \frac{1}{3} y^3 \Big|_0^x \, dx = \int_0^4 \frac{1}{3} x^3\,dx = \frac{1}{12} x^4 \Big|_0^4$$

$$= \frac{256}{12} = \frac{64}{3}$$

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An object occupies the region bounded above by the sphere $x^2+y^2+z^2=32$ and below by the upper nappe of the cone $z^2=x^2+y^2$. The mass density at any point of the object is equal to its distance from the xy plane. Set up a triple integral in rectangular coordinates for the total mass m of the object.

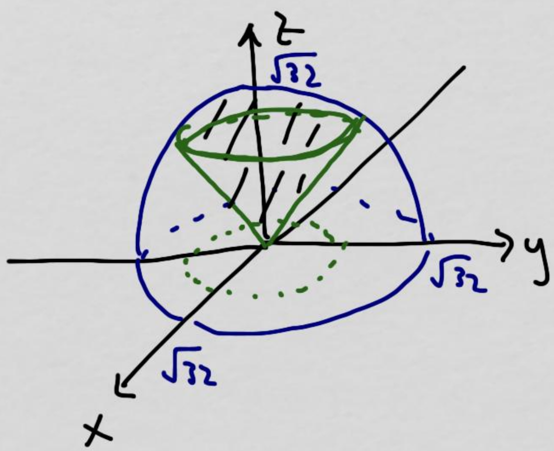
A. $\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}}\int_{-\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$

B. $\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$

C. $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{-\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$

D. $\int_{0}^{4}\int_{0}^{\sqrt{16-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z\,dz\,dy\,dx$

E. $\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} xy\,dz\,dy\,dx.$



density : $\rho(x,y) = z$ (distance from xy-plane)

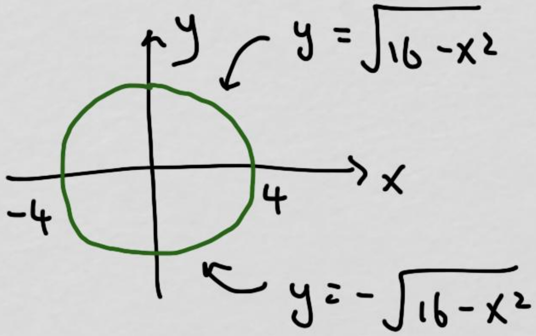
project onto the xy-plane ($dy\,dx$ in the order after dz)

intersection of $x^2+y^2+z^2=32$ and $z^2=x^2+y^2$ is projected onto xy-plane

$$x^2+y^2+\underbrace{(x^2+y^2)}_{z^2=x^2+y^2}=32$$

$$2(x^2+y^2)=32$$

$x^2+y^2=16 \rightarrow$ circle of radius 4



$$-4 \le x \le 4$$

$$-\sqrt{16-x^2} \le y \le \sqrt{16-x^2}$$

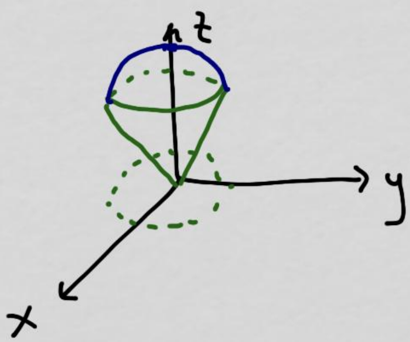
$$\sqrt{x^2+y^2} \le z \le \sqrt{32-x^2-y^2}$$

(cone) (sphere)

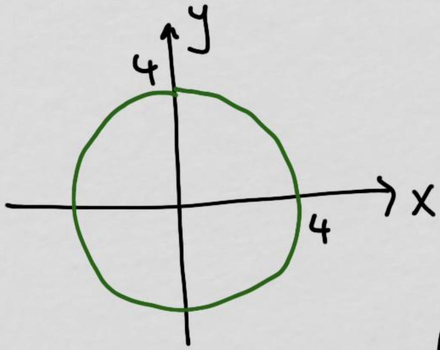
mass: $\displaystyle\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z \, dz\, dy\, dx$

Cartesian is clearly not very good.

try cylindrical:



on xy-plane:



$0 \le \theta \le 2\pi$

$0 \le r \le 4$

$\sqrt{x^2+y^2} \le z \le \sqrt{32-x^2-y^2}$

(cone) (sphere)

$\sqrt{r^2} = r$ $\qquad$ $\sqrt{32-(x^2+y^2)} = \sqrt{32-r^2}$

in cylindrical: $\displaystyle\int_0^{2\pi}\int_0^4\int_r^{\sqrt{32-r^2}} z \; r\,dz\,dr\,d\theta$

density

part of dV in cylindrical

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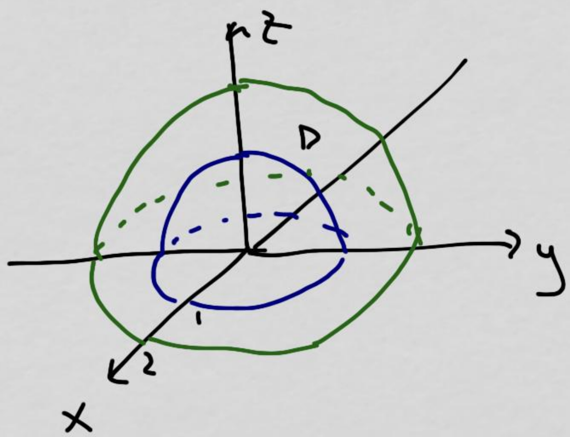
If D is the solid region above the xy-plane that is between $z = \sqrt{4 - x^2 - y^2}$ and $z = \sqrt{1 - x^2 - y^2}$, then $\iiint_D \sqrt{x^2 + y^2 + z^2}\, dV =$

A. $\frac{14\pi}{3}$ B. $\frac{16\pi}{3}$ C. $\frac{15\pi}{2}$ D. 8π E. 15π.

$\underbrace{z = \sqrt{4 - x^2 - y^2}}_{\text{upper half of sphere of radius 2}} \rightarrow z^2 = 4 - x^2 - y^2 \rightarrow x^2 + y^2 + z^2 = 4$

$z = \sqrt{1 - x^2 - y^2}$ is the under half of sphere of radius 1

we are dealing with two spheres $\longrightarrow$ spherical coordinates



D is between them

$0 \le \theta \le 2\pi$

$0 \le \phi \le \pi/2$ (xy-plane)

$1 \le \rho \le 2$

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$$\iiint_D \sqrt{x^2+y^2+z^2}\, dV$$

in spherical is $\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$\sqrt{x^2+y^2+z^2}$ in spherical is $\sqrt{\rho^2} = \rho$

$$\int_0^{2\pi} \int_0^{\pi/2} \int_1^2 \rho \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\pi/2} \frac{1}{4}\rho^4 \Big|_1^2 \sin\phi \, d\phi \, d\theta = \int_0^{2\pi} \int_0^{\pi/2} \frac{15}{4} \sin\phi \, d\phi \, d\theta$$

$$= \frac{15}{4} \int_0^{2\pi} d\theta = \frac{15}{4} \cdot 2\pi = \frac{15\pi}{2}$$

$$\left(\int_0^{\pi/2} \sin\phi \, d\phi = 1 \right)$$

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Find the work done by $\mathbf{F} = \frac{y}{z}\mathbf{i} + \frac{x}{z}\mathbf{j} + \frac{x}{y}\mathbf{k}$ over the curve C in the direction of increasing t.

C: $\mathbf{r}(t) = t^9\mathbf{i} + t^6\mathbf{j} + t^5\mathbf{k}$, $0 \le t \le 1$

A. $W = 20$

B. $W = 1$

C. $W = 0$

D. $W = \frac{17}{8}$

line integral $\int_c \vec{F} \cdot d\vec{r} = \int_c \vec{F} \cdot \vec{r}\,' \, dt$

if $\vec{F}$ is force vector field then $\int_c \vec{F} \cdot d\vec{r}$ gives us work done in moving something along C

$\vec{r}(t) = \langle t^9, t^6, t^5 \rangle \quad 0 \le t \le 1$

$\vec{r}\,' = \langle 9t^8, 6t^5, 5t^4 \rangle$

$$\int_c \vec{F} \cdot d\vec{r} = \int_c \vec{F} \cdot \vec{r}\,' dt = \int_0^1 \Big\langle \underbrace{\frac{t^6}{t^5}, \frac{t^9}{t^5}, \frac{t^9}{t^6}}_{\vec{F} \text{ using } x,y,z \text{ from } \vec{r}} \Big\rangle \cdot \langle 9t^8, 6t^5, 5t^4 \rangle \, dt$$

$$\int_0^1 \langle t, t^4, t^3 \rangle \cdot \langle 9t^8, 6t^5, 5t^4 \rangle \, dt$$

$$= \int_0^1 (9t^9 + 6t^9 + 5t^7)\,dt = \frac{15}{10} t^{10} + \frac{5}{8} t^7 \Big|_0^1$$

$$= \frac{15}{10} + \frac{5}{8} = \frac{3}{2} + \frac{5}{8} = \frac{17}{8}$$