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13.6 Quadric Surfaces (part 1)

equation in terms of $x, y, z \rightarrow$ surfaces

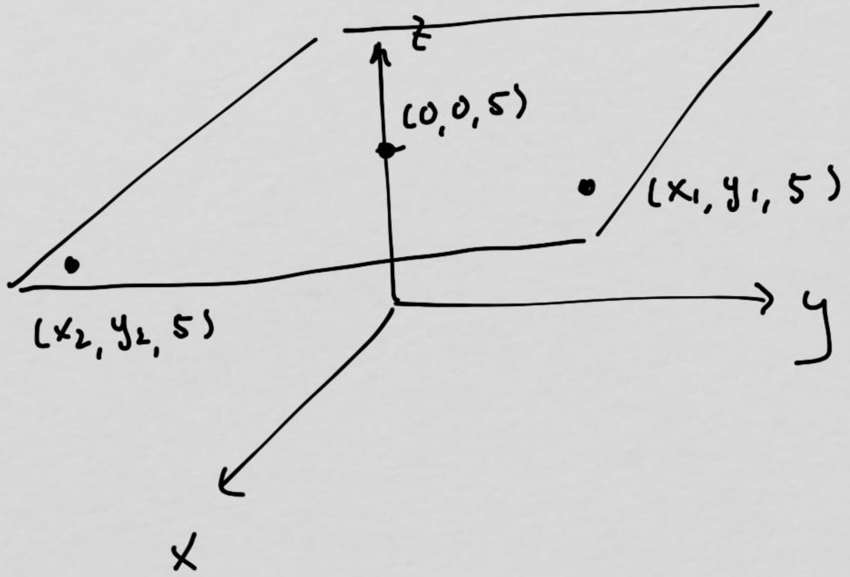
for example, $x^2+y^2+z^2=1$ is a sphere of radius 1 and center at $(0,0,0)$

and $(x-2)+2(y+3)-3(z+4)=0$ is a plane with normal vector $\langle 1, 2, -3 \rangle$ and goes through $(2,-3,-4)$

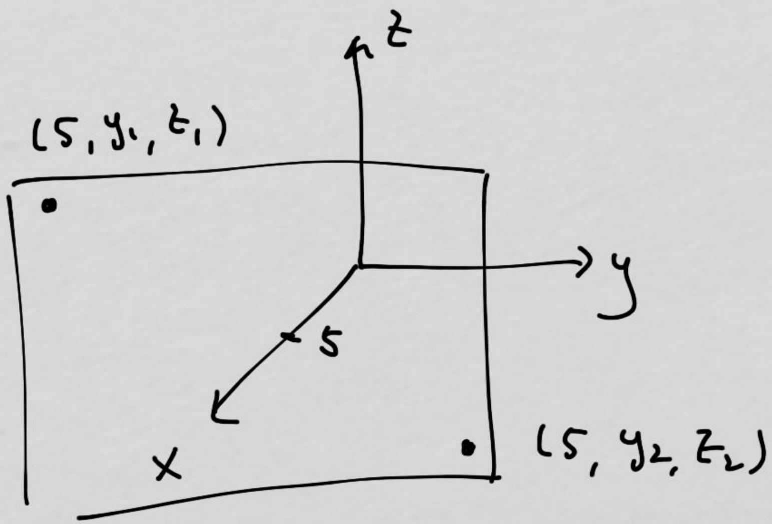
sometimes one or more of the variables is missing in the equation when that happens the missing variable is a "free variable" and can take on all values in its domain for example, $z=5$ in $\mathbb{R}^3$ has x and y missing

this means $-\infty<x<\infty$, $-\infty<y<\infty$, and the surface is made up of all points of the form $(x, y, 5)$

any real number



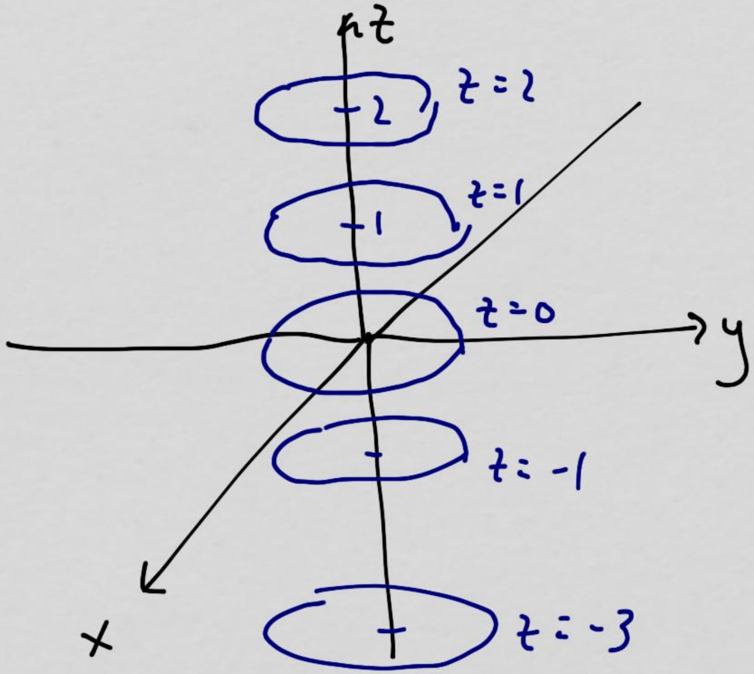
likewise, $x=5$, we get



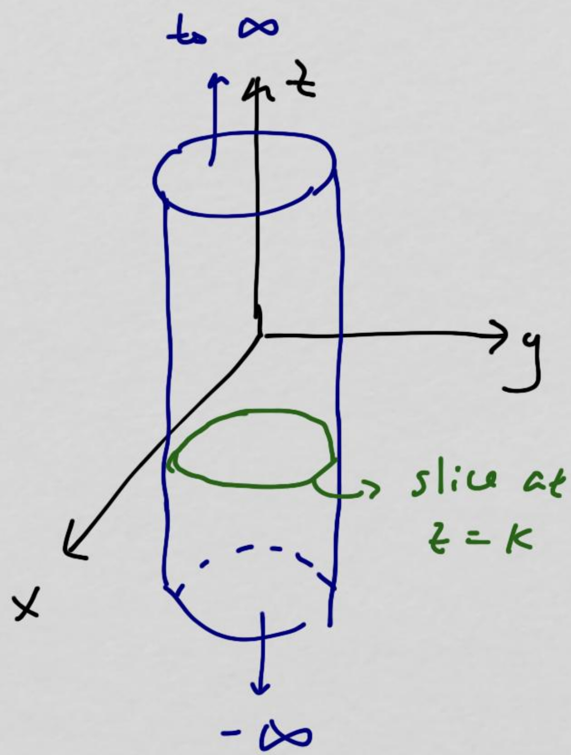
$x^2+y^2=1$ in $\mathbb{R}^3$ has no z, so $-\infty < z < \infty$

for any z, the point (x, y, z) such that $x^2+y^2=1$ is on the surface

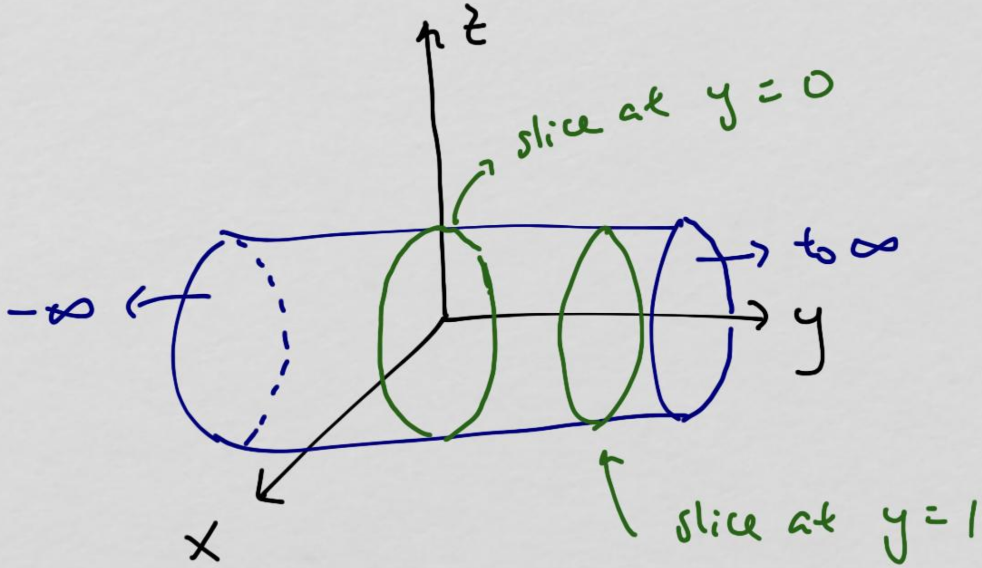
in other words, for any z we have a circle of radius 1 centered at $(0, 0, z)$ $\rightarrow$ it's a circle on the plane $z=k$



Stack them $\rightarrow$



Similarly, $x^2 + z^2 = 1$ is circular cylinder parallel to y-axis



these slices (intersection of surface with x, y, z equal to a constant)

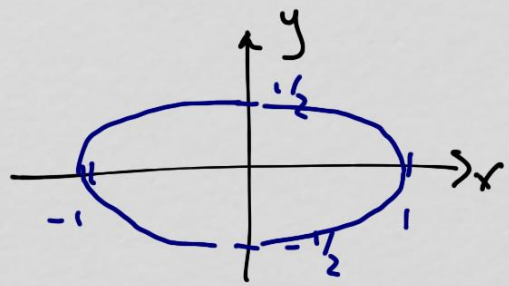
are called <u>traces</u>

intersection of a surface with xy-plane is called the xy-trace

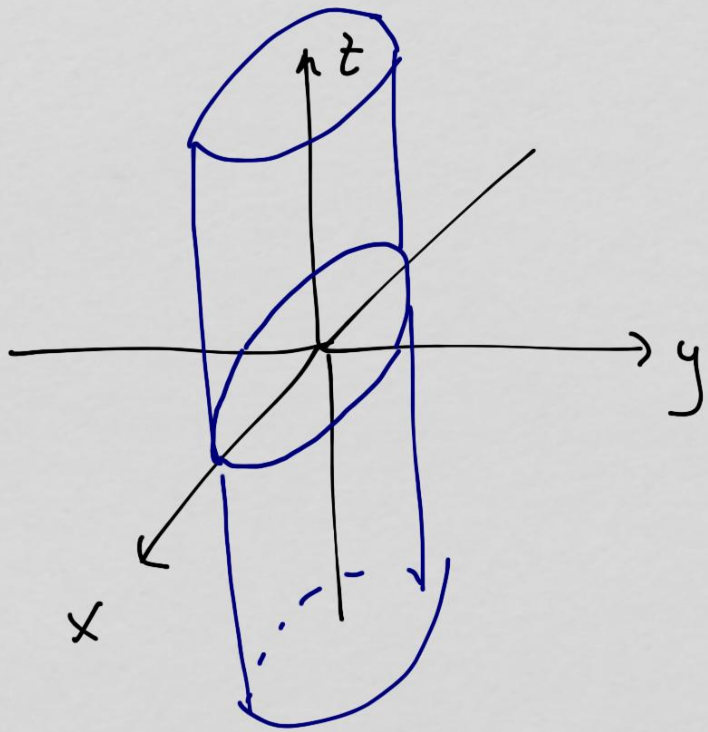
(intersection w/ yz-plane is yz-trace, intersection w/ xz-plane is xz-trace) cylinders don't have to have circular cross sections

for example, $x^2+4y^2=1$ has z missing

for each value of z, the cross section is



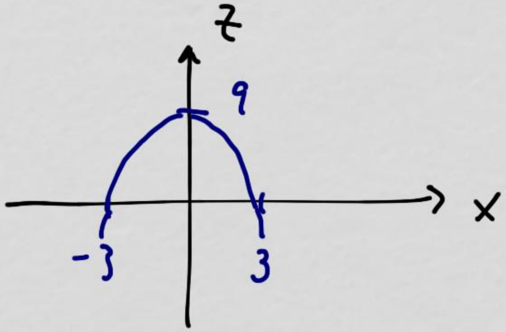
so the cylinder looks like



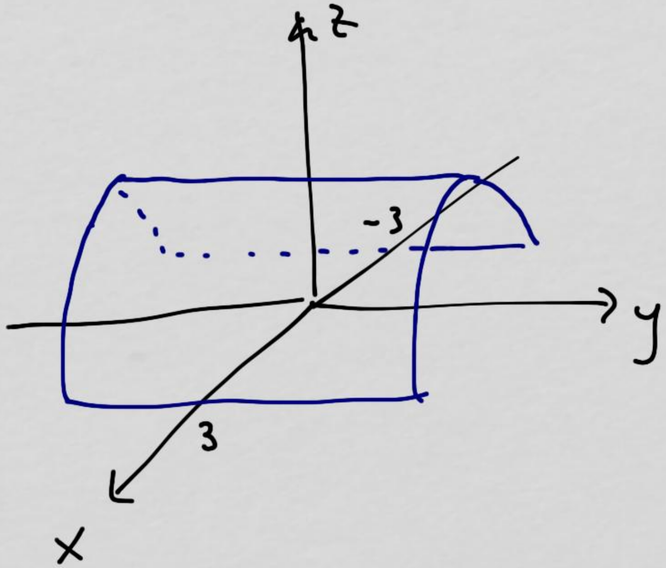
elliptical cylinder

$z = 9 - x^2$

for each y, the trace is



so the surface looks like



this is a parabolic cylinder

as long as we can identify the traces, we can stack them to form the surface

for example, $x^2+y^2+z^2=16$ (pretend we didn't know what this looks like)

x-intercepts: $x = \pm 4$

y-intercepts: $y = \pm 4$

z-intercepts: $z = \pm 4$

xy-trace $(z=0)$: $x^2+y^2=16$

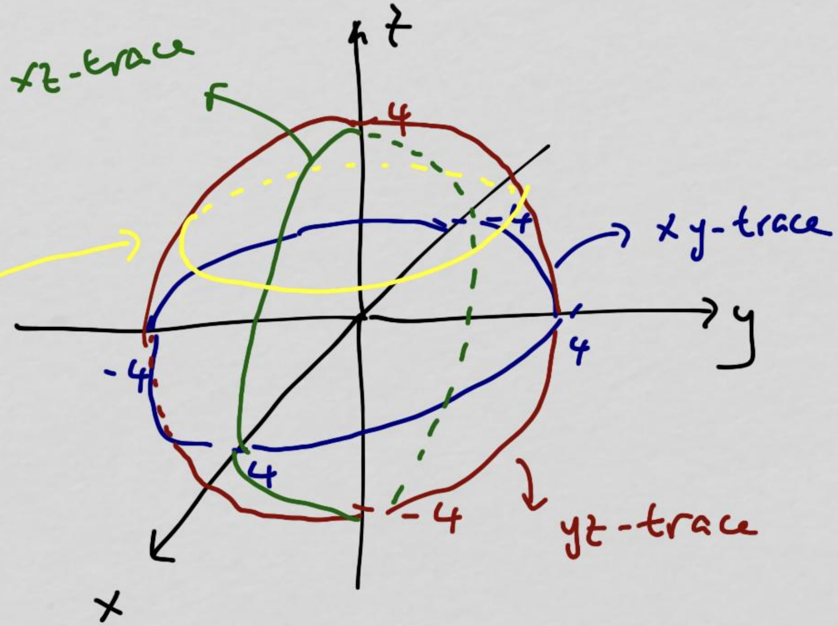
yz-trace $(x=0)$: $y^2+z^2=16$

xz-trace $(y=0)$: $x^2+z^2=16$

at $z=k$, trace is

$x^2+y^2=16-k^2$

circle w/ radius $\sqrt{16-k^2} \le 4$ $(-4 \le k \le 4)$



notice we stacked traces to form the shape

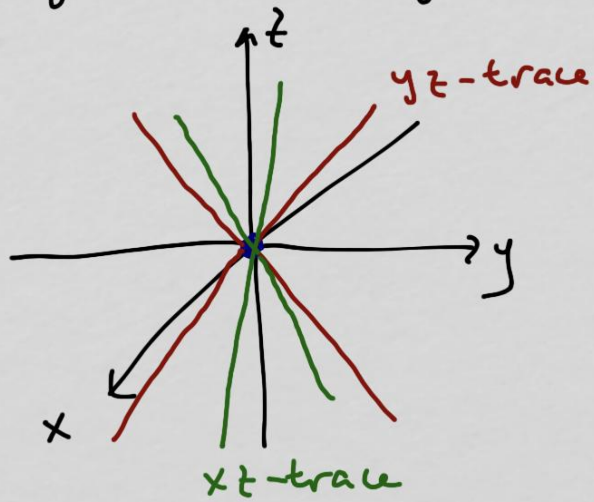
example $x^2 + y^2 = z^2$

x-intercepts : $x^2 = 0 \rightarrow x = 0$
y-intercepts : $y^2 = 0 \rightarrow y = 0$
z-intercepts : $z^2 = 0 \rightarrow z = 0$

$\Big\}$ origin

xy-trace $(z=0)$: $x^2 + y^2 = 0 \rightarrow$ circle of radius 0 (origin)
yz-trace $(x=0)$: $y^2 = z^2 \rightarrow y = \pm z$ (lines)
xz-trace $(y=0)$: $x^2 = z^2 \rightarrow x = \pm z$ (lines)

these traces and intercepts may not be enough to identify the shape



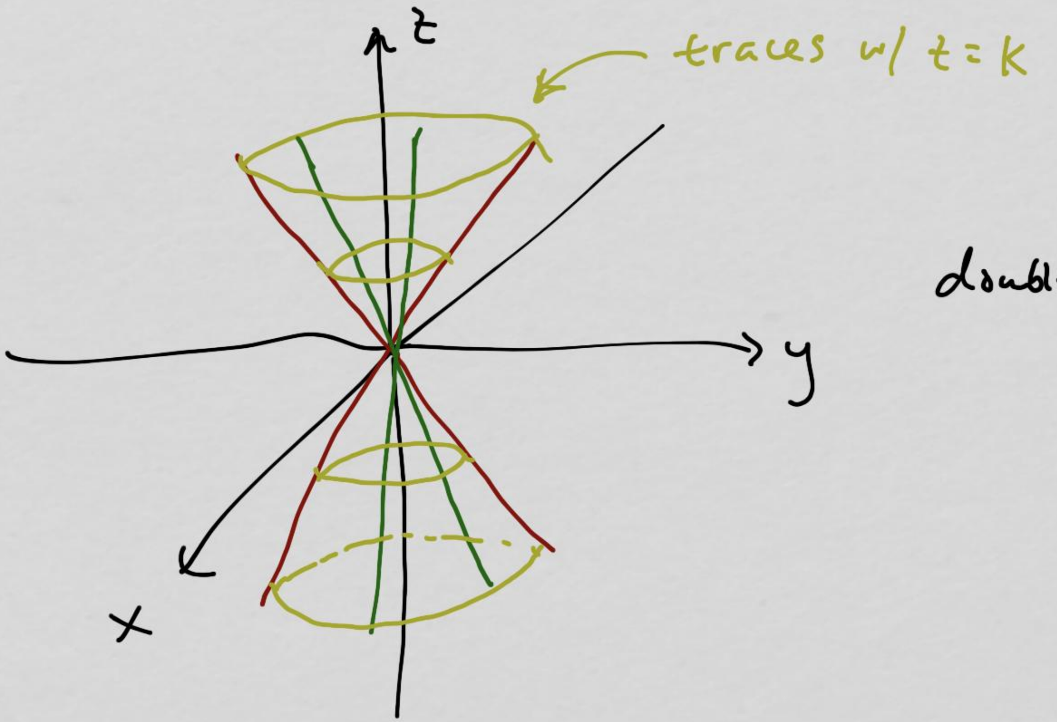
add more traces to make it easier to see

trace w/ $z=k$: $\quad x^2+y^2=k^2$

circles of radius $|k|$ $\quad -\infty<k<\infty$

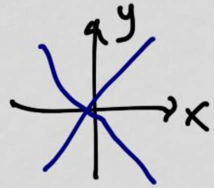
as z increases (or decreases) from $z=0$,

traces are circles of radius $|k|$



double cone

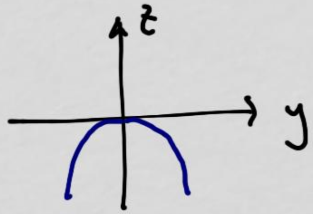
<u>example</u> $z = x^2 - y^2$



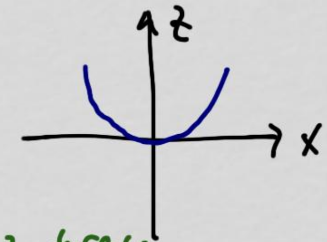
x, y, z intercepts are all 0

xy-trace $(z=0)$: $x^2 - y^2 = 0 \rightarrow y = \pm x$ (lines)

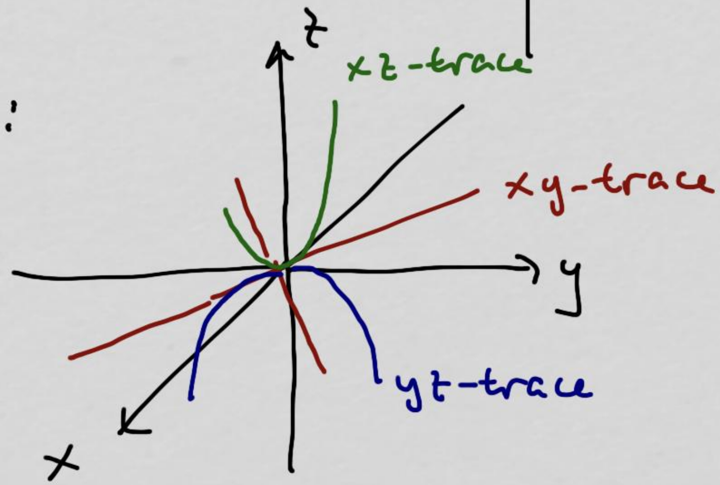
yz-trace $(x=0)$: $z = -y^2$ (parabola)



xz-trace $(y=0)$: $z = x^2$ (parabola)



what we have so far:



need more ...

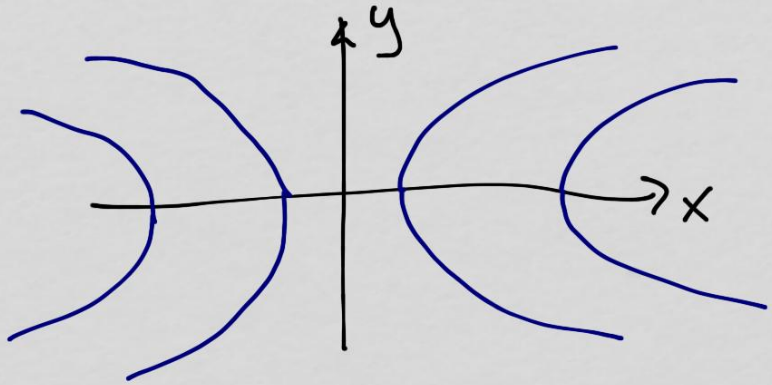
traces w/ $z = k$: $x^2 - y^2 = k$ hyperbola (squared terms have opposite signs)

if $k > 0$ ($z > 0$: above xy-plane)

x-ints: $x^2 = k > 0 \rightarrow x = \pm\sqrt{k}$

y-ints: $-y^2 = k > 0 \rightarrow$ no solutions

if $k = z > 0$, traces are hyperbolas that intersect x-axis

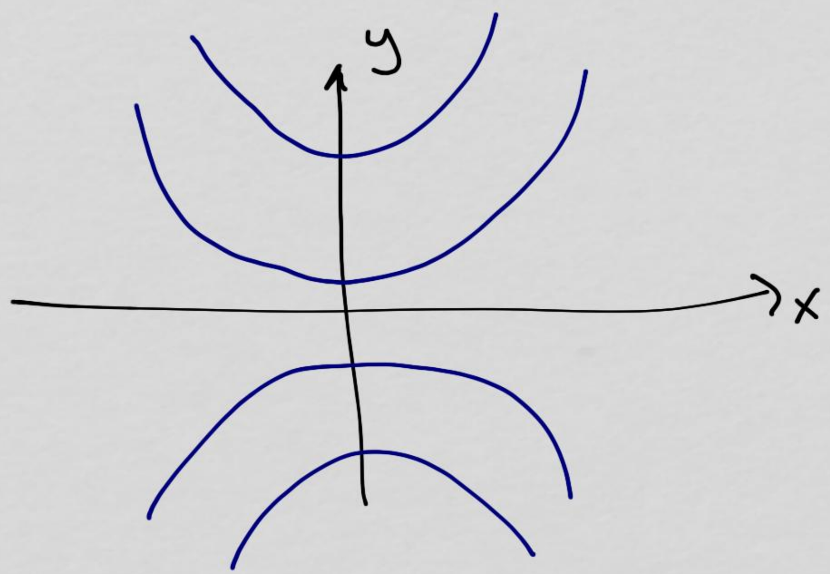


the bigger the k the farther apart the branches are if $K = z < 0$, traces are $x^2 - y^2 = K < 0$ (below xy-plane)

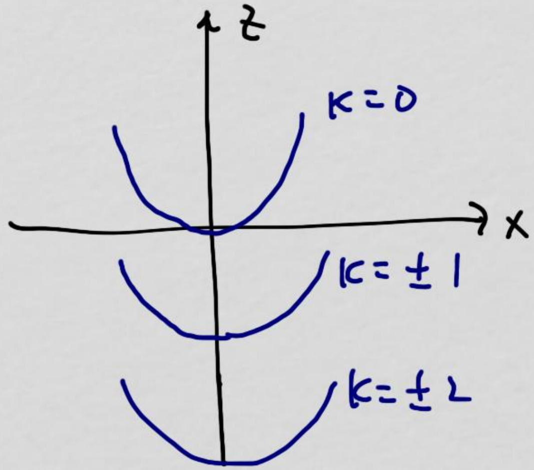
this hyperbola has y-ints but no x-ints

x-ints: $x^2 = K < 0 \rightarrow$ no solutions

y-ints: $-y^2 = K < 0 \rightarrow y = \pm\sqrt{K}$

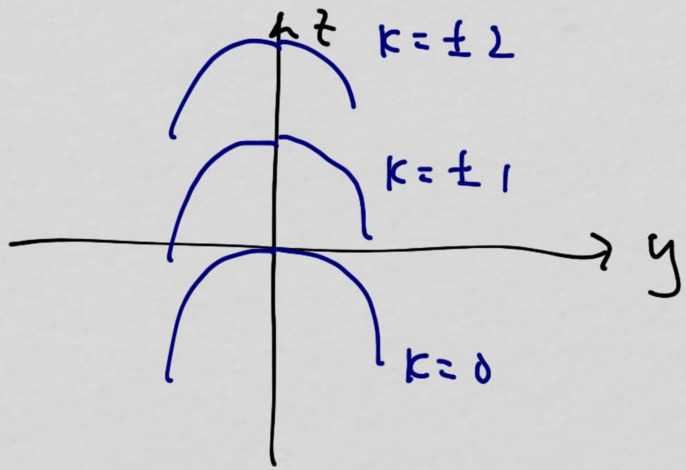


the smaller (more negative) $z = K$ is, the farther apart the branches are traces with $y=k$: $z = x^2 - k^2$ parabola with vertex at $z = -k^2$

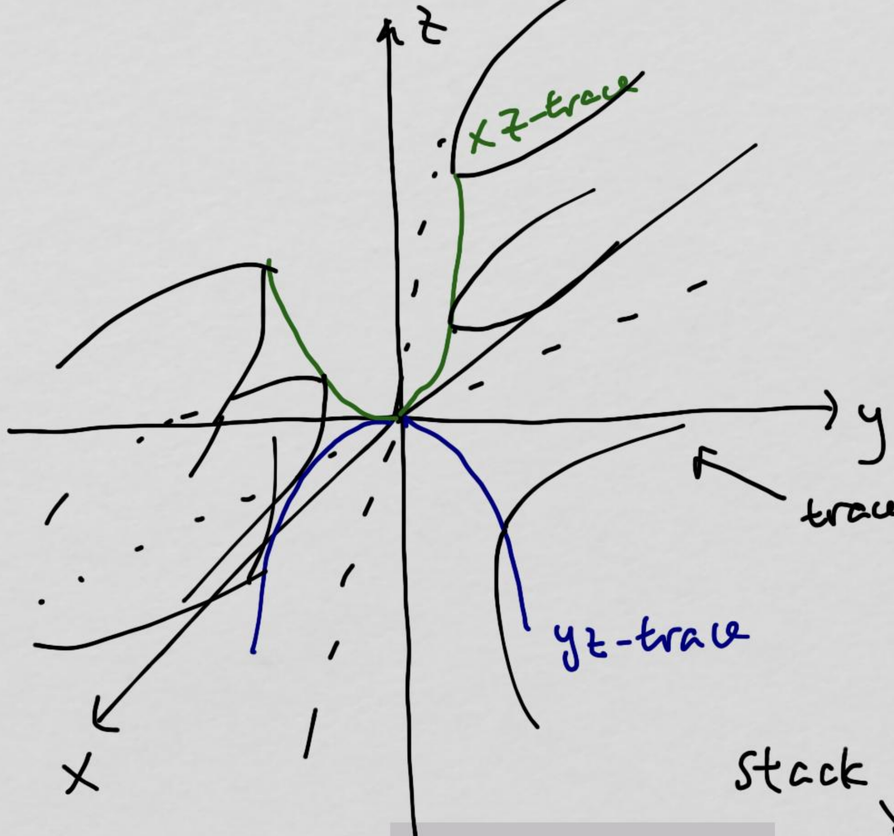


parabola gets lower as y increases or decrease from $y=0$

traces with $x=k$: $z = k^2 - y^2$ parabolas

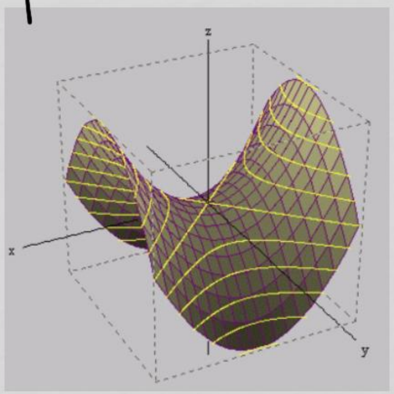


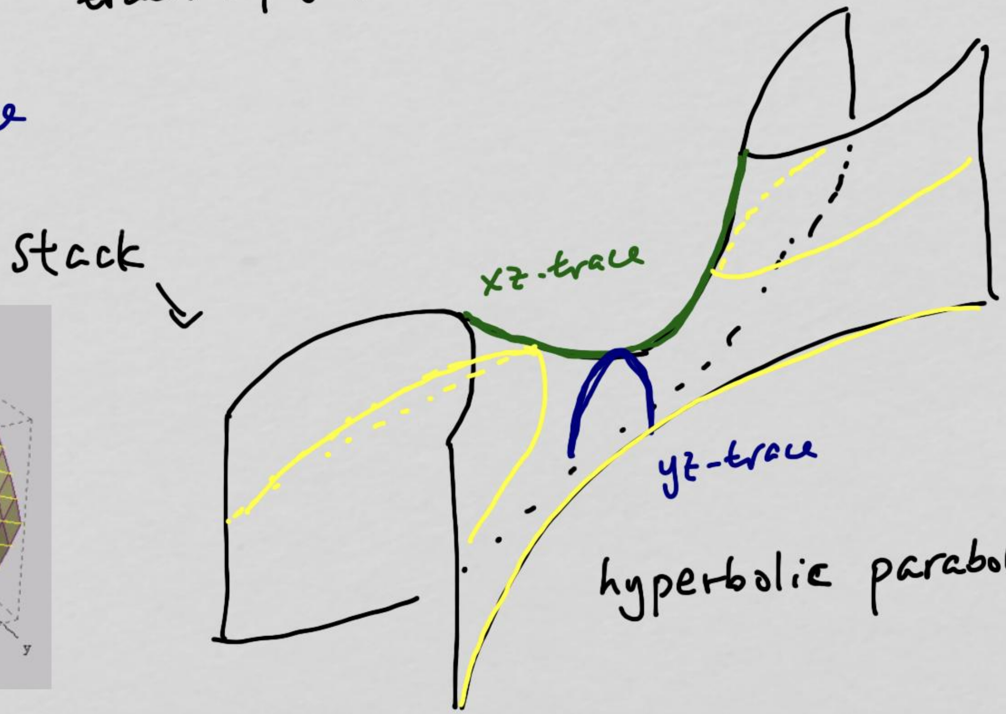
traces w/ $z=k>0$



traces w/ $z=k<0$

the xy-trace $y=\pm x$ are the asymptotes of the hyperbolas





hyperbolic paraboloid

all of these are examples of Quadric Surfaces

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

(and not A, B, C are zero)

we will look at more examples next time.