13.6 Quadric Surfaces (part 2)

more examples of $Ax^2+By^2+Cz^2+Dxy+Exz+Fyz+Gx+Hy+Iz+J=0$

(not all A,B,C are zero)

<u>example</u> $x^2+y^2-z^2=1$

x-ints: $x^2=1$, $x=\pm 1$

y-ints: $y^2=1$, $y=\pm 1$

z-ints: $-z^2=1$ no solution, so no z-ints

xy-trace $(z=0)$: $x^2+y^2=1$ circle radius 1 center $(0,0,0)$

trace w/ $z=K$: $x^2+y^2=1+K^2$ circle radius $\sqrt{1+K^2}$

center $(0,0,K)$

circle gets larger as z increases or decreases from 0

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xz-trace $(y=0)$: $x^2 - z^2 = 1$ hyperbola w/ vertices are $x = \pm 1$

(does not intersect z-axis)

trace w/ $y = k$: $x^2 - z^2 = 1 - k^2$

hyperbola w/ vertices at $x = \pm\sqrt{1-k^2}$

if $k^2 < 1$ or $\underbrace{|k| < 1}_{-1 < k < 1}$ (because then $1-k^2 > 0$)

hyperbola w/ vertices at $z = \pm\sqrt{k^2-1}$

if $|k| = |y| > 1$

why? if $|k| > 1$, then $k^2 > 1$, then $1 - k^2 < 0$

$x^2 - z^2 = \underbrace{1-k^2}_{<0}$

so has no x-ints: $x^2 = 1 - k^2 < 0$ has no solutions

but has z-ints: $-z^2 = 1 - k^2 < 0$

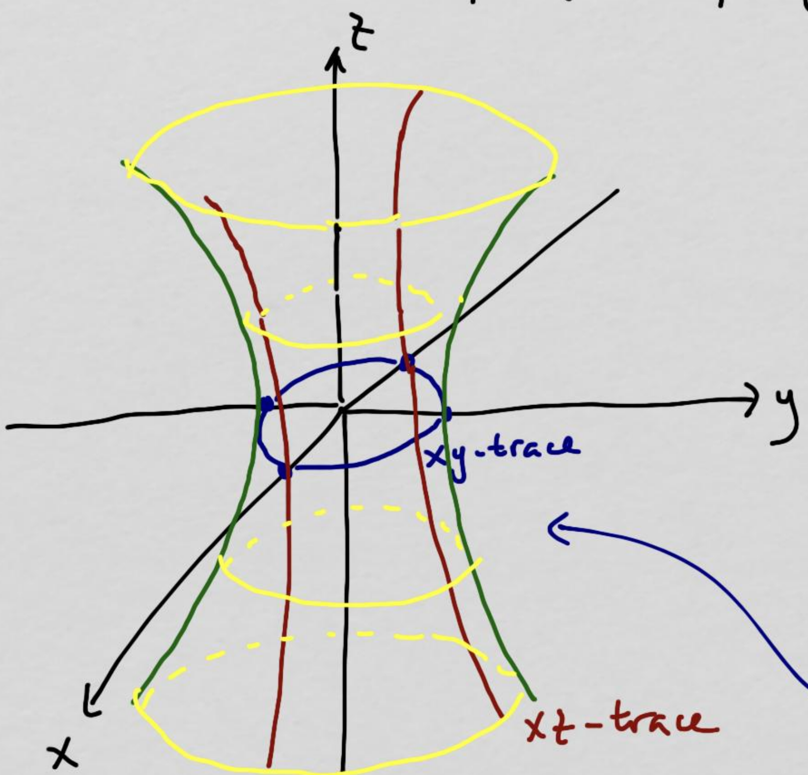
$z^2 = k^2 - 1 > 0 \rightarrow z = \pm\sqrt{k^2-1}$

yz-trace $(x=0)$: $y^2 - z^2 = 1$ hyperbola with vertices $y = \pm 1$

trace w/ $x=k$: $y^2 - z^2 = 1 - k^2$

if $|k| < 1$, hyperbola vertices on $y = \pm\sqrt{1-k^2}$

if $|k| > 1$, hyperbola vertices on $z = \pm\sqrt{k^2-1}$



trace w/ $z=k$: $x^2 + y^2 = 1 + k^2$

increasing radius as

$|k|$ increases

notice there is no z-intercepts

because traces w/ $z=k$ are

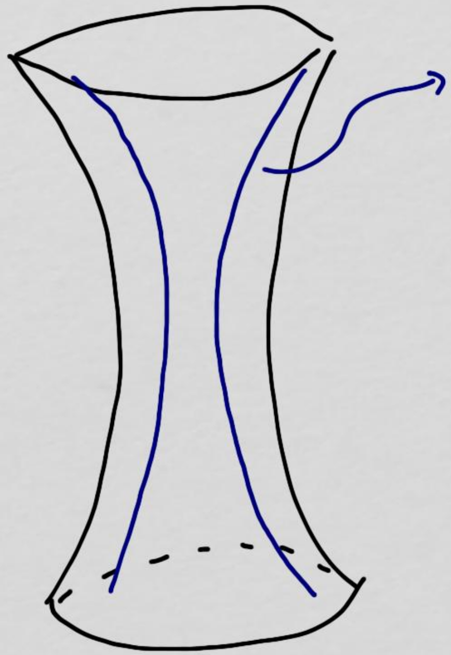
circles whose radius is no

smaller than 1

<u>Hyperboloid of one sheet</u>

where are the switching hyperbolas?

trace w/ $y = k$: $x^2 - z^2 = 1 - k^2$

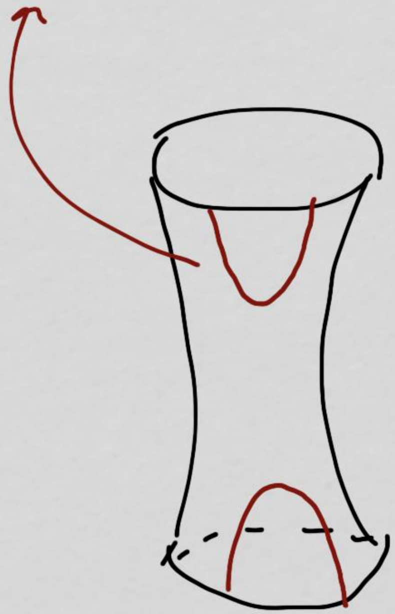


if $|k| = |y| < 1 \rightarrow -1 < y < 1$, close to the origin

if $|k| = |y| > 1 \rightarrow y > 1$ or $y < -1$

farther away from origin

(when that happens, the cut does not go through the "neck")



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<u>example</u> $-x^2-y^2+z^2=1$

x-ints: $-x^2=1$ has no solutions, so none

y-ints: $-y^2=1$ " " "

z-ints: $z^2=1 \to z=\pm 1$

xy-trace $(z=0)$: $-x^2-y^2=1 \to x^2+y^2=-1$

(looks like a circle but radius is not real)

no xy-trace

(does not intersect xy-plane)

trace w/ $z=k$: $-x^2-y^2=1-k^2$

$x^2+y^2=k^2-1$ circle of radius $\sqrt{k^2-1}$

if $k^2>1 \to |k|>1$

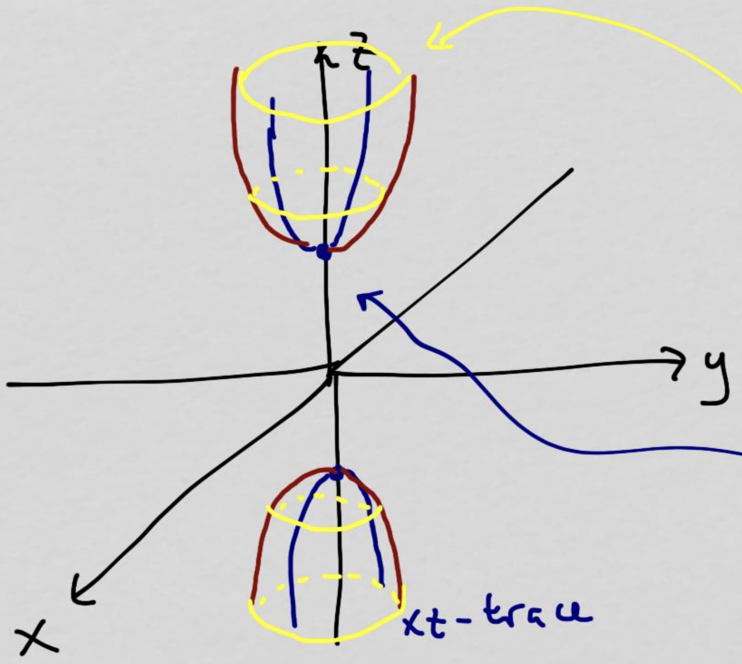
$(y>1$ or $y<-1)$

agrees w/ earlier finding of no xy-trace xz-trace ($y=0$): $-x^2+z^2=1$ hyperbola vertices on z-axis

trace w/ $y=k$: $-x^2+z^2=1+k^2$ hyperbola w/ vertice at $z=\pm\sqrt{1+k^2}$
regardless of k (no switching)

yz-trace ($x=0$): $-y^2+z^2=1$ hyperbola vertices at $z=\pm 1$

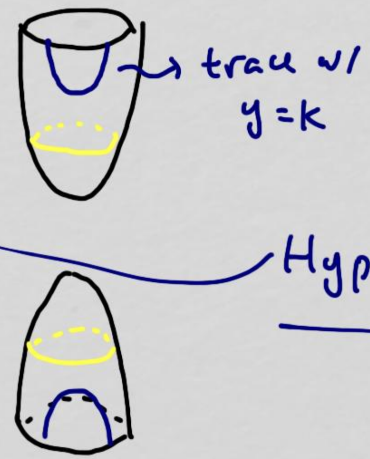
trace w/ $x=k$: $-y^2+z^2=1+k^2$ hyperbola w/ vertices at $z=\pm\sqrt{1+k^2}$
regardless of k (no switching)



trace w/ $z=k$: $x^2+y^2=k^2-1$

circles radius $\sqrt{k^2-1}$

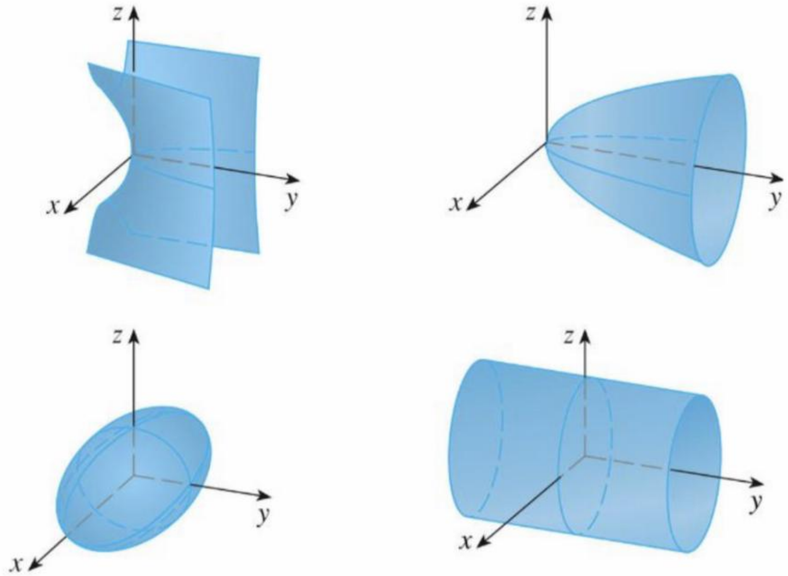
$|k|\geq 1$



Hyperboloid of two sheets

next example: from a past exam

3. Which of the following equations produces a surface that is NOT shown here?



A. $y = x^2 - z^2$

B. $y = 2x^2 + z^2$

C. $x^2 + 4y^2 + 9z^2 = 1$

D. $x^2 + 2z^2 = 1$

E. $-x^2 + y^2 - z^2 = 1$

traces!

A: xy-trace is $y = x^2$ parabola
could be the top two figures

B: xy-trace is $y = 2x^2$ parabola
could be the top two

C: $x^2+4y^2+9z^2=1$

xy-trace: $x^2+4y^2=1$ ellipse

bottom left figure

D: $x^2+2z^2=1$

xy-trace: $x^2=1$ $x=\pm 1$ maybe not very useful

but we notice y is missing, so this must be a cylinder parallel to y-axis and each slice w/ $y=k$ is an ellipse

bottom right figure

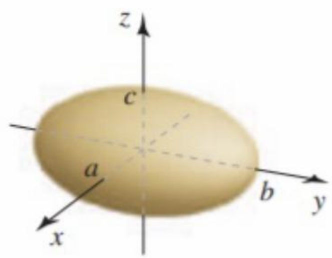
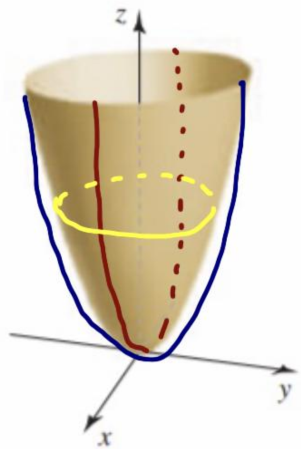
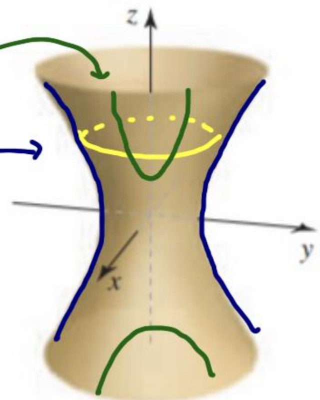
E: $-x^2+y^2-z^2=1$

xy-trace: $-x^2+y^2=1$ hyperbola w/ vertices at $y=\pm 1$

we don't see any figure like this

so, E must be the equation whose figure is NOT shown

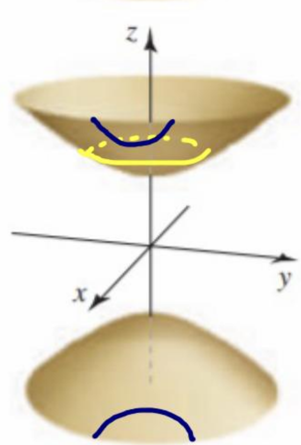
Table 13.1 from textbook — Summary of Shapes

Name	Standard Equation	Features	Graph
Ellipsoid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	All traces are ellipses. if $a^2=b^2=c^2 \rightarrow$ sphere	
Elliptic paraboloid	$z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$	Traces with $z=z_0>0$ are ellipses. Traces with $x=x_0$ or $y=y_0$ are parabolas. xz-trace and yz-trace are parabolas xy-trace is circle	
Hyperboloid of one sheet	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	Traces with $z=z_0$ are ellipses for all z_0. Traces with $x=x_0$ or $y=y_0$ are hyperbolas. slice not through neck slice through neck	

Hyperboloid of two sheets

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Traces with $z = z_0$ with $|z_0| > |c|$ are ellipses. Traces with $x = x_0$ and $y = y_0$ are hyperbolas.

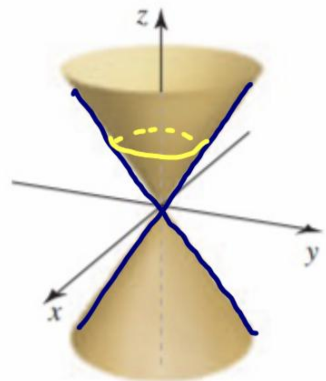


Elliptic cone

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$

Traces with $z = z_0 \neq 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are hyperbolas or intersecting lines.

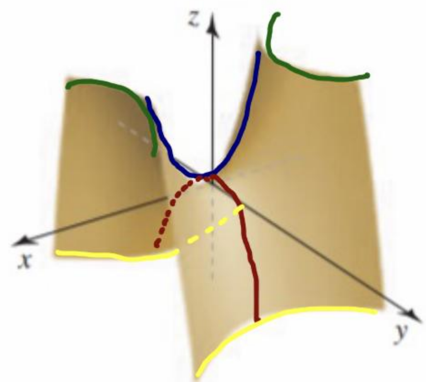
xy-trace: lines



Hyperbolic paraboloid

$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

Traces with $z = z_0 \neq 0$ are hyperbolas. Traces with $x = x_0$ or $y = y_0$ are parabolas.



memorize if we wish how signs go w/ these

or better idea: get comfortable w/ traces