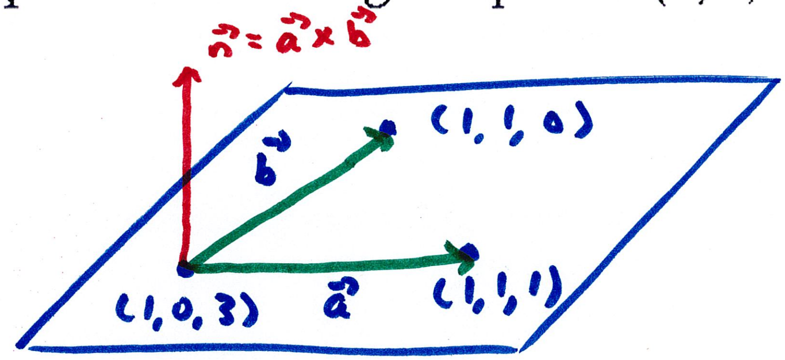
8. The equation of the plane containing the points $(1, 0, 3)$, $(1, 1, 0)$ and $(1, 1, 1)$ is:

A. $x + y = 0$
B. $x + y + z = 0$
C. $x = 1$
D. $x + 3z = 1$
E. $x - y - 3z = 3$



plane: normal vector and one point

$\vec{a} = \langle 0, 1, -2 \rangle$ $\qquad$ $\vec{b} = \langle 0, 1, -3 \rangle$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & -2 \\ 0 & 1 & -3 \end{vmatrix} = \langle -1, 0, 0 \rangle = \vec{n}$$

plane: $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$ $\qquad$ $\langle A, B, C \rangle = \vec{n}$

$$-1(x - 1) = 0 \rightarrow x - 1 = 0 \rightarrow \boxed{x = 1}$$

x from any point

3. Identify the surface $x^2 - y^2 - z = 0$.

A. hyperboloid of one sheet

B. hyperboloid of two sheets

~~C~~. sphere

D. hyperbolic paraboloid

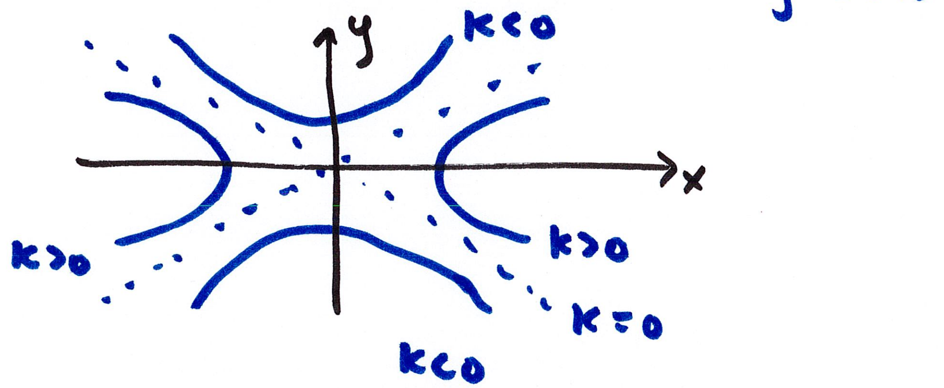
~~E~~. elliptic paraboloid

xy-trace: $x^2-y^2=0$ $x=\pm y$ lines

$z=k$: $x^2-y^2=k$ hyperbolas

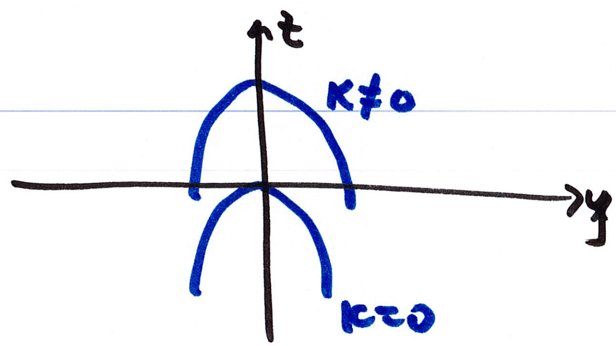
if $k>0$ vertices on x-axis

if $k<0$ " " y-axis



yz-trace: $-y^2-z=0 \to z=-y^2$ parabola

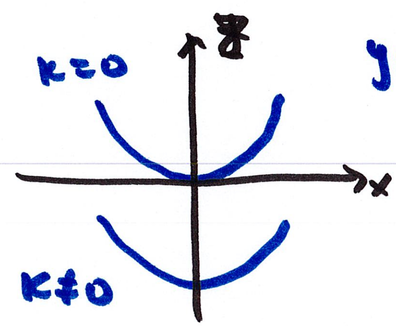
$x=k$: $z=-y^2+k^2$

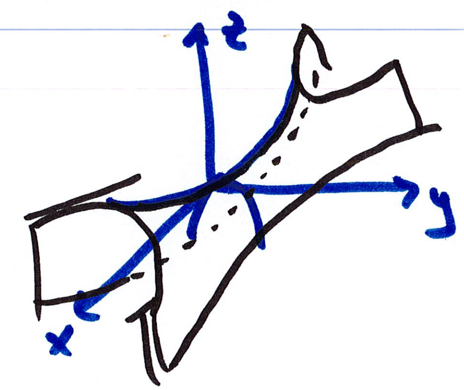


xz-trace: $x^2-z=0$

$z=x^2$

$y=k$: $z=x^2-k^2$





PROBLEM 3: Find the length of the curve given by

$$r(\vec{t}) = \langle 2t, 4\sqrt{t}, \ln t \rangle$$

for $1 \leq t \leq e$.

A. $e - 1$

B. $2e + 1$

C. $e + 1$

D. $2e - 1$

E. $4e - 3$

$$L = \int_a^b |\vec{r}\,'|\,dt \qquad s(t) = \int_a^t |\vec{r}\,'(u)|\,du$$

$$\vec{r}\,' = \langle 2, 2t^{-1/2}, \tfrac{1}{t} \rangle$$

$$|\vec{r}\,'| = \sqrt{4 + 4t^{-1} + \frac{1}{t^2}} = \sqrt{4 + \frac{4}{t} + \frac{1}{t^2}} = \sqrt{\frac{4t^2 + 4t + 1}{t^2}}$$

$$= \sqrt{\frac{(2t+1)^2}{t^2}} = \frac{2t+1}{t} = 2 + \frac{1}{t}$$

$$L = \int_1^e \left(2 + \frac{1}{t}\right) dt = 2t + \ln t \Big|_1^e = (2e + 1) - (2 + 0)$$

$$= 2e - 1$$

$\vec{r}(t) = \langle 2t, 4\sqrt{t}, \ln t \rangle \quad t \geq 1$ parametrize in terms of arc length

$$s(t) = \int_1^t |\vec{r}\,'(u)|\,du \qquad |\vec{r}\,'| = 2 + \frac{1}{t} \text{ from prev. page}$$

$$s = \int_1^t \left(2 + \frac{1}{u}\right) du = 2u + \ln|u| \Big|_1^t = 2t + \ln|t| - 2$$

length as function of t

$s = 2t + \ln|t| - 2$ solve t in terms of s

then sub t out in $\vec{r}(t)$

but can't isolate t here

8. Let $f(x, y) = \ln(2x - y)$. Using a linear approximation, the approximate value of $f(1.1, 0.9)$ is:

A. 0.3

B. -0.3

C. 0.1

D. -0.2

E. 0.2

Linear approximation: $L = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

here, $(a,b) = (1,1)$

$$f_x = \frac{1}{2x-y} \cdot 2 = \frac{2}{2x-y}$$

$$f_y = \frac{1}{2x-y} \cdot -1 = \frac{-1}{2x-y}$$

$$f(a,b) = f(1,1) = \ln(2-1) = 0$$

$$f_x(1,1) = \frac{2}{1} = 2$$

$$f_y(1,1) = \frac{-1}{1} = -1$$

$$L = 0 + 2(x-1) - (y-1)$$

$$f(1.1, 0.9) \approx L(1.1, 0.9) = 2(1.1-1) - (0.9-1)$$

$$= 0.3$$

9. The critical points of $f(x,y) = 3x^3 + 3y^3 + x^3y^3$ are:

A. $(0,0), (1,-1)$

B. $(0,0)$

C. $(1,1)$

D. $(0,0), (-3^{1/3}, -3^{1/3})$

E. $(-3^{1/3}, -3^{1/3}), (1,1)$

Critical pts: $f_x = 0$

$f_y = 0$

$f_x = 9x^2 + 3x^2y^3 = 0$

$f_y = 9y^2 + 3x^3y^2 = 0$

$3x^2(3 + y^3) = 0 \longrightarrow x = 0, \quad y = (-3)^{1/3}$

$3y^2(3 + x^3) = 0 \longrightarrow y = 0, \quad x = (-3)^{1/3}$

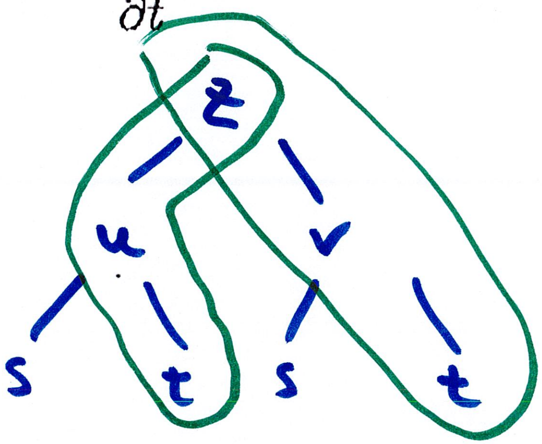
Critical pts: $(0,0), ((-3)^{1/3}, (-3)^{1/3})$

$$z=(u^2+v)^{-1} \rightarrow \frac{\partial z}{\partial v} = -(u^2+v)^{-2}\cdot(1)$$

11. If

$$z=\frac{1}{u^2+v}, \quad u(s,t)=t+s^2, \quad v(s,t)=\ln(t)$$

then $\frac{\partial z}{\partial t}$ is:



A. $\frac{-(1+\frac{1}{t})}{(t+s^2)^2+\ln t}$

~~B.~~ $\frac{-(2(t+s^2)+\frac{\ln t}{t})}{(t+s^2)^2+\ln t}$

C. $\frac{-(2(t+s^2)+\frac{1}{t})}{((t+s^2)^2+\ln t)^2}$

D. $\frac{-(u+1)}{(u^2+v)^2}$

E. $\frac{-(2u+1)}{u^2+v}$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial t} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial t}$$

$$= \frac{-2u}{(u^2+v)^2}(1) - \frac{1}{(u^2+v)^2}\left(\frac{1}{t}\right)$$

$$= \frac{-2ut-1}{t(u^2+v)^2} = \frac{-2(t+s^2)t-1}{t((t+s^2)^2+\ln t)^2} = \frac{\frac{-2(t+s^2)t-1}{t}}{((t+s^2)^2+\ln t)^2}$$

1. Evaluate the limit if it exists

difference of squares

$$\lim_{(x,y)\to(0,0)} \frac{(x^4 - y^4) + xy}{x^2 + y^2} \to \frac{0}{0} = ?$$

A. -1
B. 0
C. 1
D. 2
E. The limit does not exist

$$\lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)(x^2-y^2)+xy}{x^2+y^2}$$

$$= \lim_{(x,y)\to(0,0)} \underbrace{(x^2-y^2)}_{\text{to } 0} + \underbrace{\frac{xy}{x^2+y^2}}_{?}$$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

along $x=0$: $\lim_{y\to 0} \frac{(0)y}{0+y^2} = 0$

along $y=0$: $\lim_{x\to 0} \frac{x(0)}{x^2+0} = 0$

along $y=x$: $\lim_{x\to 0} \frac{x^2}{x^2+x^2} = \lim_{x\to 0} \frac{x^2}{2x^2} = \frac{1}{2}$

$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ DNE

therefore

$\lim_{(x,y)\to(0,0)} \frac{x^4-y^4+xy}{x^2+y^2}$ DNE

10. The directional derivative of $f(x, y) = x^3 e^{-2y}$ in the direction of greatest increase of f at the point $x = 1$, $y = 0$ is:

A. $3\vec{\imath}$

B. $3\vec{\imath} - 2\vec{\jmath}$

C. 3

D. $\sqrt{5}$

E. $\sqrt{13}$

directional derivative : $D_{\vec{u}} f = \vec{\nabla} f \cdot \vec{u}$

gradient

unit vector giving direction

$f(x,y) = x^3 e^{-2y}$

$\vec{\nabla} f = \langle f_x, f_y \rangle = \langle 3x^2 e^{-2y}, -2x^3 e^{-2y} \rangle$

$\vec{\nabla} f(1,0) = \langle 3, -2 \rangle$

the gradient is in the direction of greatest increase in f and its magnitude is the greatest directional derivative

so, here we want $|\vec{\nabla} f| = |\langle 3, -2 \rangle| = \sqrt{13}$