Find the maximum value of $x + y$ along the curve defined by $x^2 + 2y^2 = 6$.

A. 3

B. $4\sqrt{3}$

C. $2\sqrt{6}$

D. 4

E. $2\sqrt{2}$

Constrained optimization

→ Lagrange multipliers

$f = x + y$

$g = x^2 + 2y^2 - 6 = 0$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

$\langle 1, 1 \rangle = \lambda \langle 2x, 4y \rangle$

$$\begin{cases} 1 = \lambda \cdot 2x \\ 1 = \lambda \cdot 4y \end{cases}$$

find relationship between x and y

then plug into g

→ $\lambda \cdot 2x = \lambda \cdot 4y$

$\boxed{x = 2y}$ → sub into g: $x^2 + 2y^2 = 6$

$4y^2 + 2y^2 = 6$ → $y^2 = 1$ $y = 1, -1$

$x = 2y$ so $x = 2, -2$

points to check: $(2, 1)$, $(-2, -1)$

plug into $f = x + y$

$f(2,1) = 3 \rightarrow$ max

$f(-2,-1) = -3 \rightarrow$ min

If the order of integration is reversed, which of the following integrals is equal to $\int_0^{\sqrt{\pi}} \int_{y^2}^{\pi} (\sin x^2)\,dx\,dy$?

A. $\int_0^{\pi} \int_{\sqrt{x}}^{\pi} (\sin x^2)\,dy\,dx$

B. $\int_0^{\pi} \int_0^{\sqrt{x}} (\sin x^2)\,dy\,dx$

C. $\int_{\sqrt{x}}^{\sqrt{\pi}} \int_0^{\pi} (\sin x^2)\,dy\,dx$

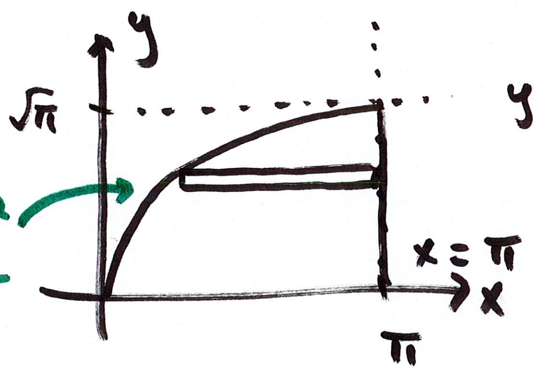
D. $\int_0^{\pi} \int_x^{\sqrt{x}} (\sin x^2)\,dy\,dx$

E. $\int_0^{\sqrt{\pi}} \int_x^{\pi} (\sin x^2)\,dy\,dx$

$0 \le y \le \sqrt{\pi}$

left $\to$ $y^2 \le x \le \pi$ $\leftarrow$ right

Sketch the region described

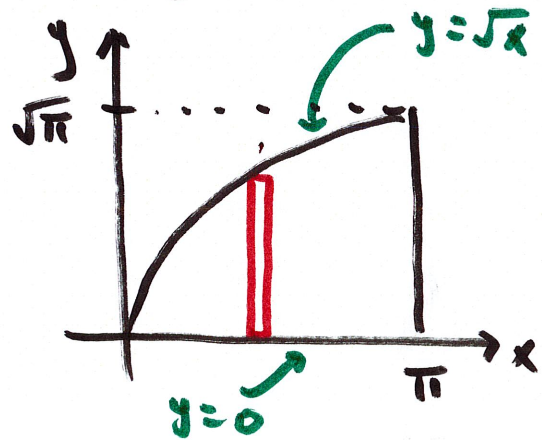


$y=\sqrt{\pi}$

Type II

$x = y^2 \longleftrightarrow y = \sqrt{x}$

switch to Type I



$0 \le x \le \pi$

$0 \le y \le \sqrt{x}$

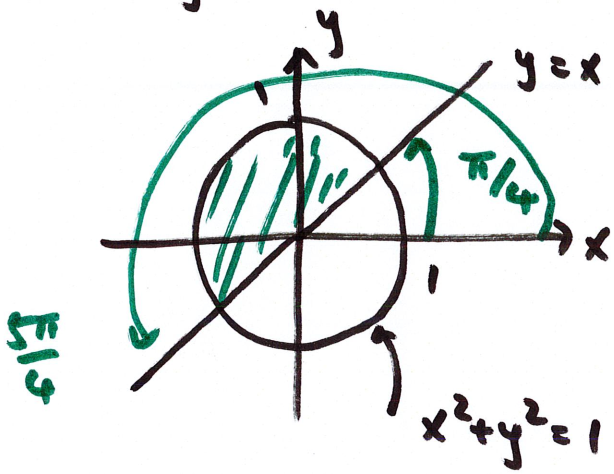
$$\int_{0}^{\pi} \int_{0}^{\sqrt{x}} (\sin x^2)\,dy\,dx$$

x y

If R is the region in the xy-plane inside the circle $x^2 + y^2 = 1$ and above the line $y = x$, then $\iint_R x\,dA$ expressed in polar coordinates is:

sketch R

$x^2+y^2=1$ circle radius 1 center origin



A. $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^1 r\cos\theta\,dr\,d\theta$

B. $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^1 r^2\cos\theta\,dr\,d\theta$

C. $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_0^{r\cos\theta} r^2\cos\theta\,dr\,d\theta$

D. $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_0^{r^2} r^2\cos\theta\,dr\,d\theta$

E. $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_0^1 r^2\cos\theta\,dr\,d\theta$

so to polar

origin

$0 \le r \le 1$ edge of circle radius 1

$\frac{\pi}{4} \le \theta \le \frac{5\pi}{4}$

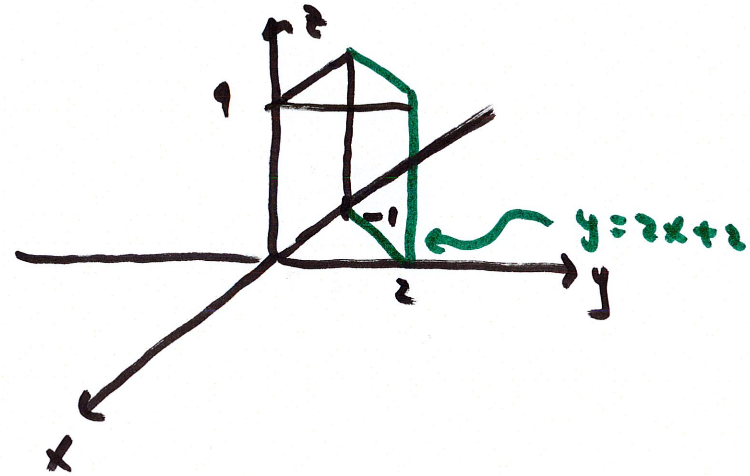
$$\underbrace{\int_{\pi/4}^{5\pi/4}}_{\theta} \underbrace{\int_0^1}_{r} \underbrace{r\cos\theta}_{x} \underbrace{r\,dr\,d\theta}_{dA \text{ in polar}}$$

Rewrite the following integral using the indicated order of integration and then evaluate the resulting integral.

$$\int_0^9 \int_{-1}^0 \int_0^{2x+2} dy\,dx\,dz \text{ in the order } \underline{dz\,dx\,dy}$$

$$\int_0^9 \int_{-1}^0 \int_0^{2x+2} dy\,dx\,dz = \int_{\square}^{\square} \int_{\square}^{\square} \int_{\square}^{\square} dz\,dx\,dy = \square$$ (Simplify your answer.)

3D view:

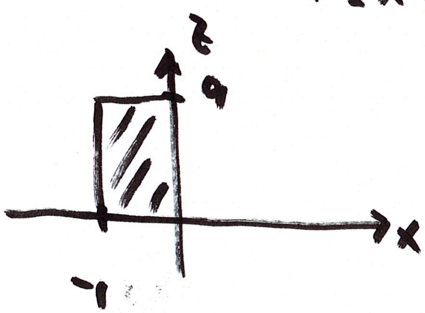


$$\underbrace{\int_0^9 \int_{-1}^0}\int_0^{2x+2} dy\,\underbrace{dx\,dz}$$

last two - "floor"

floor: $0 \le z \le 9$

$-1 \le x \le 0$



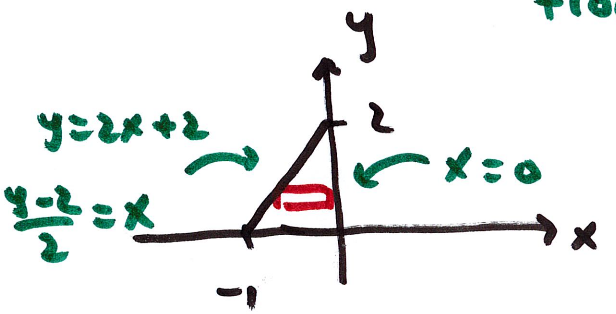
y ("ceiling" into screen)

$0 \le y \le 2x+2$

xz-plane

plane

new order: $dz\,\underbrace{dx\,dy}_{\text{"floor"}}$



dy outside: y bounded by constants

$$0 \le y \le 2$$

$$\frac{y-2}{2} \le x \le 0$$

z bounded by planes parallel to xy-plane at $z=0$, $z=9$

so $\quad 0 \le z \le 9$

$$\int_0^2 \int_{\frac{y-2}{2}}^{0} \int_0^9 dz\,dx\,dy$$

$y \qquad x \qquad z$

The mass of an object occupying the region bounded above by the plane $z = 2$ and below by the upper nappe of the cone $z^2 = x^2 + y^2$ with mass density at each point equal to $x^2 + y^2 + z^2$ is given by:

A. $\displaystyle\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

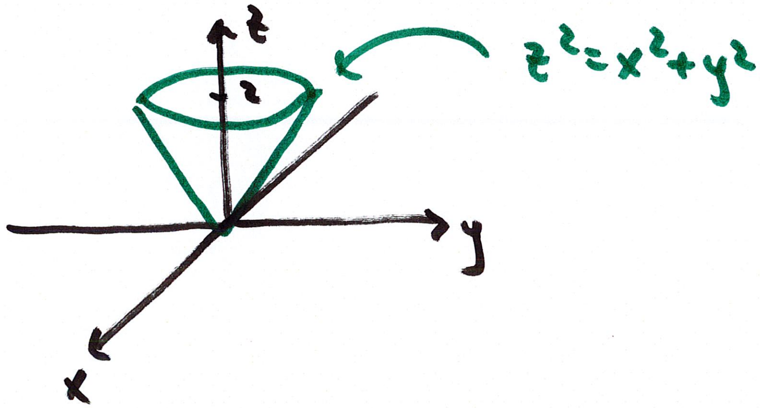
B. $\displaystyle\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{2\sec\phi} \rho^4 \sin\phi \, d\rho \, d\phi \, d\theta$

C. $\displaystyle\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{2} \rho^4 \sin\phi \, d\rho \, d\phi \, d\theta$

D. $\displaystyle\int_0^{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\int_0^{2\sec\phi} \rho^3 \sin^2\phi \, d\rho \, d\phi \, d\theta$

E. $\displaystyle\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{2\sec\phi} \rho^3 \sin^2\phi \, d\rho \, d\phi d\theta$

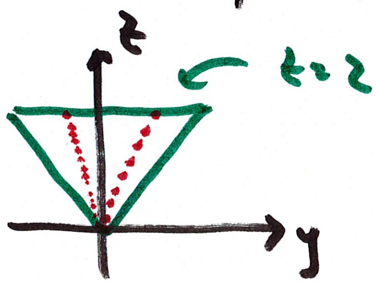
answers are all in spherical



all the way around z-axis so

$\boxed{0 \le \theta \le 2\pi}$

ρ: from origin to $z=2$

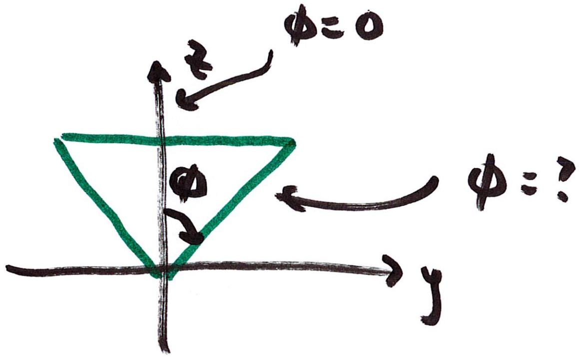


accumulate points from origin to the edge of volume

$z = \rho\cos\phi = 2$

$\rho = \frac{2}{\cos\phi} = 2\sec\phi$

so, $\boxed{0 \le \rho \le 2\sec\phi}$



$$\boxed{0 \le \phi \le \pi/4}$$

cone : $z^2 = x^2 + y^2$

on this perspective : $z^2 = y^2$

$z = y \rightarrow$ slope 1

bisects first quadrant

so, $\phi = \pi/4$

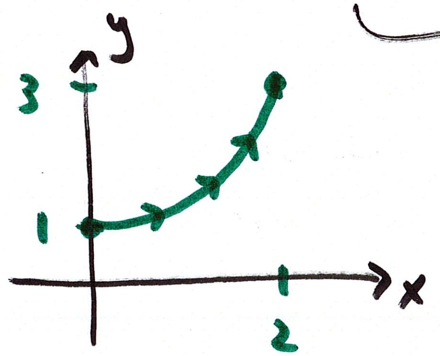
density : $d = x^2 + y^2 + z^2 = \rho^2$

mass: $$\underbrace{\int_0^{2\pi}}_{\theta} \underbrace{\int_0^{\pi/4}}_{\phi} \underbrace{\int_0^{2\sec\phi}}_{\rho} \underbrace{\rho^2}_{\text{density}} \overbrace{\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta}^{dV}$$

$\int_C \vec{F}\cdot\vec{T}\,ds = \int_C \vec{F}\cdot\vec{r}\,'\,dt = \int_C \vec{F}\cdot d\vec{r} = \int_C f\,dx + g\,dy$ $\quad \vec{F} = \langle f, g\rangle$

$\vec{r} = \langle x, y\rangle$

16. If C is the curve $\underbrace{y = \dfrac{x^2}{2} + 1}$ from $(0,1)$ to $(2,3)$, then $\displaystyle\int_C 3x\,ds =$



$\hookrightarrow y = \frac{1}{2}x^2 + 1$

make $x = t$

then $y = \frac{1}{2}t^2 + 1$

A. $\dfrac{8}{3}$

B. $\dfrac{10}{3}$

C. $\sqrt{5}$

D. $\sqrt{5} - 1$

E. $5\sqrt{5} - 1$

parametrize C: $\vec{r}(t) = \langle t, \frac{1}{2}t^2 + 1\rangle$

$0 \le t \le 2$

next, $ds = |\vec{r}\,'|\,dt$

$\vec{r}\,' = \langle 1, t\rangle \qquad |\vec{r}\,'| = \sqrt{1+t^2} \qquad ds = \sqrt{1+t^2}\,dt$

$\int_C 3x\,ds = \int_0^2 3t\sqrt{1+t^2}\,dt$ $\qquad u = 1+t^2$, $du = 2t\,dt$

x of $\vec{r}(t)$

$= \int_1^5 \frac{3}{2}u^{1/2}\,du = u^{3/2}\Big|_1^5 = (5)^{3/2} - 1 = 5\sqrt{5} - 1$

Let $\vec{F}(x,y) = 3x^2\vec{i} - \vec{j}$. Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where C is the top half of the circle of radius 2 centered at $(0,0)$, starting at $(2,0)$ and ending at $(-2,0)$.

A. 16

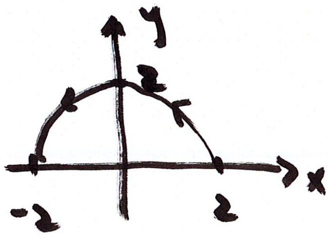
B. 8

C. 0

D. -8

E. -16

$\vec{F} = \langle 3x^2, -1\rangle$



$\vec{r}(t) = \langle \underset{x}{2\cos t}, \underset{y}{2\sin t}\rangle$

$0 \le t \le \pi$

(circle : $\vec{r}(t) = \langle R\cos t, R\sin t\rangle$
radius R)

$d\vec{r} = \langle -2\sin t, 2\cos t\rangle dt$

$\vec{F} = \langle 3x^2, -1\rangle = \langle 3(2\cos t)^2, -1\rangle$

$= \langle 12\cos^2 t, -1\rangle$

$\int_C \vec{F}\cdot d\vec{r} = \int_0^{\pi} \langle 12\cos^2 t, -1\rangle \cdot \langle -2\sin t, 2\cos t\rangle dt$

$= \int_0^{\pi} (-24\cos^2 t\sin t - 2\cos t)\, dt$

$= \int_0^{\pi} -24\cos^2 t \sin t\, dt - \int_0^{\pi} 2\cos t\, dt = \cdots = -16$

$u = \cos t$

$du = -\sin t\, dt$