The Tangent plane to $z = \ln(x-4y)$ at $(9, 2, 0)$ contains $(2, 1, a)$.
what is a?

A) -3

B. 2

C. -2

D. $\ln 2$

E. -8

tangent plane : $\cancel{F = \ln(x-4y)}$

$$F = \ln(x-4y) - z$$

$$\vec{\nabla} F = \left\langle \frac{1}{x-4y}, \frac{-4}{x-4y}, -1 \right\rangle$$

at $(9, 2, 0)$

$\vec{\nabla} F = \langle 1, -4, -1 \rangle$ normal to the tangent plane

plane : $\vec{n} \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$

$$\langle 1, -4, -1 \rangle \cdot \langle x-9, y-2, z \rangle = 0$$

$$x - 9 - 4y + 8 - z = 0$$

$\underset{\nearrow 2}{x} - 4\underset{\uparrow 1}{y} - \underset{\nwarrow a}{z} = 1$ eq. of tangent plane

$2 - 4 - a = 1 \rightarrow a = -3$

Find the directional derivative of $f(x,y) = xe^{y^2} + e^{x+y}$

at $(0,0)$ in the direction of $3\vec{i} - 4\vec{j}$

$D_{\vec{u}} f(x,y) = \vec{\nabla} f(x,y) \cdot \vec{u}$ unit vector giving the direction

$\vec{\nabla} f = \langle e^{y^2} + e^{x+y}, \; 2xye^{y^2} + e^{x+y} \rangle$

at $(0,0)$ $\vec{\nabla} f = \langle 2, 1 \rangle$

$\vec{u} = \dfrac{\langle 3, -4 \rangle}{|\langle 3, -4 \rangle|} = \dfrac{\langle 3, -4 \rangle}{5} = \langle \frac{3}{5}, -\frac{4}{5} \rangle$

$D_{\vec{u}} f(0,0) = \langle 2, +1 \rangle \cdot \langle \frac{3}{5}, -\frac{4}{5} \rangle = 2/5$

Evaluate the line integral $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = \langle y, -x, xy\rangle$

where C is parametrized by $\vec{r}(t) = \sin t\,\vec{i} + \cos t\,\vec{j} + t\,\vec{k}$ $\quad 0 \le t \le \pi$

$$\int_C \vec{F}\cdot d\vec{r} = \cancel{\int_C |\vec{F}\cdot\vec{r}\,'|\,dt} = \int_C \vec{F}\cdot\vec{r}\,'\,dt$$

$$\vec{r}\,' = \langle \cos t, -\sin t, 1\rangle$$

$$\vec{F} = \langle y, -x, xy\rangle = \langle \cos t, -\sin t, \sin t\cos t\rangle$$

$$\int_0^{\pi} \langle \cos t, -\sin t, \sin t\cos t\rangle \cdot \langle \cos t, -\sin t, 1\rangle\,dt$$

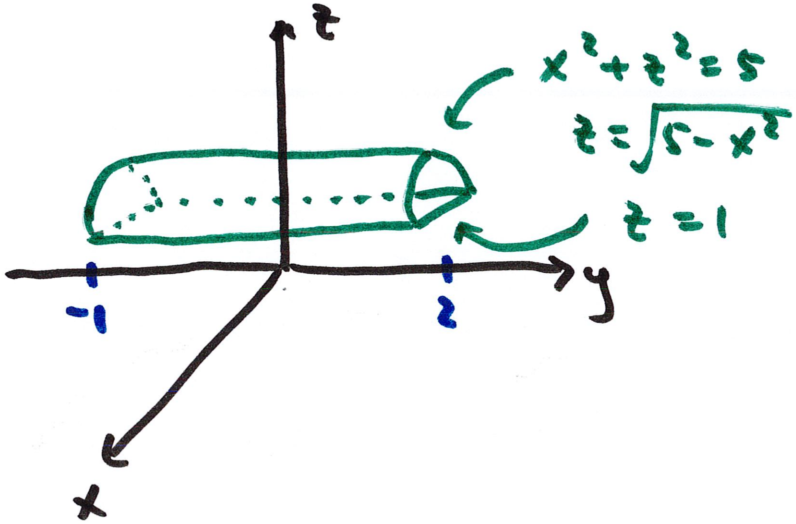
$$= \int_0^{\pi} (\cos^2 t + \sin^2 t + \sin t\cos t)\,dt = \int_0^{\pi} 1 + \underbrace{\sin t\cos t\,dt}_{\substack{u = \sin t \\ du = \cos t\,dt}}$$

$$= \pi$$

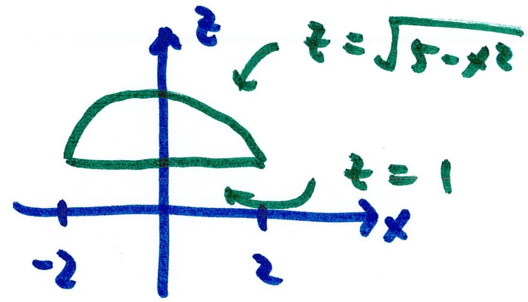
Suppose E is the space bounded above by the cylinder $x^2+z^2=5$

below by $z=1$ and on the sides by the planes $y=-1$ and $y=2$

Find $\iiint_E z\,dV$



choose xz-plane projection as the "floor"



$-2 \le x \le 2$

$1 \le z \le \sqrt{5-x^2}$

$-1 \le y \le 2$

$$\underbrace{\int_{-1}^{2}}_{y} \underbrace{\int_{-2}^{2}}_{x} \underbrace{\int_{1}^{\sqrt{5-x^2}}}_{z} z\,dz\,dx\,dy = \cdots = 16$$

find x bounds: $x^2+z^2=5$

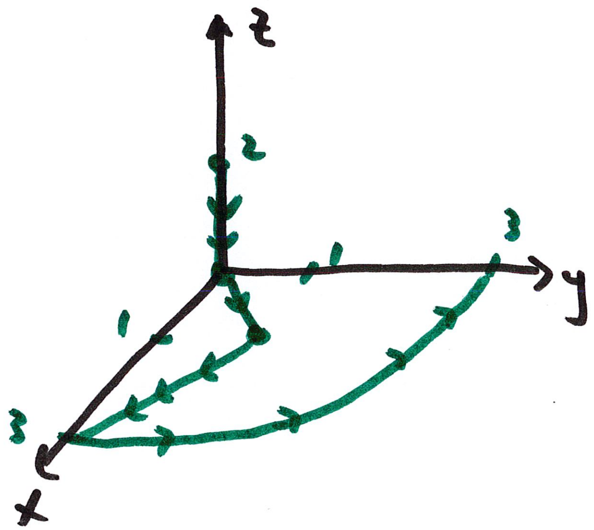
at $z=1 \rightarrow x^2+1=5$

$x^2=4$

$x=\pm 2$

The oriented curve C consists of line segment from (0, 0, 2) to (0, 0, 0), the line segment from (0, 0, 0) to (1, 1, 0) the line segment from (1, 1, 0) to (3, 0, 0) then a circular segment from (3, 0, 0) to (0, 3, 0)

Find $\int_C \vec{F} \cdot d\vec{r}$ w/ $\vec{F} = \langle ye^x, e^x, 2z \rangle$



options: parametrize the path

Fundamental Theorem of Line Integrals

$$\int_C \vec{F} \cdot d\vec{r} = \int_C \vec{\nabla}\phi \cdot d\vec{r} = \phi(\text{end}) - \phi(\text{start})$$

$\vec{F}$ must be conservative

is $\vec{F}$ conservative? Is curl $\vec{F} = \vec{0}$?

$$\text{curl } \vec{F} = \vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ye^x & e^x & 2z \end{vmatrix}$$

$= \langle 0, 0, e^x - e^x \rangle = \langle 0, 0, 0 \rangle$ so $\vec{F}$ is $\vec{\nabla}\phi$

find ϕ

$\vec{F} = \langle ye^x, e^x, 2z \rangle = \langle \phi_x, \phi_y, \phi_z \rangle$

$\phi_x = ye^x \rightarrow \phi = \int ye^x \, dx = ye^x + f(y,z)$

$\phi_y = e^x \qquad \phi_y = e^x + \frac{\partial f}{\partial y} = e^x \rightarrow \frac{\partial f}{\partial y} = 0$

$\phi_z = 2z \qquad \phi_z = \frac{\partial f}{\partial z} = 2z \rightarrow f = z^2 + g(y)$

$\frac{\partial f}{\partial y} = \frac{dg}{dy} = 0 \rightarrow g = C$

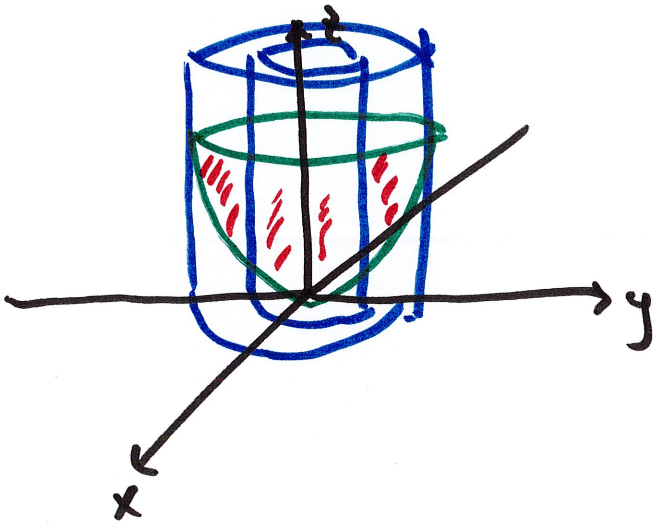
so, $\phi = ye^x + z^2 + C$

$\int_C \vec{F} \cdot d\vec{r} = \phi(\text{end}) - \phi(\text{start})$

$\phi(\text{end})$ at $(0,3,0)$; $\phi(\text{start})$ at $(0,0,2)$

$= (3 + C) - (4 + C) = -1$

Find the surface area of the part of the paraboloid $z = \frac{x^2}{2} + \frac{y^2}{2}$ that lies between the cylinders $x^2+y^2=8$ and $x^2+y^2=24$

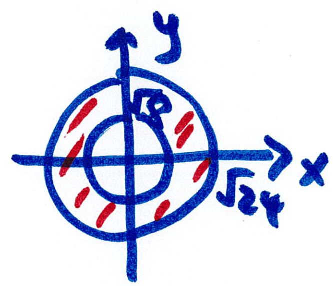


surface area : $\iint_S dS = \iint_R |\vec{r}_u \times \vec{r}_v| \, dA$

parameterize paraboloid : cylindrical

$u = r, \quad v = \theta$

$\vec{r}(u,v) = \langle u\cos v, \ u\sin v, \ \frac{1}{2}u^2 \rangle$



$\sqrt{8} \le u \le \sqrt{24}$

$0 \le v \le 2\pi$

$z = \frac{1}{2}(x^2+y^2)$

$= \frac{1}{2}(r^2)$

$\vec{r}_u = \langle \cos v, \sin v, u \rangle$

$\vec{r}_v = \langle -u\sin v, u\cos v, 0 \rangle$

$\vec{r}_u \times \vec{r}_v = \langle -u^2\cos v, -u^2\sin v, u \rangle$

$|\vec{r}_u \times \vec{r}_v| = \sqrt{u^4+u^2} = u\sqrt{u^2+1}$

$$\int_0^{2\pi} \int_{\sqrt{8}}^{\sqrt{24}} \underbrace{u\sqrt{u^2+1}\,du}_{\substack{\text{Subs: } w = u^2+1 \\ dw = 2u\,du}}\,dv = \cdots = \frac{196\pi}{3}$$