The number and value of the absolute max of the function $f(x,y) = x^2 - xy + y^2$ on the domain $\underbrace{2x^2+2y^2 \le 1}$

A. Two max with value 1

B. Two max " " 1/2

C. Four max " " 1/2

D. " " " 3/4

E. Two max " " 3/4

two conditions: $2x^2+2y^2 < 1$

$2x^2+2y^2 = 1$

find critical pts of $f(x,y)$ inside the circle

Lagrange multipliers for points on the circle

critical pts: $f_x = 0 \quad f_y = 0$

$f_x = 2x - y = 0 \rightarrow y = 2x$

$f_y = 2y - x = 0$

$4x - x = 0 \rightarrow x = 0,\ y = 0$

$\boxed{\text{cp: } (0,0)}$ inside $2x^2+2y^2=1$? yes, so keep it now solve max $f(x,y) = x^2 - xy + y^2$ subject to $g(x,y) = 2x^2 + 2y^2 - 1 = 0$

Solve: $\vec{\nabla} f = \lambda \vec{\nabla} g$

$$\langle 2x - y, 2y - x \rangle = \lambda \langle 4x, 4y \rangle$$

$$\left.\begin{array}{ll} 2x - y = \lambda \cdot 4x & \rightarrow \quad \lambda = \dfrac{2x-y}{4x} \\ 2y - x = \lambda \cdot 4y & \rightarrow \quad \lambda = \dfrac{2y-x}{4y} \end{array}\right\} \quad \frac{2x-y}{x} = \frac{2y-x}{y}$$

$$2xy - y^2 = 2xy - x^2$$

$$\underbrace{x^2 = y^2}_{\text{sub into } g(x,y)} \qquad y = \pm x$$

$$2x^2 + 2y^2 = 1$$

$$2x^2 + 2x^2 = 1$$

$$x^2 = \frac{1}{4}$$

$$x = \pm \frac{1}{2}, \quad y = \pm \frac{1}{2}$$

points to check:

$\left(\frac{1}{2}, \frac{1}{2}\right), \left(-\frac{1}{2}, -\frac{1}{2}\right)$

$\left(\frac{1}{2}, -\frac{1}{2}\right), \left(-\frac{1}{2}, \frac{1}{2}\right)$

critical pt from earlier: $(0,0)$

$f(0,0) = 0$

$f(\frac{1}{2}, \frac{1}{2}) = \frac{1}{4}$

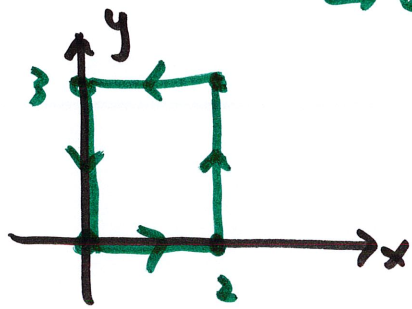
$$\left.\begin{aligned} f(\tfrac{1}{2}, -\tfrac{1}{2}) &= 3/4 \\ f(-\tfrac{1}{2}, \tfrac{1}{2}) &= 3/4 \end{aligned}\right\} \text{ two max of } 3/4$$

$f(-\frac{1}{2}, -\frac{1}{2}) = 1/4$

Use Green's Theorem to evaluate $\int_C x^2 dy$ where C is the boundary of the rectangle with vertices $(0,0)$, $(2,0)$, $(2,3)$, $(0,3)$ ~~okite~~ oriented counterclockwise.

↳ usual Green's Theorem assumption

A. 4
B. 8
C. 12
D. 16
E. 24



↳ $\vec{F} = \langle f, g \rangle$

then $\iint_R (g_x - f_y)\, dA = \oint_C \vec{F} \cdot d\vec{r}$

$= \oint_C f\,dx + g\,dy$

we need to identify the vector field $\vec{F}$ first

$\oint_C f\,dx + g\,dy = \int_C x^2 dy \rightarrow f = 0,\ g = x^2$ so $\vec{F} = \langle \overset{f}{0}, \overset{g}{x^2} \rangle$

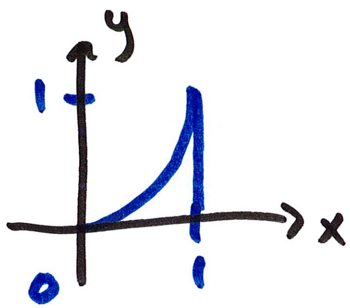
replace $\int_C x^2 dy$ with $\underbrace{g_x - f_y = 2x}$

$\iint_R 2x\, dA = \int_0^2 \int_0^3 2x\, dy\, dx = \cdots = 12$

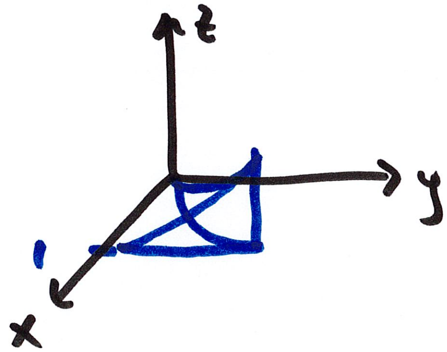
Rewrite the iterated integral $\int_0^1 \int_0^{x^2} \int_0^y f(x,y,z)\,dz\,dy\,dx$

in the order $dx\,dz\,dy$

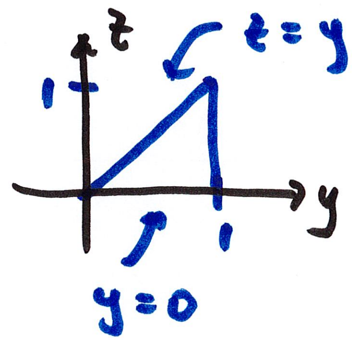
$$\int_0^1 \int_0^{x^2} \int_0^y f(x,y,z)\,dz\,\underbrace{dy\,dx}_{\text{"floor" is xy-plane}} \rightarrow \begin{array}{l} 0 \le x \le 1 \\ 0 \le y \le x^2 \\ 0 \le z \le y \end{array}$$



then with $0 \le z \le y$ we can get a 3D view

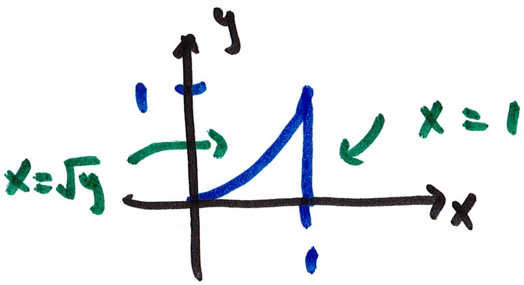


now go to $\cancel{\underbrace{dz\,dy\,dx}_{\text{"floor" is}}}$ $\quad dx\,\underbrace{dz\,dy}_{\text{"floor" is yz-plane}}$



y is bounded by constants: $0 \le y \le 1$

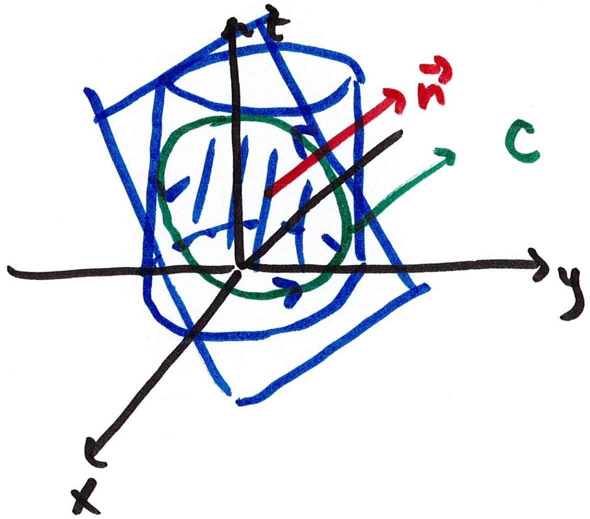
$0 \le z \le y$



so $\sqrt{y} \le x \le 1$

new integral: $\displaystyle\int_0^1 \int_0^y \int_{\sqrt{y}}^1 f(x,y,z)\,dx\,dz\,dy$

Consider the curve C : $\vec{r}(t) = \langle \cos t, \sin t, 1 - \cos t - \sin t \rangle$ $\quad 0 \le t \le 2\pi$
which is the intersection of the cylinder $x^2 + y^2 = 1$ with the plane
$x + y + z = 1$. If $\vec{F} = \langle y + \sin x, z + \sin y, x + \cos z \rangle$
find $\int_C \vec{F} \cdot d\vec{r}$

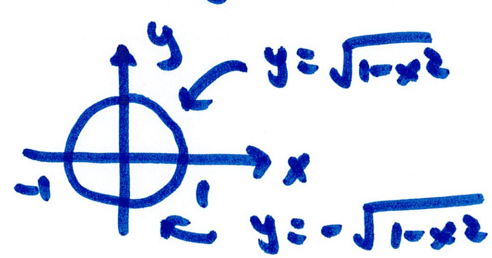


possible choices : do $\int_C \vec{F} \cdot d\vec{r}$
we have $\vec{F}$
but substituting x, y, z of $\vec{r}$
in $\vec{F}$ ends up with a mess

Stokes' : $\int_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl} \vec{F} \cdot d\vec{S}$
S is any surface w/ C as the boundary

let's let S be the ellipse enclosed by C

$\vec{r}(u, v) = \langle u, v, 1 - u - v \rangle$
x $\nearrow$ $\quad$ $\uparrow$ y



$-1 \leq u \leq 1$

$-\sqrt{1-u^2} \leq v \leq \sqrt{1-u^2}$

$\vec{r}_u = \langle 1, 0, -1 \rangle$ $\quad\quad$ $\vec{r}_u \times \vec{r}_v = \langle 1, 1, 1 \rangle$ is this oriented correctly?

$\vec{r}_v = \langle 0, 1, -1 \rangle$

yes, it is upward which agrees w/ C's orientation

$$\text{curl } \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y+\sin x & z+\sin y & x+\cos z \end{vmatrix} = \langle -1, -1, -1 \rangle$$

$$\iint_S \text{curl } \vec{F} \cdot d\vec{S} = \int_{-1}^{1} \int_{-\sqrt{1-u^2}}^{\sqrt{1-u^2}} \langle -1, -1, -1 \rangle \cdot \langle 1, 1, 1 \rangle \, dv \, du$$

$$= -3 \underbrace{\int_{-1}^{1} \int_{-\sqrt{1-u^2}}^{\sqrt{1-u^2}} dv \, du}_{\substack{\text{area of circle} \\ \text{radius 1}}} = -3 \cdot \pi (1)^2 = -3\pi$$