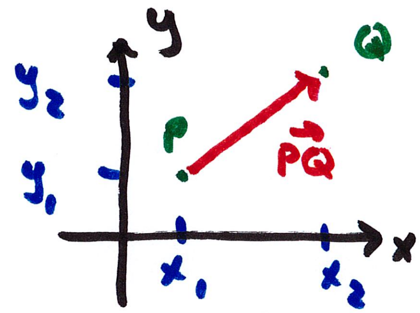
13.1 - 13.4 Review of Vectors

basic operations

$P(x_1, y_1)$, $Q(x_2, y_2)$

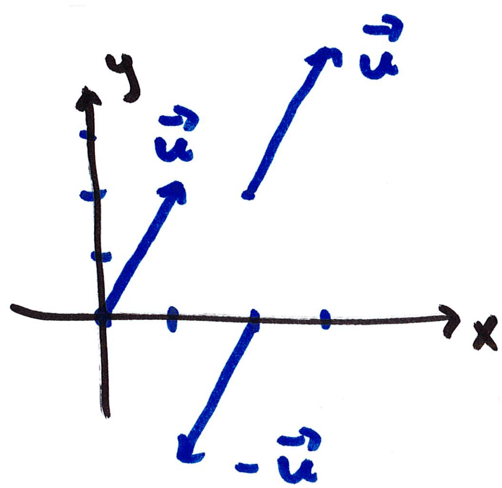


vector from P to Q $\quad \vec{PQ} = \langle x_2 - x_1, y_2 - y_1 \rangle = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j}$

Q to P $\quad \vec{QP} = \langle x_1 - x_2, y_1 - y_2 \rangle = (x_1 - x_2)\vec{i} + (y_1 - y_2)\vec{j}$

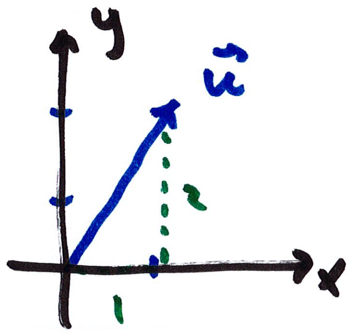
note $\vec{PQ} = -\vec{QP}$ negative sign reverses direction

for example: $\vec{u} = \langle 1, 2 \rangle$ one in x-direction from starting pt
two in y-direction



$-\vec{u} = \langle -1, -2 \rangle$

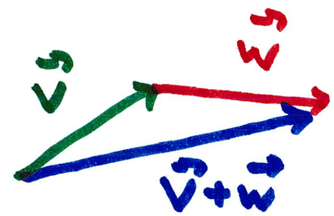
$\swarrow$ one left $\qquad \searrow$ two down magnitude: length of vector



length: $|\vec{u}| = \sqrt{1^2 + 2^2} = \sqrt{5}$

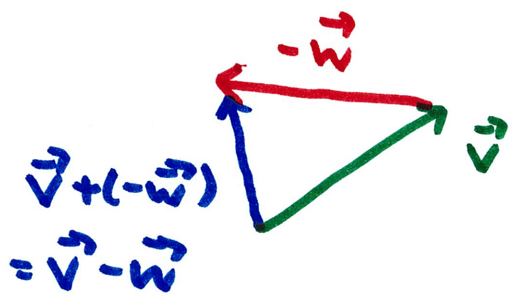
adding/subtracting:

$\vec{v} = \langle 1, 2, 3 \rangle$ $\qquad$ $\vec{w} = \langle 4, 5, 6 \rangle$



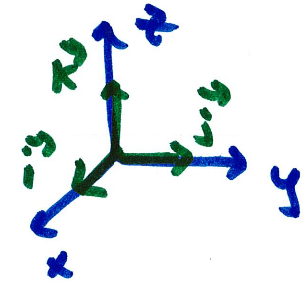
$\vec{v}+\vec{w} = \langle 1+4, 2+5, 3+6 \rangle = \langle 5, 7, 9 \rangle$

$\vec{v}-\vec{w} = \langle 1-4, 2-5, 3-6 \rangle = \langle -3, -3, -3 \rangle$



$\vec{v}+(-\vec{w})$

unit vector: magnitude is 1   for example $(\vec{i}, \vec{j}, \vec{k})$

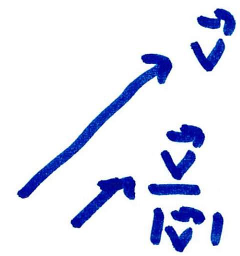


$\vec{v} = \langle 1, 2, 3 \rangle$ is NOT a unit vector

because $|\vec{v}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14} \neq 1$

a unit vector w/ same direction as $\vec{v}$

$$\frac{\vec{v}}{|\vec{v}|} = \frac{\langle 1, 2, 3 \rangle}{\sqrt{14}} = \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$$



for example, if we want a vector of length 2 parallel to $\vec{v}$:

$$2\frac{\vec{v}}{|\vec{v}|} = \left\langle \frac{2}{\sqrt{14}}, \frac{4}{\sqrt{14}}, \frac{6}{\sqrt{14}} \right\rangle$$

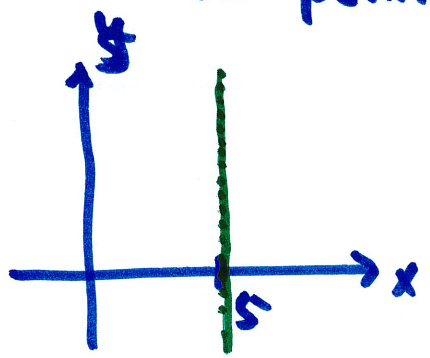
vector length 3 in opposite direction as $\vec{v}$

$$-3\frac{\vec{v}}{|\vec{v}|}$$

most shapes in 2D ($\mathbb{R}^2$) are similar to their counterparts in 3D ($\mathbb{R}^3$)

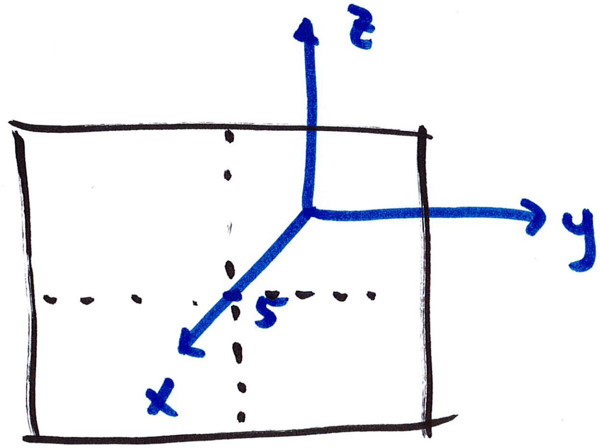
$x = 5$ in $\mathbb{R}^2$ is a collection of points w/ $x = 5$, y all reals

all points of the form $(5, c)$



some real #

in 3D, same idea: all points of the form $(5, c, d)$



circle becomes a sphere

$$(x-h)^2 + (y-k)^2 = r^2$$

circle center : (h, k)
radius : r

3D : $$(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2$$

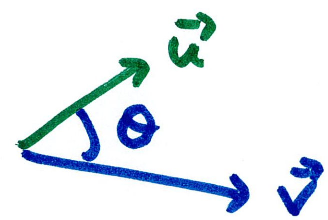
sphere center : (h, k, l)
radius r

<u>vector dot product</u>

$\vec{u} = \langle 1, 2, 3 \rangle \qquad \vec{v} = \langle 4, 5, 6 \rangle$

$\vec{u} \cdot \vec{v} = 1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6 = 32$ <u>a scalar</u>

another formula : $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos \theta$



note if $\vec{u} \perp \vec{v} \qquad \vec{u} \cdot \vec{v} = 0$

also $\vec{v} \cdot \vec{u} = \vec{u} \cdot \vec{v}$

Cross product

$\vec{u} = \langle 2, 1, 1 \rangle \qquad \vec{v} = \langle 5, 0, 1 \rangle$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ 5 & 0 & 1 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & 1 \\ 5 & 1 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & 1 \\ 5 & 0 \end{vmatrix}$$

$$= \vec{i}\,(1 \cdot 1 - 0 \cdot 1) - \vec{j}\,(2 \cdot 1 - 5 \cdot 1) + \vec{k}\,(2 \cdot 0 - 1 \cdot 5)$$

$$= (1)\,\vec{i} + (3)\,\vec{j} - 5\,\vec{k} = \langle 1, 3, -5 \rangle$$ vector

$\vec{u} \times \vec{v}$ is $\perp$ to BOTH $\vec{u}$ and $\vec{v}$

$$\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$$