13.6 Quadric Surfaces (part 1)

in $\mathbb{R}^2$ equations like $y = x^2$ are <u>curves</u>

in $\mathbb{R}^3$ equations in terms of x, y, and z, the graphs are <u>surfaces</u>

for example, $x^2 + y^2 + z^2 = 1$ is a sphere

$(x-2) + 2(y+3) - 3(z+4) = 0$ is a plane

normal vector: $\langle 1, 2, -3 \rangle$

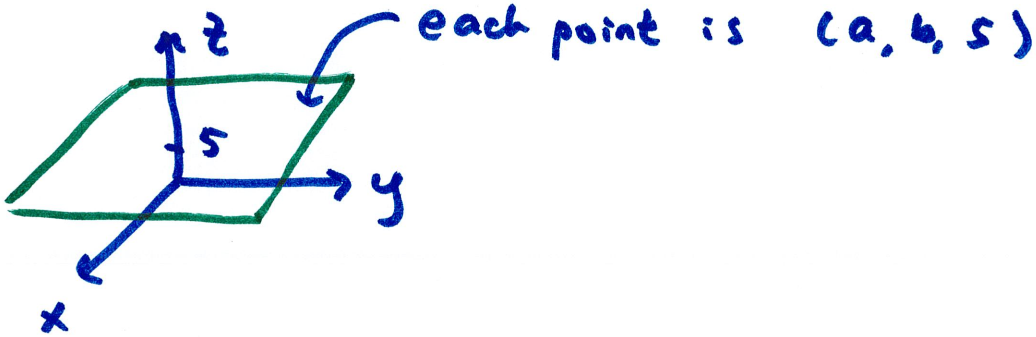
through point: $(2, -3, -4)$

Sometimes a variable is missing

→ the missing variable is a "free variable"

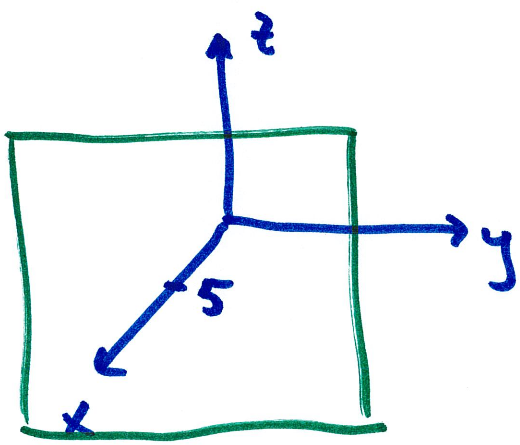
not related to others, so can be anything for example, $z = 5$ in $\mathbb{R}^3$ is missing x, y

surface made of points $(a, b, 5)$ where $-\infty < a < \infty$, $-\infty < b < \infty$



likewise, $x = 5$ in $\mathbb{R}^3$ is missing y, z

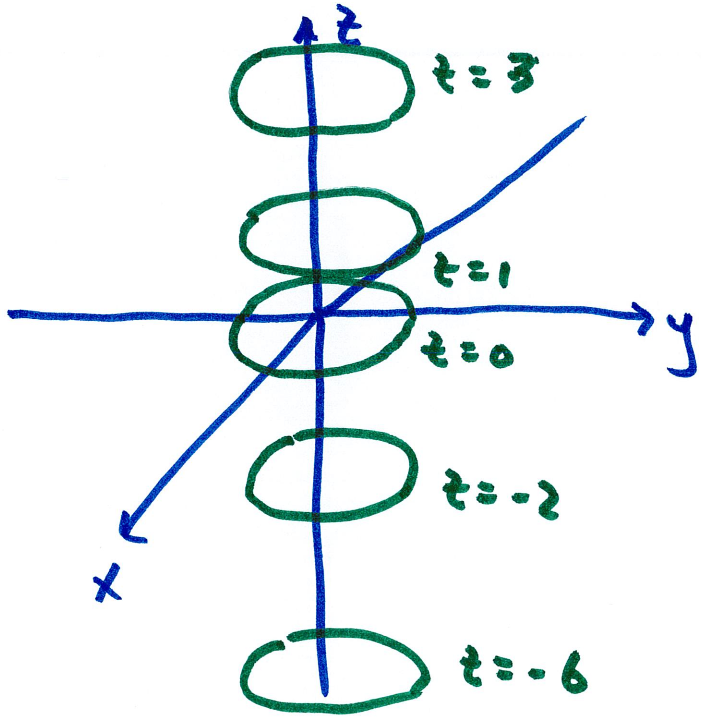
surface contains points $(5, a, b)$ $-\infty < a < \infty$, $-\infty < b < \infty$



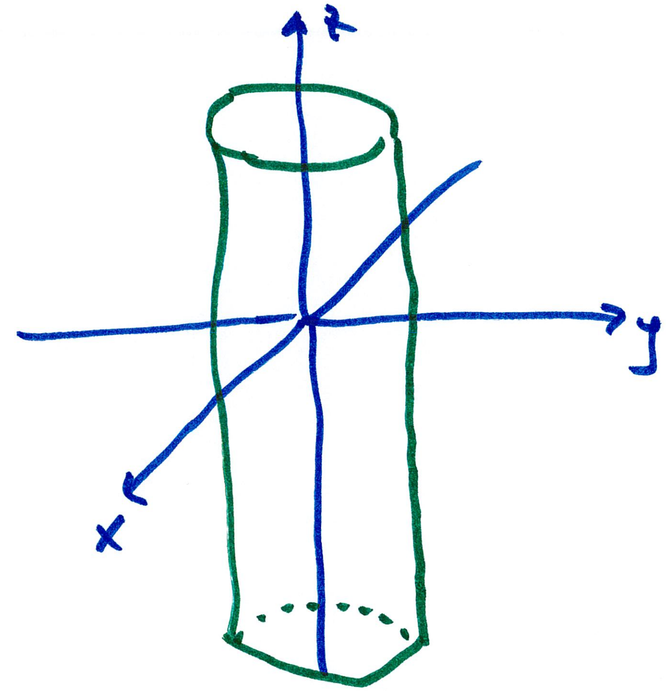
$x^2+y^2=1$ in $\mathbb{R}^3$ contains no z

Surface made of points whose x and y satisfy $x^2+y^2=1$
but $-\infty < z < \infty$

for any z, we have circle radius 1 center $(0,0,z)$

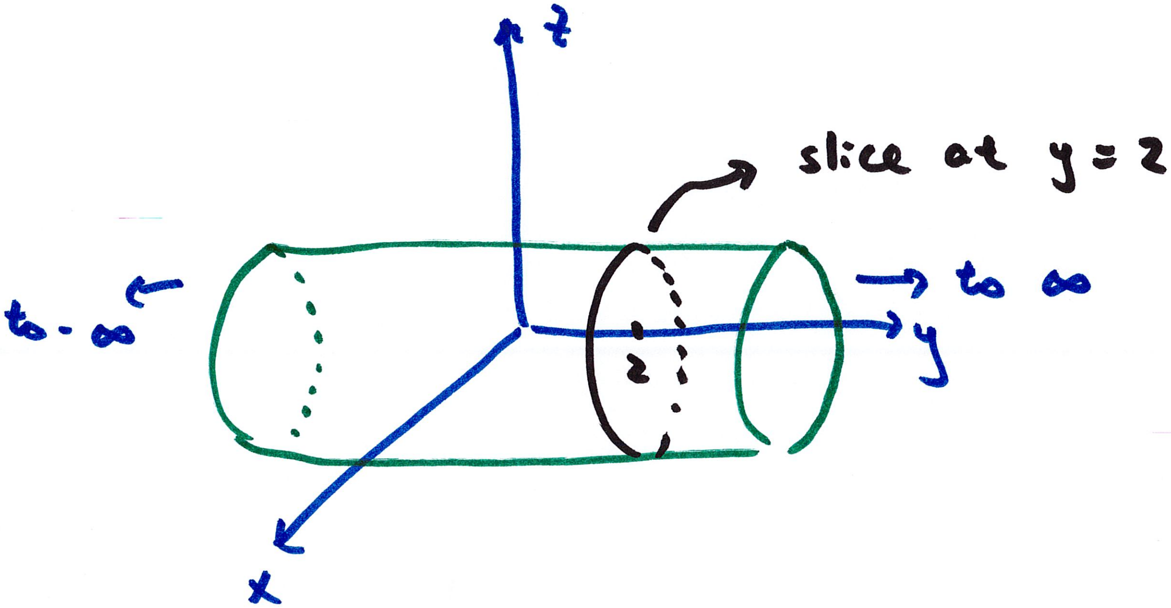


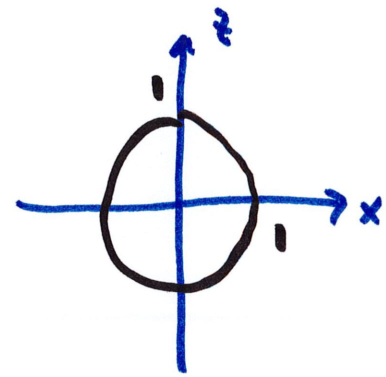
Stack them →



circular cylinder

$x^2 + z^2 = 1$ in $\mathbb{R}^3$ is missing y





these slices at x, y, z equal to ~~th~~ some constant are called <u>traces</u>

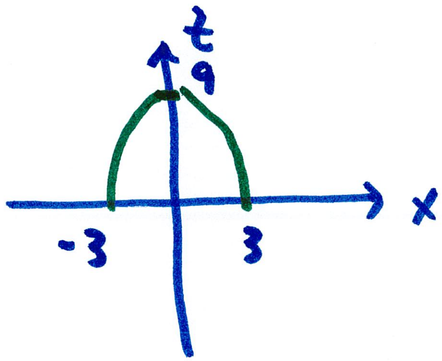
intersection of surface with the <u>xy-plane</u> is called the <u>xy-trace</u>

" " " <u>xz-plane</u> " " <u>xz-trace</u>

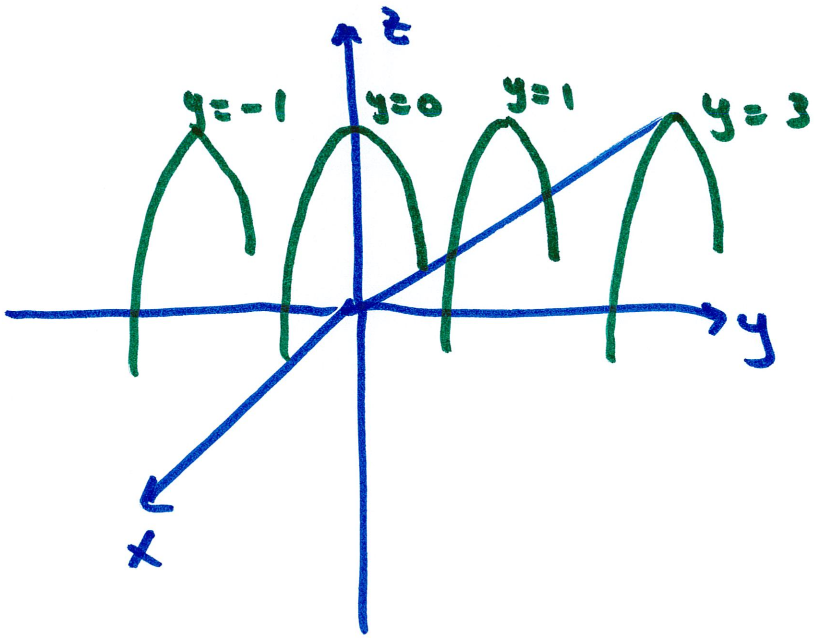
" " " <u>yz-plane</u> " " <u>yz-trace</u>

example $z = 9 - x^2$ has no y so $-\infty < y < \infty$

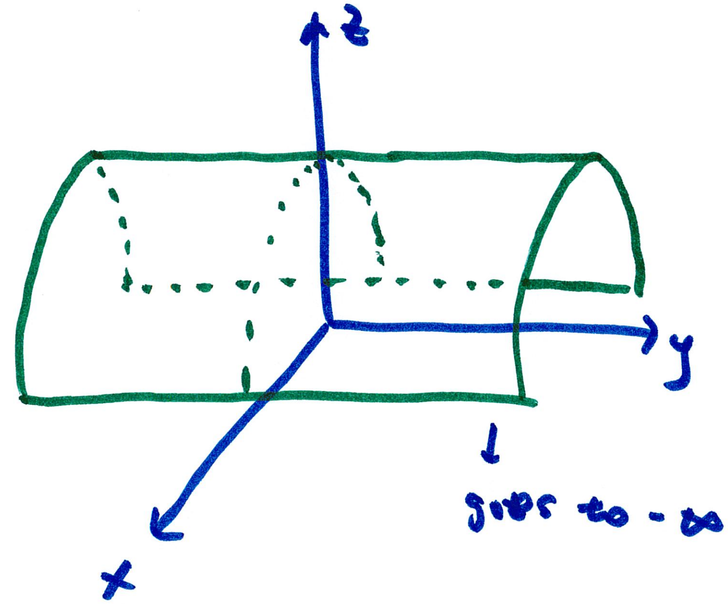
for each y. $z = 9 - x^2$ is a parabola



same cross section for all y



stack →



parabolic cylinder

stacking traces is an important skill to sketch surfaces

$x^2+y^2+z^2=16$ pretend we didn't know this is a sphere

x-ints: $x^2=16 \to x=\pm 4$

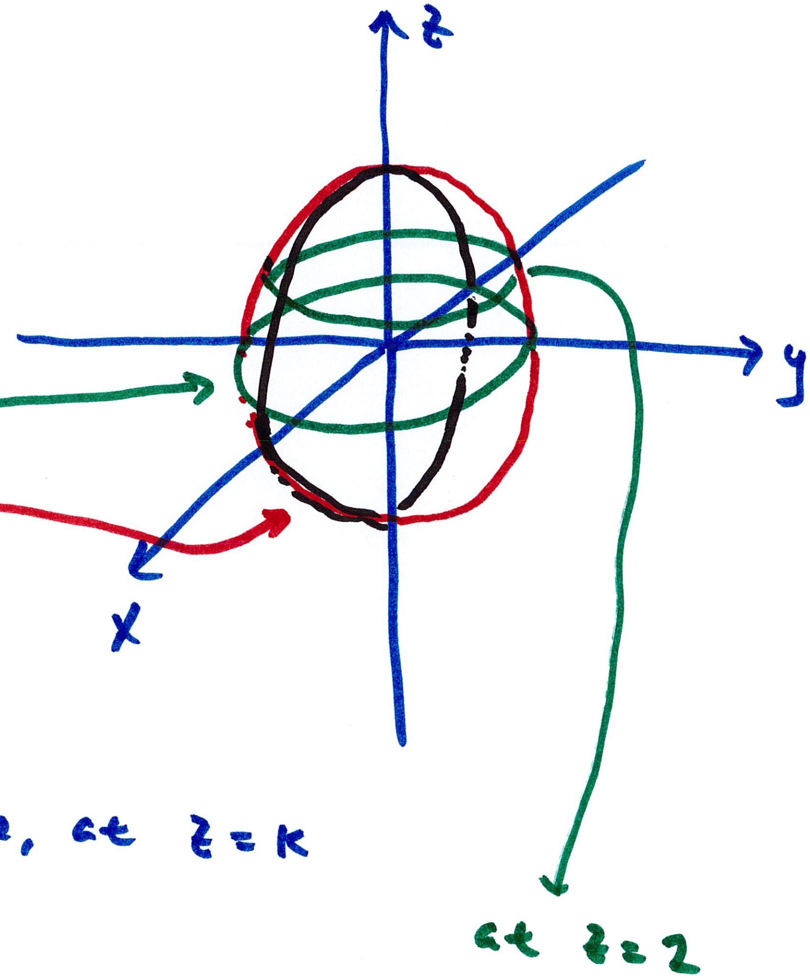
y-ints: $y=\pm 4$

z-ints: $z=\pm 4$

xy-trace $(z=0)$: $x^2+y^2=16$

yz-trace $(x=0)$: $y^2+z^2=16$

xz-trace $(y=0)$: $x^2+z^2=16$



↳ can take more slices at, for example, at $z=k$

$x^2+y^2+k^2=16$

$x^2+y^2=16-k^2$ circle radius $\sqrt{16-k^2}$ $-4\le k\le 4$

example $x^2+y^2=z^2$

x-ints: $x=0$

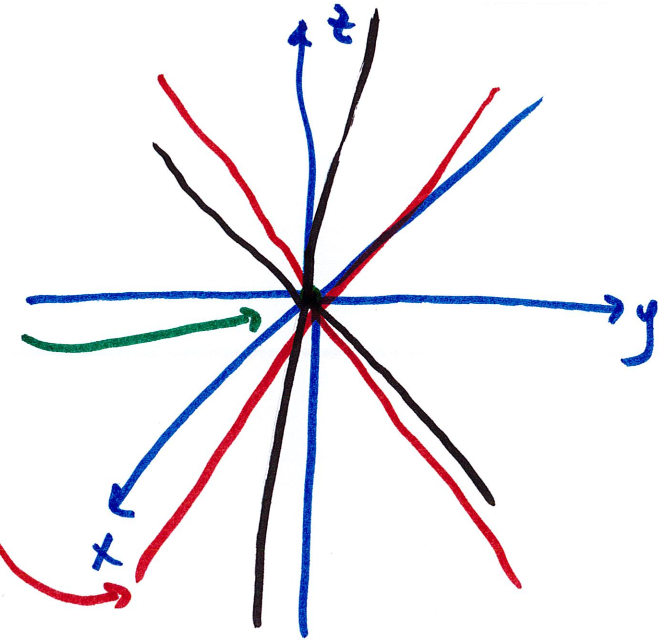
y-ints: $y=0$

z-ints: $z=0$

xy-trace $(z=0)$: $x^2+y^2=0$ (point at origin)

yz-trace $(x=0)$: $y^2=z^2 \rightarrow y=\pm z$ two lines

xz-trace $(y=0)$: $x^2=z^2 \rightarrow x=\pm z$

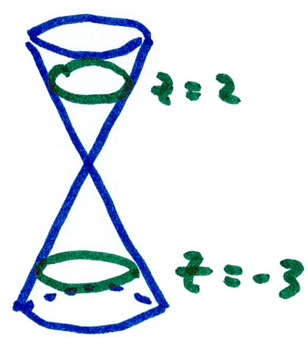


slice at $z=k$

$x^2+y^2=k^2$ circle radius $|k|$ $-\infty<k<\infty$

circle gets bigger as $z\to\infty$ or $z\to-\infty$

stack them:



double cone

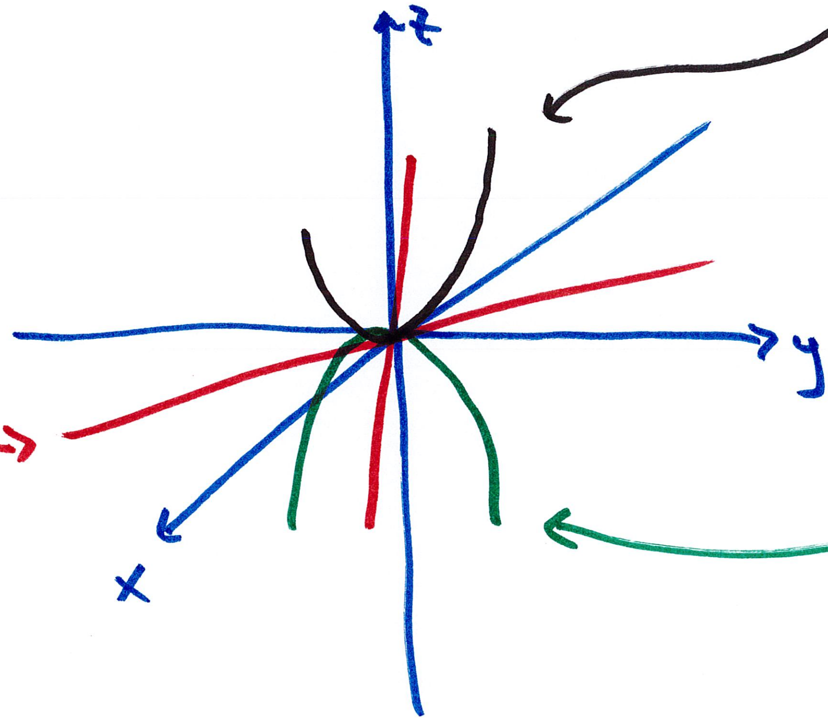
example $z = x^2 - y^2$

x, y, z ints are 0

xy-trace $(z=0)$: $y = \pm x$ lines

yz-trace $(x=0)$: $z = -y^2$ parabola

xz-trace $(y=0)$: $z = x^2$ parabola

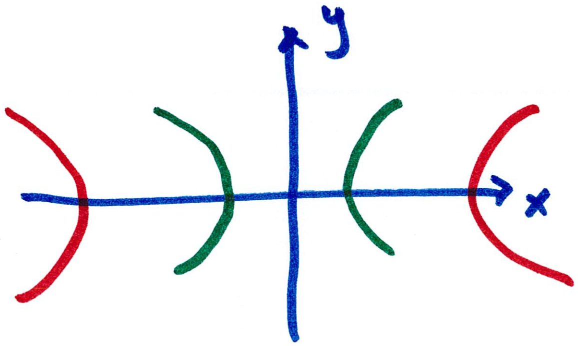


need more slices

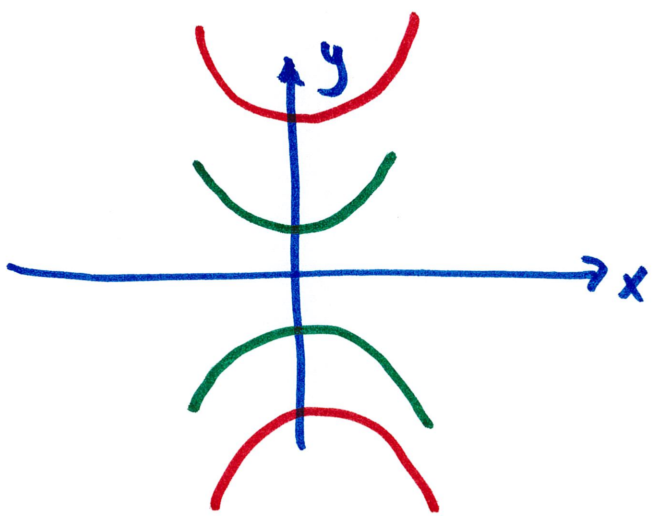
more slices at $z = k$

$x^2 - y^2 = k$

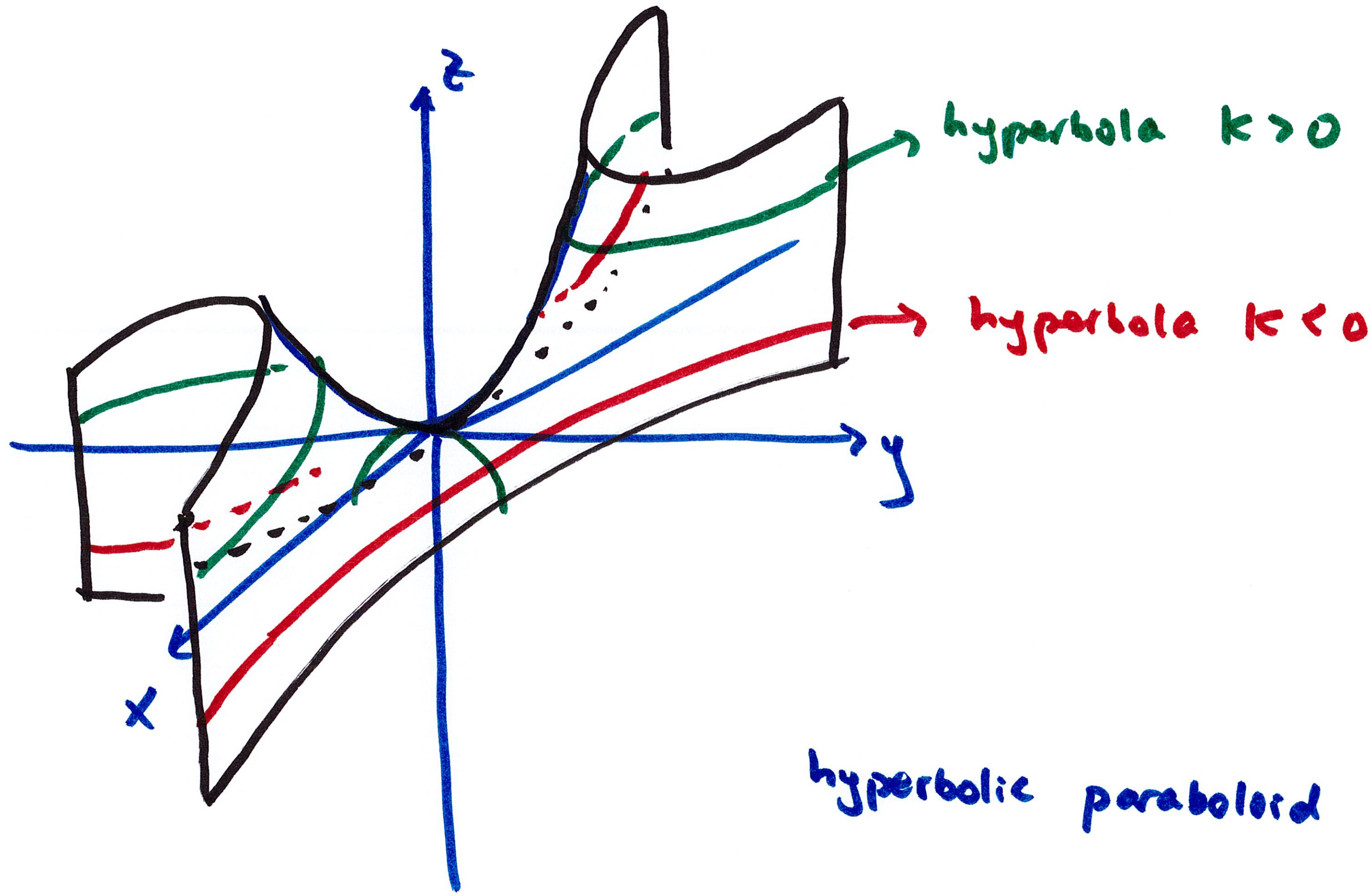
if $k > 0$, this is a hyperbola w/ intercepts on x-axis
branches more apart as k increases



if $k < 0$, intercepts on y-axis



Slices at $y=k$ and $x=k$ are all parabolas



hyperbolic paraboloid