13.6 Quadric Surfaces (part 2)

example $x^2+y^2-z^2=1$

x-ints: $x=\pm 1$

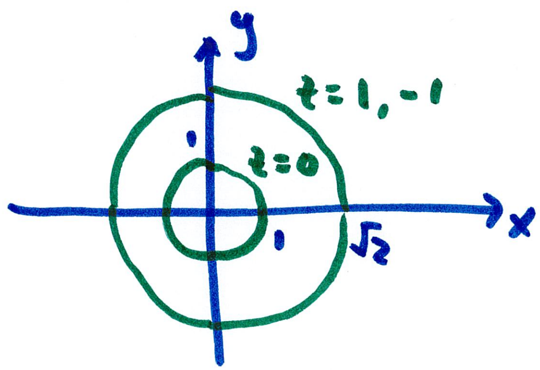
y-ints: $y=\pm 1$

z-ints: $-z^2=1 \rightarrow$ no solutions, so no z-ints.

xy-traces $(z=0)$: $x^2+y^2=1$ circle radius 1

traces at $z=k$: $x^2+y^2=1+k^2$ circle radius $\sqrt{1+k^2}$

circle gets bigger as $z\to\infty$ or $z\to-\infty$



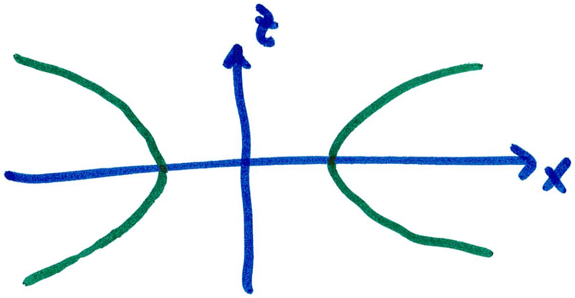
xz-trace $(y=0)$ $\quad x^2 - z^2 = 1$

hyperbola vertices are on x-axis

if $z=0$, $x^2=1 \rightarrow x = \pm 1$

if $x=0$, $-z^2 = 1 \rightarrow$ no solutions

branches of hyperbola go through x-axis



slices at $y=k$: $\quad x^2 - z^2 = 1-k^2$

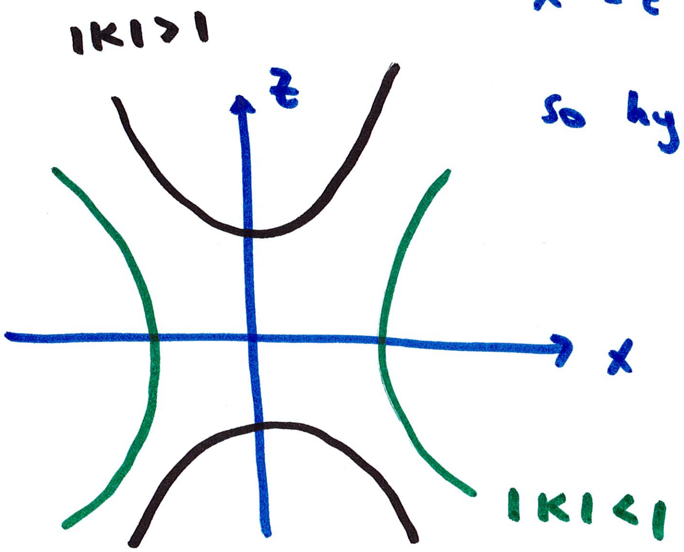
if $|k|<1$ $\quad (-1<k<1)$ $\quad 1-k^2>0$

so hyperbola goes through x-axis

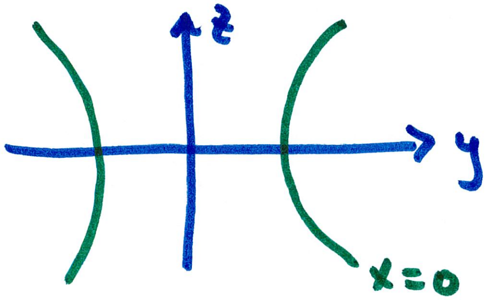
if $|k|>1$ $\quad 1-k^2<0$

$$x^2 - z^2 = \underbrace{1-k^2}_{<0} \rightarrow z^2 - x^2 = \underbrace{k^2-1}_{>0}$$

so hyperbolas go through z-axis



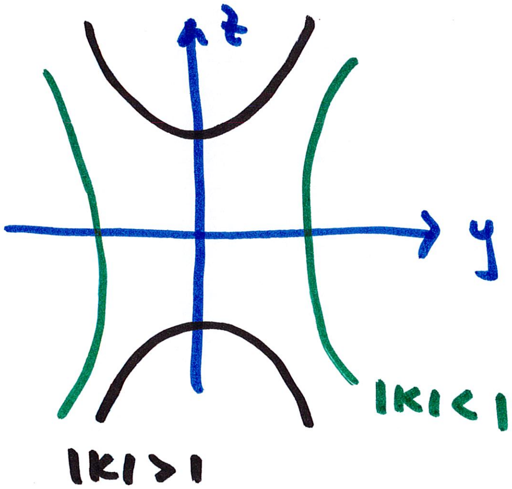
yz-trace $(x=0)$: $y^2 - z^2 = 1$ hyperbola vertices on y-axis

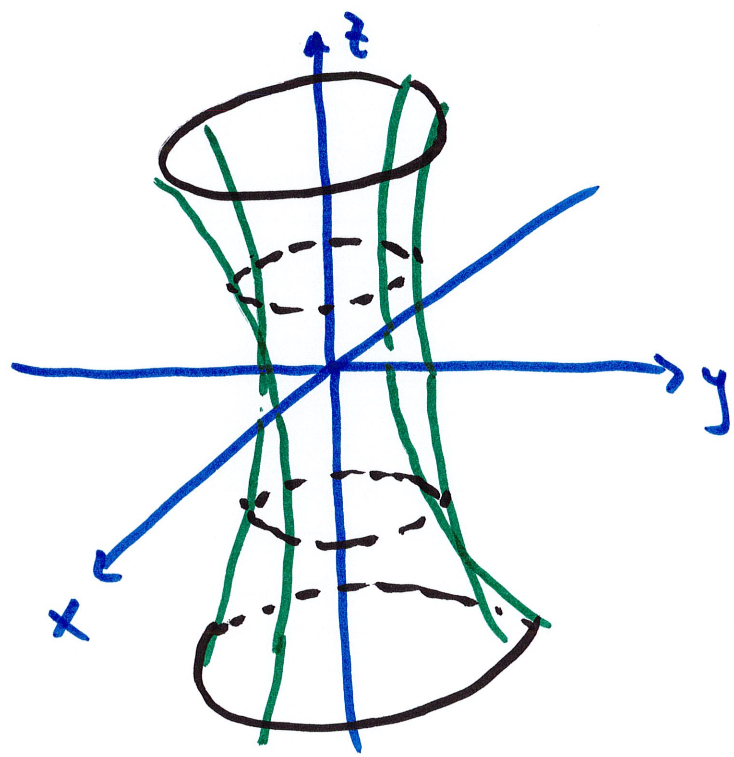


traces w/ $x=k$: $y^2 - z^2 = 1 - k^2$

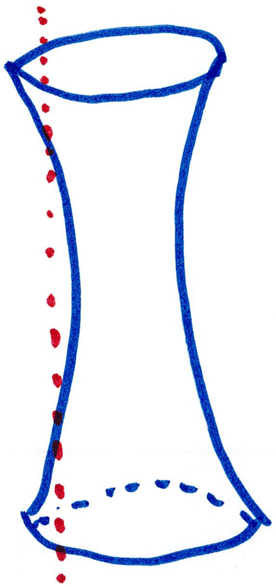
if $|k| < 1$ then hyperbolas open left/right

if $|k| > 1$ " " " up/down





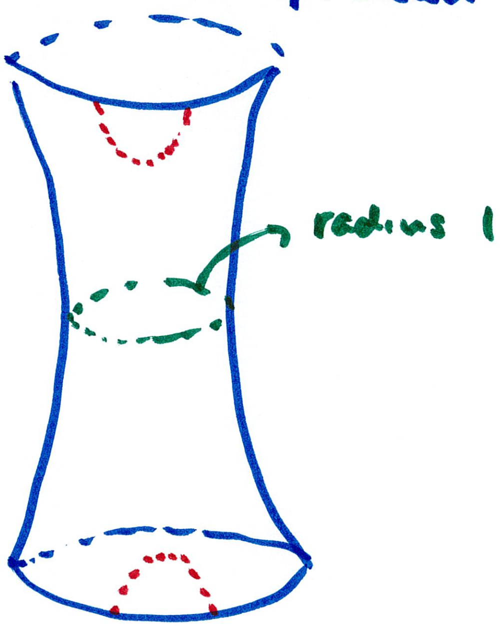
close to origin, side views see hyperbolas opening L/R looks like



Hyperboloid of One sheet

where are the switching hyperbolas?

hyperbolic slices are up/down when x and y are > 1 (far from origin)



slice w/ x or y > 1 does not go through this circle

(slices look like the red hyperbola on the left)

<u>example</u> $-x^2-y^2+z^2=1$

x-ints: $-x^2=1$ none

y-ints: $-y^2=1$ none

z-ints: $z^2=1$ $z=\pm 1$

xy-trace $(z=0)$: $-x^2-y^2=1$

$x^2+y^2=-1$ no such shape

this surface does NOT tough xy-plane

trace w/ $z=k$: $-x^2-y^2=1-k^2$

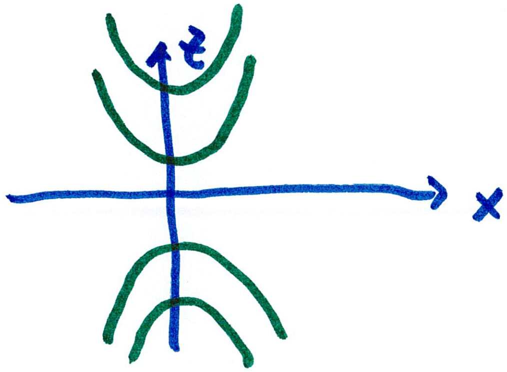
$x^2+y^2=k^2-1$ circle radius $\sqrt{k^2-1}$

~~F~~ $k^2 \geq 1$ $k \geq 1$ or $k \leq -1$

slices are circles at $z \geq 1$ or $z \leq -1$ but nothing in between

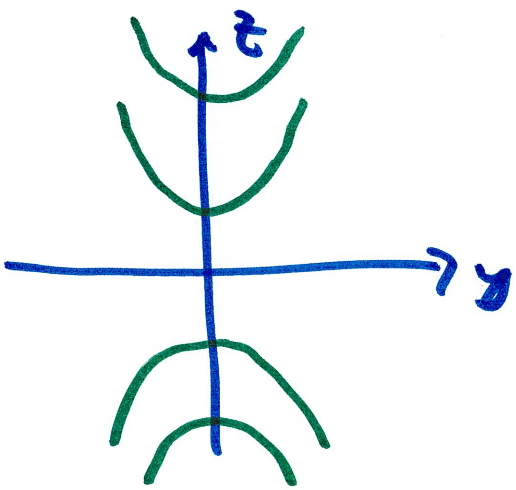
xz-trace ($y=0$) : $-x^2+z^2=1$ hyperbola vertices are at $z=\pm 1$
opens up / down

trace w/ $y=k$: $-x^2+z^2=1+k^2$ vertices always on z-axis since
$1+k^2 \geq 0$ no switching hyperbolas



yz-trace ($x=0$) : $-y^2+z^2=1$ hyperbola w/ vertices on z-axis
up/down

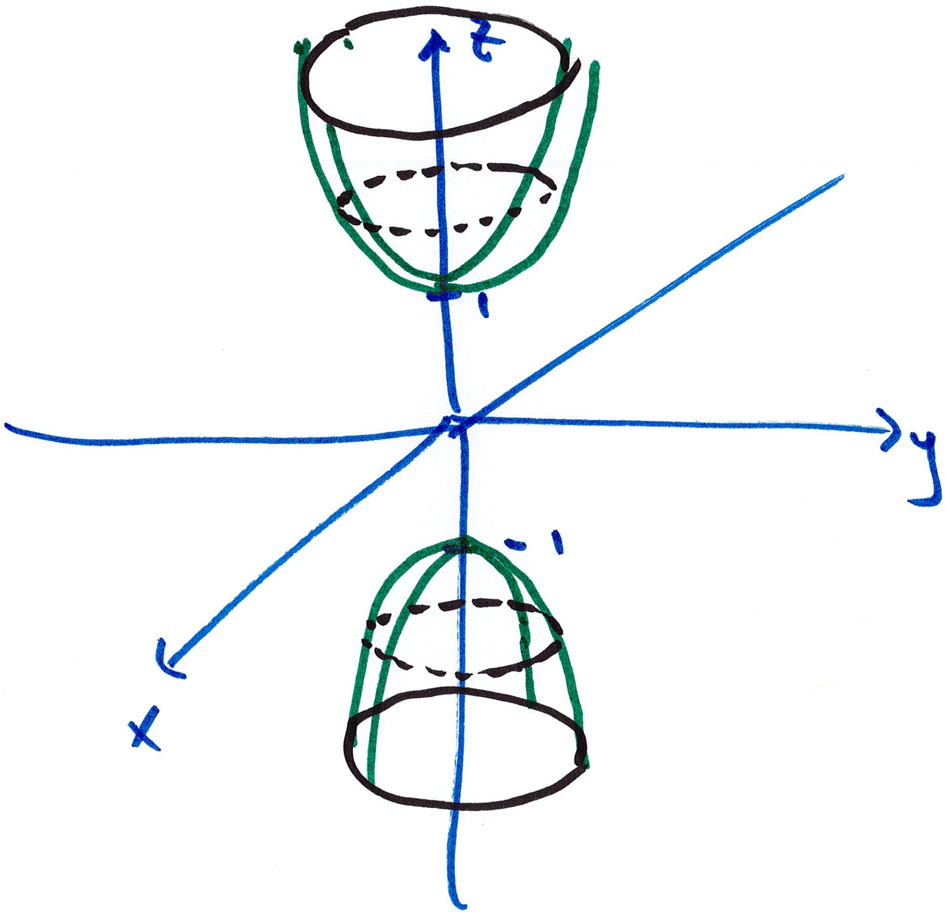
trace w/ $x=k$: $-y^2+z^2=1+k^2$ always up/down, like w/ xz-traces
no switching

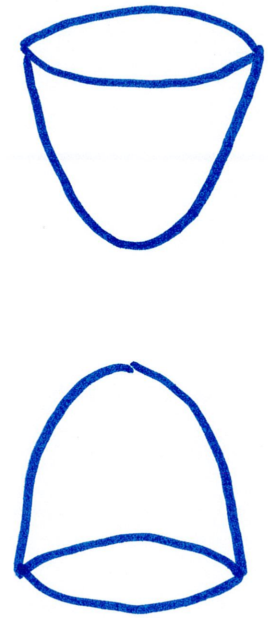


put them together: side views are hyperbolas (up/down)

top/bottom views are circles

"gap" between $z=1$ and $z=-1$



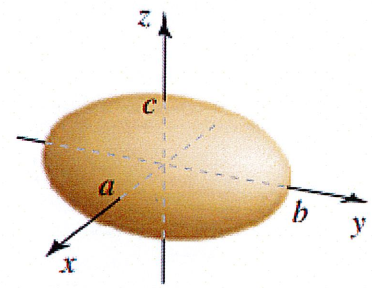
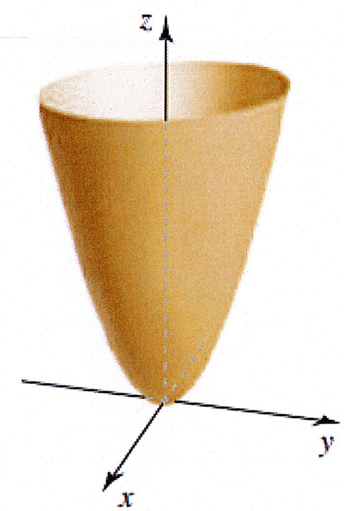
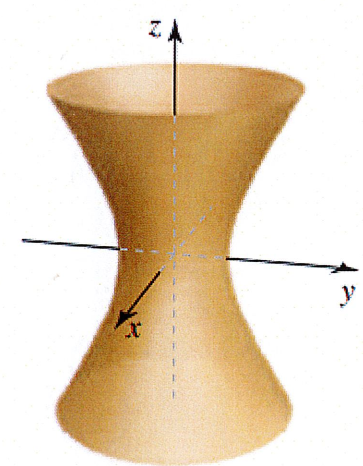


<u>Hyperboloid of</u>

<u>Two sheets</u>

Quadric Surfaces : $Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$

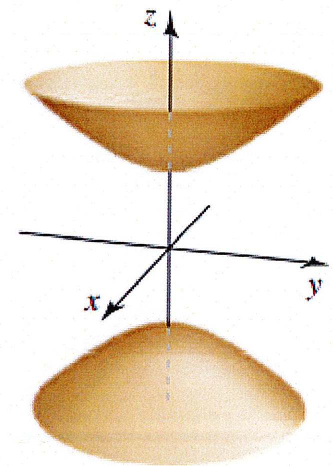
Table 13.1

Name	Standard Equation	Features	Graph
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	All traces are ellipses.	
Elliptic paraboloid	$z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	Traces with $z = z_0 > 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are parabolas.	
Hyperboloid of one sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$	Traces with $z = z_0$ are ellipses for all z_0. Traces with $x = x_0$ or $y = y_0$ are hyperbolas.	

Hyperboloid of two sheets

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

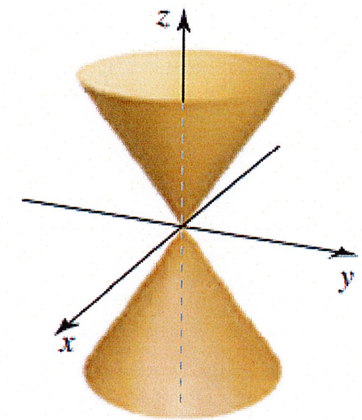
Traces with $z = z_0$ with $|z_0| > |c|$ are ellipses. Traces with $x = x_0$ and $y = y_0$ are hyperbolas.



Elliptic cone

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$

Traces with $z = z_0 \neq 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are hyperbolas or intersecting lines.



Hyperbolic paraboloid

$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

Traces with $z = z_0 \neq 0$ are hyperbolas. Traces with $x = x_0$ or $y = y_0$ are parabolas.