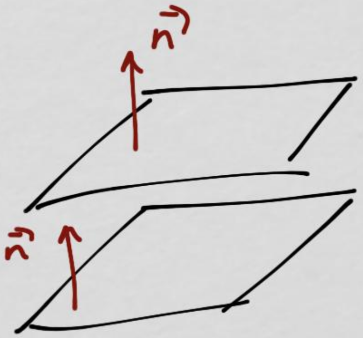
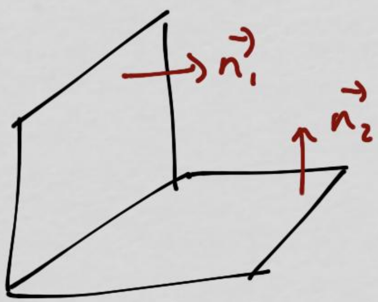
two planes are parallel if their normal vectors are parallel



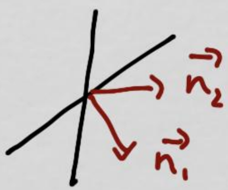
(floor and ceiling of your room)

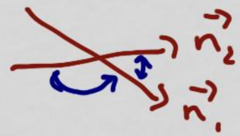
two planes are orthogonal if their normal vectors are orthogonal



(floor and a wall of your room)

the angle between two planes is the <u>smallest</u> angle between normal vectors





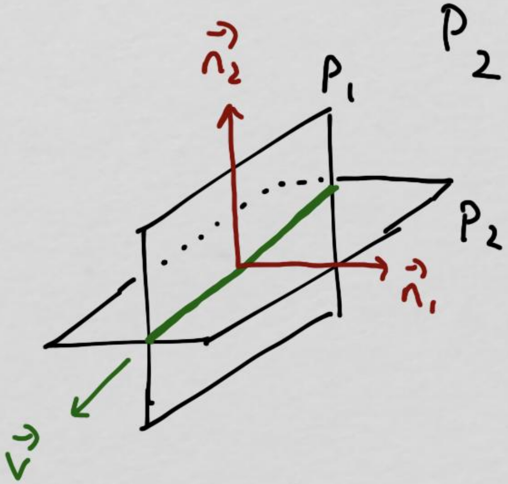
we use the smaller angle

the intersection of two planes is a line (look at the place where the floor of your room meets a wall)

<u>example</u> Find the equation of the line of the intersection of the planes

$P_1 : -2x - y + 4z = 1 \qquad \vec{n}_1 = \langle -2, -1, 4 \rangle$

$P_2 : x + y + z = 1 \qquad \vec{n}_2 = \langle 1, 1, 1 \rangle$



note the direction vector of the line is orthogonal to both $\vec{n}_1$ and $\vec{n}_2$

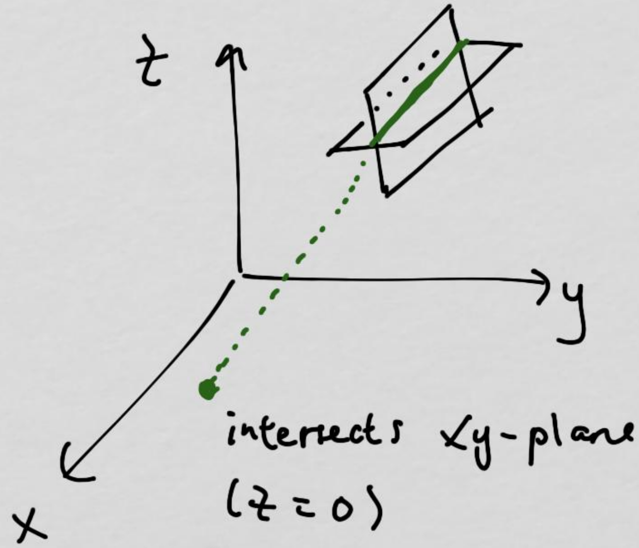
so, $\vec{v} = \vec{n}_1 \times \vec{n}_2$ or $\vec{n}_2 \times \vec{n}_1$

in this example, let's use $\vec{v} = \vec{n}_1 \times \vec{n}_2 = \langle -5, 6, -1 \rangle$

then we need a point on the line

this is a bit tricky

easiest way : find intersection of line with a coordinate plane (xy-plane, xz-plane, or yz-plane)



let's find its intersection w/ xy-plane

$\rightarrow$ $z=0$ sub into plane equations

$P_1 : -2x-y+4z=1 \rightarrow -2x-y=1$

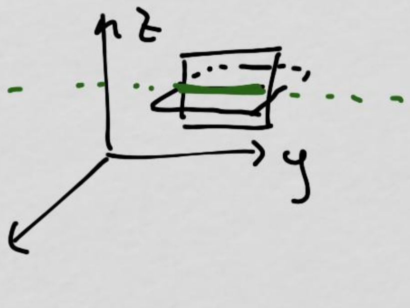
$P_2 : x+y+z=1 \rightarrow x+y=1$

solve $\begin{cases} -2x-y=1 \\ x+y=1 \end{cases}$ for x and y $\longrightarrow$ $x=-2,\ y=3$

so, the intersection of line w/ xy-plane is $(-2, 3, 0)$

therefore, its equation is $\boxed{\vec{r}(t) = \langle -2, 3, 0\rangle + t\,\langle -5, 6, -1\rangle}$

note that the line may not have an intersection with a chosen coordinate plane



never intersects xy-plane

if this were the case, there would be no solution in the system of equations from the planes

choose a different coordinate plane