Exam 1

Thu. 2/25 6:30 pm

MA 261 Exam 1 Location

RECITATION TA	EXAM ROOM
LI	WTHR 200
HOGLE	WTHR 200
GRANADOS	EE 129
HIATT	EE 129
ENYEART	Hiler Theater (WALC)
HARDWICK	MTHW 210
GLENN SECTIONS 753, 761	RHPH 172
GLENN SECTION 769	EE 170
SMITH SECTIONS 777, 785	LILY G126
SMITH SECTION 793	EE 170

- Bring your charged laptop
- Bring scratch papers
- Arrive by 6:20 pm

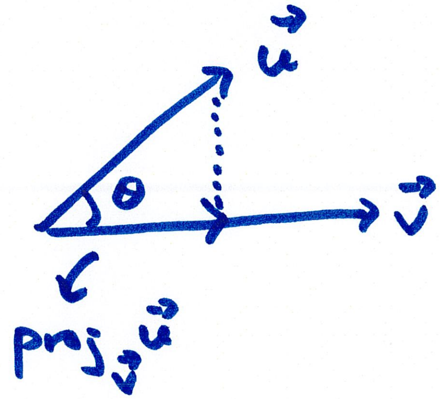
Find $\text{proj}_{\mathbf{v}}\mathbf{u}$.

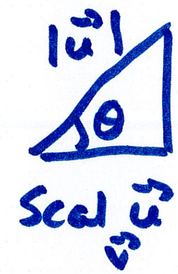
$\mathbf{v} = 3\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, $\mathbf{u} = 10\mathbf{i} + 11\mathbf{j} + 2\mathbf{k}$

- A. $\frac{75}{19}\mathbf{i} - \frac{25}{19}\mathbf{j} + \frac{75}{19}\mathbf{k}$ ✓
- B. $\frac{50}{3}\mathbf{i} + \frac{55}{3}\mathbf{j} + \frac{10}{3}\mathbf{k}$
- C. $\frac{141}{19}\mathbf{i} - \frac{47}{19}\mathbf{j} + \frac{141}{19}\mathbf{k}$
- D. $\frac{10}{9}\mathbf{i} + \frac{11}{9}\mathbf{j} + \frac{2}{9}\mathbf{k}$

$\vec{V} = 3\vec{i} - \vec{j} + 3\vec{k} \qquad \vec{u} = 10\vec{i} + 11\vec{j} + 2\vec{k}$

$\text{proj}_{\vec{v}}\vec{u}$ vector projection of $\vec{u}$ onto $\vec{v}$





$$\text{scal}_{\vec{v}}\vec{u} = |\vec{u}|\cos\theta$$

$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$$

$$\text{scal}_{\vec{v}}\vec{u} = \cancel{|\vec{u}|}\,\frac{\vec{u}\cdot\vec{v}}{\cancel{|\vec{u}|}|\vec{v}|}$$

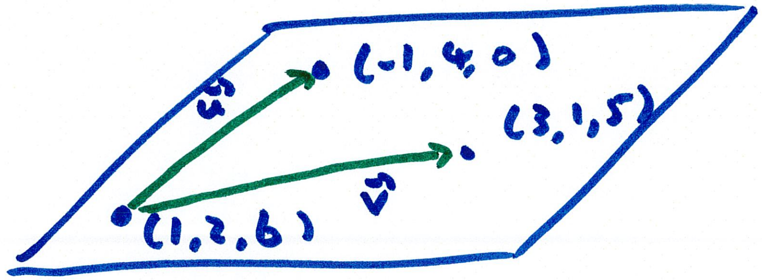
$$\text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}\,\frac{\vec{v}}{|\vec{v}|} = \frac{(30-11+6)}{\sqrt{19}}\,\frac{\langle 3,-1,3\rangle}{\sqrt{19}}$$

$$= \frac{25}{19}\langle 3,-1,3\rangle$$

Write the equation for the plane.

The plane passing through the points (1,2,6), (− 1,4,0), and (3,1,5)

- ○ A. $2x+4y+5z=40$
- ○ B. $6x+11y-z=22$
- ○ C. $9x-2y+8z=53$
- ✓ D. $4x+7y+z=24$



$\vec{u} = \langle -2, 2, -6 \rangle \qquad \vec{v} = \langle 2, -1, -1 \rangle$

$\vec{u} \times \vec{v}$ or $\vec{v} \times \vec{u}$ is normal to plane

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 2 & -6 \\ 2 & -1 & -1 \end{vmatrix} = \langle -8, -14, -2 \rangle = \vec{n}$$

plane: $\vec{n} \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$

$\langle -8, -14, -2 \rangle \cdot \langle x+1, y-4, z-0 \rangle = 0$

$-8x - 8 - 14y + 56 - 2z = 0$

$-8x - 14y - 2z = -48$ match D by dividing by -2

Given an acceleration vector, initial velocity $\langle u_0, v_0 \rangle$, and initial position $\langle x_0, y_0 \rangle$, find the velocity and position vectors for $t \geq 0$.

$\mathbf{a}(t) = \langle 2,4 \rangle$, $\langle u_0, v_0 \rangle = \langle 4,4 \rangle$, $\langle x_0, y_0 \rangle = \langle 0,3 \rangle$

What is the velocity vector?

$\mathbf{v}(t) = \langle \square, \square \rangle$, for $t \geq 0$

What is the position vector?

$\mathbf{r}(t) = \langle \square, \square \rangle$, for $t \geq 0$

$\vec{a}(t) = \langle 2, 4 \rangle$

$\vec{v}(0) = \langle u_0, v_0 \rangle = \langle 4, 4 \rangle$

$\vec{r}(0) = \langle x_0, y_0 \rangle = \langle 0, 3 \rangle$

$\vec{V}(t) = \int a(t)\,dt = \int \langle 2, 4 \rangle dt = \langle 2t + C_1, 4t + C_2 \rangle$

$\vec{v}(0) = \langle u_0, v_0 \rangle = \langle 4, 4 \rangle = \langle 0 + C_1, 0 + C_2 \rangle \qquad C_1 = 4, \; C_2 = 4$

so, $\vec{v}(t) = \langle 2t + 4, 4t + 4 \rangle$

$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle t^2 + 4t + b_1, 2t^2 + 4t + b_2 \rangle$

$\vec{r}(0) = \langle x_0, y_0 \rangle = \langle 0, 3 \rangle = \langle 0 + 0 + b_1, 0 + 0 + b_2 \rangle \qquad b_1 = 0, \; b_2 = 3$

so $\vec{r}(t) = \langle t^2 + 4t, 2t^2 + 4t + 3 \rangle$

Determine whether the following curve uses arc length as a parameter. If not, find a description that uses arc length as a parameter.

$\mathbf{r}(t) = \langle -4\cos t, -4\sin t\rangle$, for $0 \le t \le \pi$

Choose the correct answer below.

- A. $\mathbf{r}_1(s) = \langle -4\cos s, -4\sin s\rangle$, for $0 \le s \le 4\pi$
- B. $\mathbf{r}_1(s) = \langle -\cos s, -\sin s\rangle$, for $0 \le s \le \frac{\pi}{4}$
- ✓ C. $\mathbf{r}_1(s) = \left\langle -4\cos\frac{s}{4}, -4\sin\frac{s}{4}\right\rangle$, for $0 \le s \le 4\pi$
- D. $\mathbf{r}_1(s) = \left\langle -\cos\frac{s}{4}, -\sin\frac{s}{4}\right\rangle$, for $0 \le s \le \pi$
- E. The given curve uses arc length as a parameter.

$\vec{r}(t) = \langle -4\cos t, -4\sin t\rangle \quad 0 \le t \le \pi$

if "t" is actually arc length "s"

then $|\vec{r}\,'| = 1$

$\vec{r}\,' = \langle 4\sin t, -4\cos t\rangle$

$|\vec{r}\,'| = \sqrt{16\sin^2 t + 16\cos^2 t} = \sqrt{16} = 4 \ne 1$

so, "t" is not "s"

arc length: $s(t) = \int_a^t |\vec{r}\,'(u)|\,du = \int_0^t 4\,du$

start of t interval

$s(t) = 4t \rightarrow t = \frac{s}{4}$

$0 \le t \le \pi$

$0 \le s \le 4\pi$

$\vec{r}(s) = \langle -4\cos\frac{s}{4}, -4\sin\frac{s}{4}\rangle$

Find the curvature of the curve **r**(t).

r(t) = (4 + ln (sec t))**i** + (2 + t)**k**, $-\pi/2 < t < \pi/2$

- A. $\kappa = 1 - \cos t$
- B. $\kappa = \sin t$
- C. $\kappa = \cos t$ ✓
- D. $\kappa = -\cos t$

$$\vec{r}(t) = \langle 4 + \ln(\sec t),\ 0,\ 2+t \rangle \qquad -\tfrac{\pi}{2} < t < \tfrac{\pi}{2}$$

$$\kappa = \left| \frac{d\vec{T}}{ds} \right| \qquad \vec{T} : \text{unit tangent vector}$$

$$\vec{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \frac{\langle \tan t,\ 0,\ 1 \rangle}{\sqrt{\tan^2 t + 1}} = \frac{\langle \tan t,\ 0,\ 1 \rangle}{\sec t} = \langle \sin t,\ 0,\ \cos t \rangle$$

$$\vec{T}\,' = \langle \cos t,\ 0,\ -\sin t \rangle$$

$$\kappa = \left| \frac{d\vec{T}}{ds} \right| = \left| \frac{d\vec{T}/dt}{ds/dt} \right| = \left| \frac{\vec{T}\,'}{\vec{r}\,'} \right| = \frac{1}{\sec t} = \cos t$$

$$\frac{|\vec{T}\,'|}{|\vec{r}\,'|} = \frac{\sqrt{\cos^2 t + \sin^2 t}}{\sec t}$$

The level curves of $f(x, y) = e^{x^2+y^2-y}$ are

A. circles

B. parabolas

C. hyperbolas

D. lines

E. (noncircular) ellipses

$z = f(x,y) = C$ (constant)

$C = e^{x^2+y^2-y}$ $\quad C > 0$

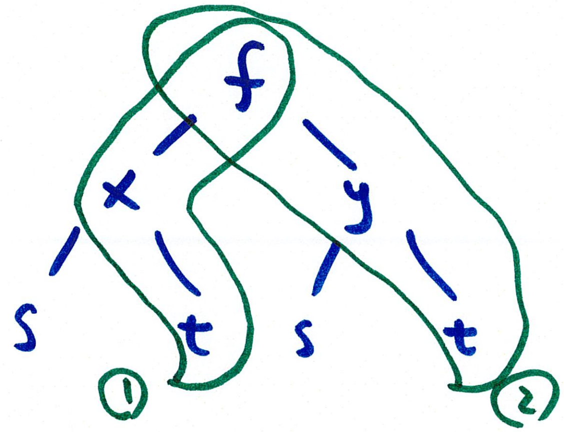
$\underbrace{\ln C}$ $= x^2+y^2-y = K$ $\rightarrow$ $x^2+y^2-y = K$ shape?

C is constant

so ln C is also constant

circle

Let $f(x,y) = 2x^2y + xy^3$ and $x = g(s,t)$, $y = h(s,t)$ are functions of s and t. Suppose $g(1,2) = 1$, $h(1,2) = -1$ and $\frac{\partial g}{\partial t}(1,2) = 2$, $\frac{\partial h}{\partial t}(1,2) = 1$. Then at $(s,t) = (1,2)$, $\boxed{\frac{\partial f}{\partial t}}$ equals



A. 0

B. -10

C. 10

~~D.~~ 5

E. -5

$$\frac{\partial f}{\partial t} = \underbrace{\frac{\partial f}{\partial x}\frac{\partial x}{\partial t}}_{1} + \underbrace{\frac{\partial f}{\partial y}\frac{\partial y}{\partial t}}_{2}$$

$$= (4xy + y^3)\frac{\partial g}{\partial t} + (2x^2 + 3xy^2)\frac{\partial h}{\partial t}$$

$$= (-4-1)(2) + (2+3)(1) = -5$$

at $\overset{s\ \ t}{(1,2)}$

$x = g(1,2) = 1$

$y = h(1,2) = -1$

$\frac{\partial g}{\partial t} = 2 \quad \frac{\partial h}{\partial t} = 1$

The limit $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4+y^4}{x^2+y^2}$ is equal to

A. 1

B. 0

C. 1/2

D. 2

E. Does not exist

along $x=0$ $\quad \lim\limits_{y\to 0} \dfrac{y^4}{y^2} = \lim\limits_{y\to 0} y^2 = 0$

along $y=0$ $\quad \lim\limits_{x\to 0} \dfrac{x^4}{x^2} = \lim\limits_{x\to 0} x^2 = 0$

along $y=x$ $\quad \lim\limits_{x\to 0} \dfrac{2x^4}{2x^2} = \lim\limits_{x\to 0} x^2 = 0$

still, can't say for sure limit is 0

find a path-independent way

$$(x^2+y^2)^2 = x^4+2x^2y^2+y^4$$

$$x^4+y^4 = (x^2+y^2)^2 - 2x^2y^2$$

$$\lim_{(x,y)\to(0,0)} \frac{x^4+y^2}{x^2+y^2}$$

$$= \lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)^2-2x^2y^2}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \underbrace{(x^2+y^2)}_{0} - \underbrace{\frac{2x^2y^2}{x^2+y^2}}_{0}$$

$(\text{small }\#)^4$

$(\text{small }\#)^2$

numerator << denom

$= 0$