MA 261 Exam 2 Location

(Thursday, 4/8, 6:30 pm)

RECITATION TA	**EXAM ROOM**
LI	WTHR 200
HOGLE	WTHR 200
GRANADOS	EE 129
HIATT	EE 129
ENYEART	Hiler Theater (WALC)
HARDWICK	MTHW 210
GLENN SECTIONS 753, 761	RHPH 172
GLENN SECTION 769	EE 170
SMITH SECTIONS 777, 785	LILY G126
SMITH SECTION 793	EE 170

Consider the function $f(x,y) = 2x^4 - 2x^2y + y^2 + 3$ and the point $P(-1,3)$.

a. Find the unit vectors that give the direction of steepest ascent and steepest descent at P.

b. Find a vector that points in a direction of no change in the function at P.

a. What is the unit vector in the direction of steepest ascent at P?

↳ direction of gradient vector at P

$\langle \square, \square \rangle$

(Type exact answers, using radicals as needed.)

What is the unit vector in the direction of steepest descent at P?

↳ opposite of $\vec{\nabla} f$ at P

$\langle \square, \square \rangle$

(Type exact answers, using radicals as needed.)

b. Which of the following vectors is in a direction of no change of the function at P?

- A. $\langle 0,4 \rangle$
- B. $\langle -4,-4 \rangle$
- C. $\langle 4,0 \rangle$
- D. $\langle -4,4 \rangle$

$f(x,y) = 2x^4 - 2x^2y + y^2 + 3 \qquad P(-1, 3)$

$\vec{\nabla} f = \langle f_x, f_y \rangle = \langle 8x^3 - 4xy, -2x^2 + 2y \rangle$

at $(-1, 3)$ $\quad \vec{\nabla} f = \langle -8+12, -2+6 \rangle = \underbrace{\langle 4, 4 \rangle}$

direction of steepest ascent

make it unit vector

$\dfrac{\langle 4, 4 \rangle}{|\langle 4, 4 \rangle|} = \dfrac{\langle 4, 4 \rangle}{\sqrt{32}} = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

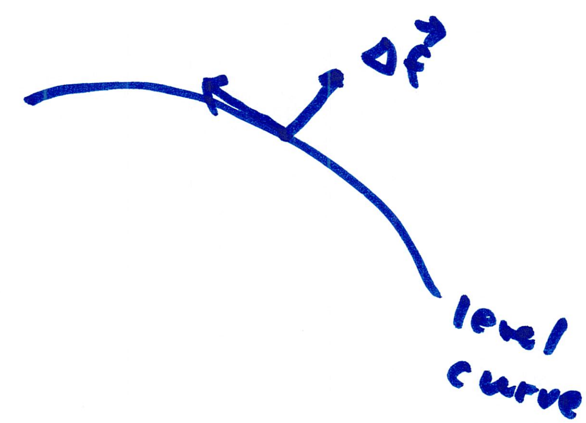
direction of steepest descent: $-\vec{\nabla} f$

unit vector: $\langle -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$

b). no change: dot with $\vec{\nabla} f = 0$

$\rightarrow$ along a level curve $\rightarrow$ no change in f

so, $\langle -4, 4 \rangle$



The function $f(x, y) = \frac{1}{3}x^3 - 4xy + y^2 - 9x$ has:

A. a relative minimum and a relative maximum

B. two saddle points

C. two relative minima

D. a relative maximum and a saddle point

E. a relative minimum and a saddle point

critical pts: $f_x = 0 \rightarrow x^2 - 4y - 9 = 0$

$f_y = 0 \rightarrow -4x + 2y = 0$

$\hookrightarrow y = 2x$

sub into $x^2 - 4y - 9 = 0$

$x^2 - 8x - 9 = 0$

$(x - 9)(x + 1) = 0$

$x = -1, \quad x = 9$

$y = -2, \quad y = 18$

$(-1, -2), (9, 18)$

$D = f_{xx} f_{yy} - (f_{xy})^2$

$$\left.\begin{aligned} f_{xx} &= 2x \\ f_{yy} &= 2 \\ f_{xy} &= -4 \end{aligned}\right\} \quad D = 4x - 16$$

$D(-1,-2) < 0 \rightarrow$ saddle point

$D(9,18) > 0$ and $f_{xx}(9,18) > 0 \rightarrow$ min

Find the maximum value of $x + y$ along the curve defined by $x^2 + 2y^2 = 6$.

A. 3

B. $4\sqrt{3}$

C. $2\sqrt{6}$

D. 4

E. $2\sqrt{2}$

max $f(x,y) = x+y$

Subject to $g(x,y) = x^2+2y^2-6=0$

Lagrange multipliers: $\vec{\nabla} f = \lambda \vec{\nabla} g$

$\langle 1, 1\rangle = \lambda \langle 2x, 4y\rangle$

$$\left.\begin{aligned} 1 &= \lambda \cdot 2x \to \lambda = \frac{1}{2x} \\ 1 &= \lambda \cdot 4y \to \lambda = \frac{1}{4y} \end{aligned}\right\} \quad \frac{1}{2x} = \frac{1}{4y} \to 2x = 4y \to x = 2y$$

Sub into $g(x,y) = x^2+2y^2-6=0$

$4y^2+2y^2-6=0 \to y = \pm 1$ so $x = \pm 2$ $\underline{\text{same signs}}$

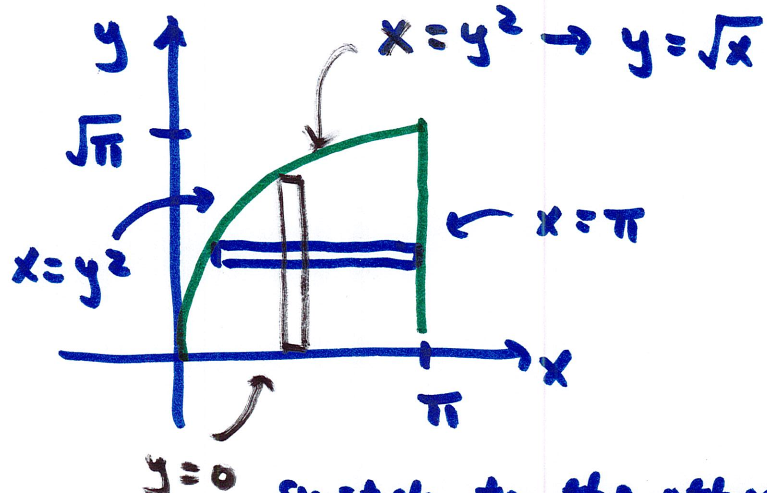
points: $(2,1), (-2,-1)$

$\underset{\text{max}}{f(2,1)=3}$ $\quad f(-2,-1) = -3$

If the order of integration is reversed, which of the following integrals is equal to $\int_0^{\sqrt{\pi}} \int_{y^2}^{\pi} (\sin x^2) dx\, dy$?

$0 \le y \le \sqrt{\pi}$

left $\rightarrow$ $y^2 \le x \le \pi$ $\leftarrow$ right



A. $\int_0^{\pi} \int_{\sqrt{x}}^{\pi} (\sin x^2) dy\, dx$

B. $\int_0^{\pi} \int_0^{\sqrt{x}} (\sin x^2) dy\, dx$

C. $\int_{\sqrt{x}}^{\sqrt{\pi}} \int_0^{\pi} (\sin x^2) dy\, dx$

D. $\int_0^{\pi} \int_x^{\sqrt{\pi}} (\sin x^2) dy\, dx$

E. $\int_0^{\sqrt{\pi}} \int_x^{\pi} (\sin x^2) dy\, dx$

switch to the other type

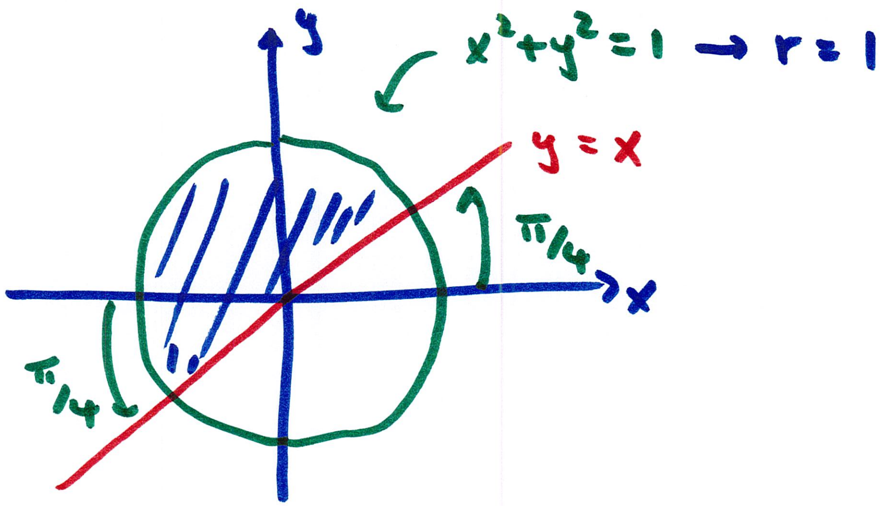
x bounded by constants: $0 \le x \le \pi$

y bounded by curves above and below

$0 \le y \le \sqrt{x}$

$$\int_0^{\pi} \int_0^{\sqrt{x}} (\sin x^2)\, dy\, dx$$

If R is the region in the xy-plane inside the circle $x^2+y^2=1$ and above the line $y=x$, then $\iint_R x\,dA$ expressed in polar coordinates is:



A. $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^1 r\cos\theta\,dr\,d\theta$

B. $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^1 r^2\cos\theta\,dr\,d\theta$

C. $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_0^{r\cos\theta} r^2\cos\theta\,dr\,d\theta$

D. $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_0^{r^2} r^2\cos\theta\,dr\,d\theta$

E. $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_0^{1} r^2\cos\theta\,dr\,d\theta$

polar: $0 \le r \le 1$

$\frac{\pi}{4} \le \theta \le \frac{5\pi}{4}$

$$\iint_R x\,dA = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_0^1 r\cos\theta \cdot r\,dr\,d\theta$$

The mass of an object occupying the region bounded above by the plane $z = 2$ and below by the upper nappe of the cone $z^2 = x^2 + y^2$ with mass density at each point equal to $x^2 + y^2 + z^2$ is given by:

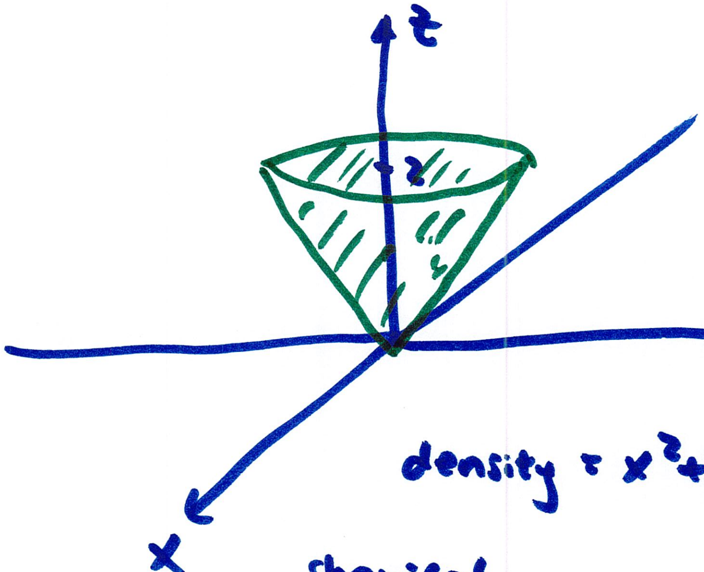
A. $\displaystyle\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{2} \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$

B. $\displaystyle\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{2\sec\phi} \rho^4 \sin\phi \, d\rho\, d\phi\, d\theta$

C. $\displaystyle\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{2} \rho^4 \sin\phi \, d\rho\, d\phi\, d\theta$

D. $\displaystyle\int_0^{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\int_0^{2\sec\phi} \rho^3 \sin^2\phi \, d\rho\, d\phi\, d\theta$

E. $\displaystyle\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_0^{2\sec\phi} \rho^3 \sin^2\phi \, d\rho\, d\phi d\theta$



density $= x^2+y^2+z^2 = \rho^2$

sherical

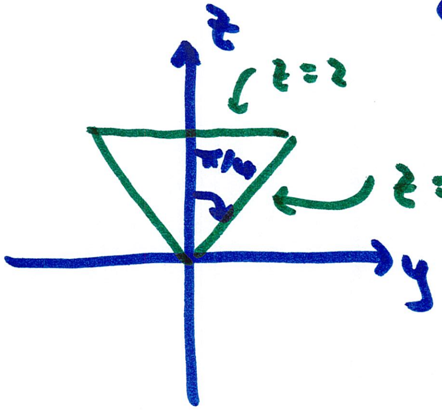
$0 \le \theta \le 2\pi$

$0 \le \phi \le \pi/4$

$0 \le \rho \le$ → equivalent of $z=2$ in spherical

$\rho\cos\phi = 2$

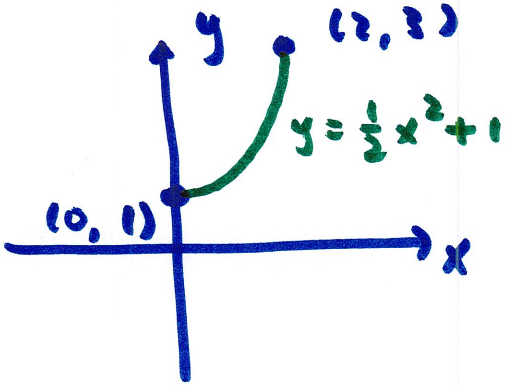
$\rho = \frac{2}{\cos\phi} = 2\sec\phi$



$z=y \rightarrow \pi/4$ from axes

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{2\sec\phi} \underbrace{\rho^2}_{\text{density}}\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$$

16. If C is the curve $y = \frac{x^2}{2} + 1$ from $(0, 1)$ to $(2, 3)$, then $\int_C 3x\, ds =$



A. $\frac{8}{3}$

B. $\frac{10}{3}$

C. $\sqrt{5}$

D. $\sqrt{5} - 1$

E. $5\sqrt{5} - 1$

parametrize C: $\vec{r}(t) = \langle t, \frac{1}{2}t^2 + 1 \rangle$ $\quad 0 \le t \le 2$

x

$y = \frac{1}{2}x^2 + 1$

w/ $x = t$

$ds = |\vec{r}\,'|\, dt$ $\qquad \vec{r}\,' = \langle 1, t \rangle$

$= \sqrt{1+t^2}\, dt$

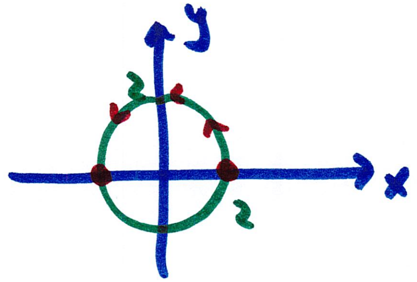
$\int_C 3x\, ds = \int_0^2 3t\sqrt{1+t^2}\, dt$ $\qquad u = 1 + t^2$, $du = 2t\,dt$ $\qquad \int_1^5 \frac{3}{2} u^{1/2}\, du$

x from $\vec{r}(t)$

$= \frac{3}{2} \cdot \frac{2}{3} u^{3/2} \Big|_1^5 = (5)^{3/2} - 1$

Let $\vec{F}(x,y) = 3x^2\vec{i} - \vec{j}$. Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where C is the top half of the circle of radius 2 centered at $(0,0)$, starting at $(2,0)$ and ending at $(-2,0)$.

$\vec{F} = \langle 3x^2, -1 \rangle$



note direction is not specified → assume ccw

but perhaps that is hinting that the actual path is not important → $\vec{F}$ conservative?

A. 16

B. 8

C. 0

D. −8

E. −16

$\vec{F} = \langle \underset{f}{3x^2}, \underset{g}{-1} \rangle$

conservative if $f_y = g_x$

$f_y = 0, \quad g_x = 0$

find ϕ such that $\vec{F} = \vec{\nabla}\phi$

$\langle 3x^2, -1 \rangle = \langle \phi_x, \phi_y \rangle$

$\phi_x = 3x^2 \rightarrow \phi = \int 3x^2\,dx = x^3 + a(y)$

$\phi_y = -1$

$\phi_y = \frac{da}{dy}$ must be equal to $\phi_y = -1$

$$\frac{da}{dy} = -1 \rightarrow a(y) = -y + c$$

so, $\phi(x,y) = x^3 - y + c$

start at $(2, 0)$ end at $(-2, 0)$

so, $\int_c \vec{F} \cdot d\vec{r} = \phi(-2, 0) - \phi(2, 0)$

$$= (-8 + c) - (8 + c) = -8 - 8 = -16$$