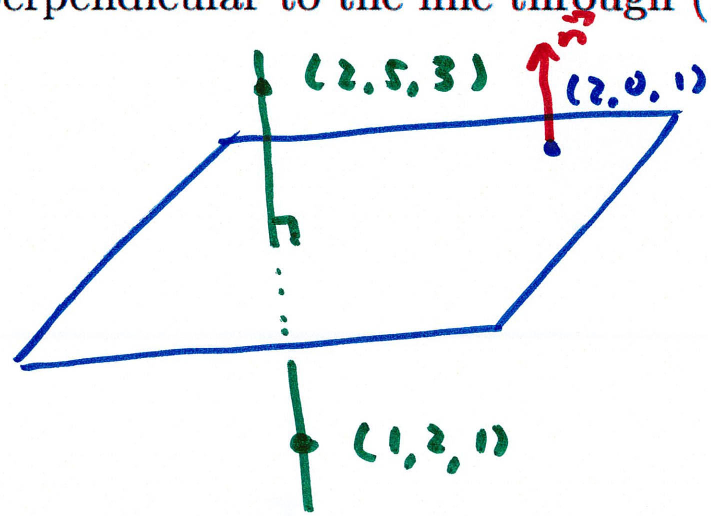
Find an equation of the plane that contains the point $(2, 0, 1)$ and that is perpendicular to the line through $(1, 2, 1)$ and $(2, 5, 3)$.



A. $x + 2y + z = 3$

B. $2x + 5y + 3z = 7$

C. $x + 3y + 2z = 4$

D. $x + 2y + z = -3$

E. $x + 3y + 2z = -4$

plane: need normal vector and one point

normal vector: direction vector of line through $\underbrace{(1,2,1), (2,5,3)}_{\vec{v} = \langle 1, 3, 2 \rangle}$

$\vec{n} = \langle 1, 3, 2 \rangle$

plane: $\vec{n} \cdot \langle \underset{\downarrow}{\overset{2}{x - x_0}}, \underset{\downarrow}{\overset{0}{y - y_0}}, \underset{\downarrow}{\overset{1}{z - z_0}} \rangle = 0$

$\langle 1, 3, 2 \rangle \cdot \langle x - 2, y, z - 1 \rangle = 0$

$x - 2 + 3y + 2z - 2 = 0 \qquad x + 3y + 2z = 4$

The equation of the tangent plane to the graph of the function $f(x,y) = x - \frac{y^2}{2}$ at $(1, 2, -1)$ is

A. $2x + y + 4z = 0$

B. $x + 4y = 9$

C. $x - 2y - z = 2$

D. $-x + 2y + z = 2$

E. $x - y - 2z = 1$

let $\vec{F} = \underbrace{f(x,y)}_{z} - x + \frac{y^2}{2} = 0$

$\vec{F} = z - x + \frac{1}{2}y^2$

$\vec{\nabla} F = \langle -1, y, 1 \rangle$ normal to surface so can be used to find tangent plane

at $(1, 2, -1)$ $\vec{\nabla}\vec{F} = \langle -1, 2, 1 \rangle = \vec{n}$

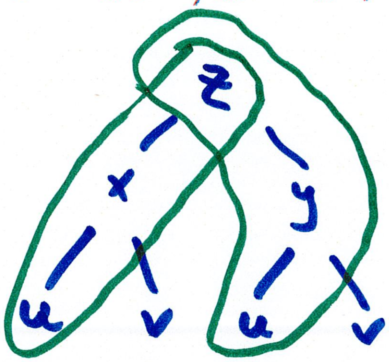
tangent plane: $\vec{n} \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$

$\langle -1, 2, 1 \rangle \cdot \langle x-1, y-2, z+1 \rangle = 0$

$-x + 1 + 2y - 4 + z + 1 = 0$

$-x + 2y + z = 2$

If $z = xe^{y^2}$, $x = u + v$, $y = u^2 - v$, find $\dfrac{\partial z}{\partial u}$ where $u = 2$ and $v = 3$.



A. $2e^2$

B. $45e$

C. $20e^2$

D. $41e$

E. $17e$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

$$= e^{y^2}(1) + 2yxe^{y^2}(2u)$$

when $u=2,\ v=3$

$x = u+v = 5 \qquad y = u^2 - v = 1$

$$= e(1) + 40e = 41e$$

The position of a particle is given by $\mathbf{r}(t) = \langle 2t, 1-2t, 5+t\rangle$, starting when $t=0$. After the particle has gone a *distance* of 3, the x-coordinate is

A. $\frac{1}{3}$
B. 3
C. $\frac{1}{2}$
D. 2
E. 1

reparametrize $\vec{r}(t)$ with respect to arc length

$$s(t) = \int_a^t |\vec{r}\,'(u)|\,du \qquad \vec{r}\,' = \langle 2, -2, 1\rangle$$

starting t

$$|\vec{r}\,'| = \sqrt{4+4+1} = 3$$

$$s(t) = \int_0^t 3\,du = 3t \quad \rightarrow t = \tfrac{1}{3}s$$

so, $\vec{r}(s) = \langle \frac{2}{3}s, 1-\frac{2}{3}s, 5+\frac{1}{3}s\rangle$

after having gone distance of 3 $\rightarrow s = 3$

$\vec{r}(3) = \langle 2, -\ , -\ \rangle$

The number and value of the absolute maxima of the function $f(x,y) = x^2 - xy + y^2$ on the domain $2x^2 + 2y^2 \leq 1$ is

A. Two maxima with value 1

B. Two maxima with value $\frac{1}{2}$

C. Four maxima with value $\frac{1}{2}$

D. Four maxima with value $\frac{3}{4}$

E. Two maxima with value $\frac{3}{4}$

find critical pts of f inside $2x^2+2y^2=1$

then find critical pts on $2x^2+2y^2=1$

then compare $f(x,y)$ at those places

$f_x = 0 \rightarrow 2x - y = 0 \rightarrow y = 2x$

$f_y = 0 \rightarrow 2y - x = 0$

$2(2x) - x = 0 \rightarrow 3x = 0 \rightarrow \boxed{x = 0,\ y = 0}$ critical pt inside $2x^2+2y^2=1$

on boundary of $2x^2+2y^2=1 \rightarrow x^2+y^2 = \frac{1}{2}$

$y = \left(\frac{1}{2} - x^2\right)^{1/2}$

$f(x,y) = x^2+y^2-xy$ becomes $f(x) = \frac{1}{2} - x\left(\frac{1}{2} - x^2\right)^{1/2}$ $\quad -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$

$f' = -x \cdot \frac{1}{2}\left(\frac{1}{2} - x^2\right)^{-1/2}(-2x) - \left(\frac{1}{2} - x^2\right)^{1/2} = 0$

$$x^2\left(\tfrac{1}{2}-x^2\right)^{-1/2}-\left(\tfrac{1}{2}-x^2\right)^{1/2}=0$$

multiply by $\left(\tfrac{1}{2}-x^2\right)^{1/2}$

$$x^2-\left(\tfrac{1}{2}-x^2\right)=0 \qquad 2x^2=\tfrac{1}{2} \qquad x^2=\tfrac{1}{4} \qquad x=\pm\tfrac{1}{2}$$

$$y=\pm\sqrt{\tfrac{1}{2}-x^2}$$

$$\boxed{\begin{array}{c}\left(\tfrac{1}{2},\tfrac{1}{2}\right),\ \left(\tfrac{1}{2},-\tfrac{1}{2}\right)\\ \left(-\tfrac{1}{2},-\tfrac{1}{2}\right),\ \left(-\tfrac{1}{2},\tfrac{1}{2}\right)\end{array}}$$

compare $f(x,y)=x^2-xy+y^2$ at those places

$f(0,0)=0$

$f\left(\tfrac{1}{2},\tfrac{1}{2}\right)=\tfrac{1}{4}$

$f\left(-\tfrac{1}{2},-\tfrac{1}{2}\right)=\tfrac{1}{4}$

$$\left.\begin{array}{l}f\left(\tfrac{1}{2},-\tfrac{1}{2}\right)=\tfrac{3}{4}\\ f\left(-\tfrac{1}{2},\tfrac{1}{2}\right)=\tfrac{3}{4}\end{array}\right\}\text{abs. max}$$

alternative : use Lagrange multipliers on boundary of $2x^2+2y^2=1$

max $f(x,y) = x^2 - xy + y^2$

subject to $g(x,y) = 2x^2+2y^2-1=0$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

$\langle 2x-y,\ 2y-x \rangle = \lambda \langle 4x,\ 4y \rangle$

$2x-y = \lambda \cdot 4x \qquad \lambda = \frac{2x-y}{4x}$

$2y-x = \lambda \cdot 4y \qquad \lambda = \frac{2y-x}{4y}$

$\rightarrow \frac{2x-y}{\cancel{4}x} = \frac{2y-x}{\cancel{4}y}$

$2xy - y^2 = 2xy - x^2$

$x^2 = y^2$

put into $g(x,y) = 2x^2+2y^2-1=0$

$4x^2=1 \qquad x = \pm\frac{1}{2}$

$y = \pm\frac{1}{2}$

Let $f(x, y) = (x^2 + y^2)e^x$. The function has

find critical pts

$f_x = (x^2+y^2)\cancel{e^x} + 2x\cancel{e^x} = 0$

$x^2+2x+y^2=0$

$f_y = 2y\cancel{e^x} = 0$

$y=0$

A. a local max. and a local min. point

B. two local max. points

C. a local max. and a saddle point

D. two local max. points

E. a local min. and a saddle point

$x^2+2x=0$

$x(x+2)=0$

$x=0, \ x=-2$

Critical pts: $(0,0), (-2,0)$

$D = f_{xx}f_{yy} - (f_{xy})^2$

$f_{xx} = (x^2+y^2)e^x + 2xe^x + 2xe^x + 2e^x$

$= (x^2+y^2)e^x + 4xe^x + 2e^x = (x^2+y^2+4x+2)e^x$

$f_{yy} = 2e^x$

$f_{yx} = 2ye^x$

at $(0,0)$ $f_{xx}=2$, $f_{yy}=2$, $f_{xy}=0$ $D>0$ $f_{xx}>0$, $D>0 \rightarrow$ rel. min at $(-2, 0)$

$f_{xx} = -2e^{-2} < 0$

$f_{yy} = 2e^{-2}$

$f_{yx} = 0$

$D = f_{xx} f_{yy} - (f_{xy})^2 = -4e^{-4} < 0$

saddle pt

A particle is moving with acceleration $t\vec{j} + \vec{k}$. If the velocity at time $t = 1$ is $\vec{v}(1) = \vec{i} - \frac{1}{2}\vec{j}$, then what is the velocity at time $t = 0$?

A. $\vec{i} - \vec{j} - \vec{k}$

B. $\vec{i} - \vec{j} + 2\vec{k}$

C. $\vec{i} - \frac{1}{2}\vec{j} - \vec{k}$

D. $\vec{i} + \frac{1}{2}\vec{j} - \vec{k}$

E. $-\vec{i} + \vec{j}$

$\vec{a}(t) = \langle 0, t, 1 \rangle$

$\vec{v}(t) = \int \vec{a}(t)\,dt$

$= \int \langle 0, t, 1 \rangle\,dt = \langle C_1, \frac{1}{2}t^2 + C_2, t + C_3 \rangle$

we know at $t=1$, $\vec{v}(1) = \langle 1, -\frac{1}{2}, 0 \rangle = \langle C_1, \overbrace{\frac{1}{2}t^2}^{\frac{1}{2}} + C_2, \overbrace{t}^{1} + C_3 \rangle$

$C_1 = 1, \quad C_2 = -1, \quad C_3 = -1$

$\vec{v}(t) = \langle 1, \frac{1}{2}t^2 - 1, t - 1 \rangle$

$\vec{v}(0) = \langle 1, -1, -1 \rangle$