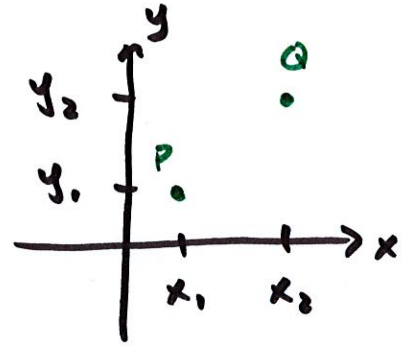
13.1 - 13.4 Review of Vectors

basic operations

$P(x_1, y_1)$, $Q(x_2, y_2)$



vector from P to Q

$$\vec{PQ} = \langle x_2 - x_1, \ y_2 - y_1 \rangle = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j}$$

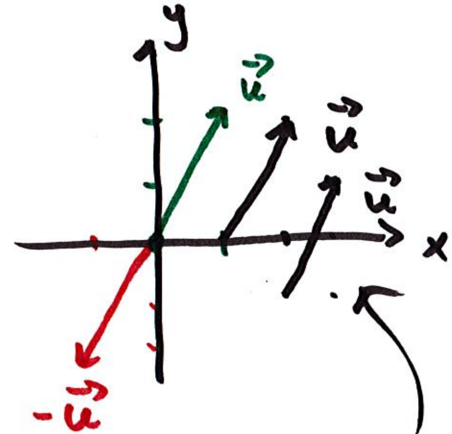
destination − starting

$$\vec{QP} = \langle x_1 - x_2, \ y_1 - y_2 \rangle = (x_1 - x_2)\vec{i} + (y_1 - y_2)\vec{j}$$

notice $\vec{PQ} = -\vec{QP}$

in general, changing the sign reverses the vector for example. $\vec{u} = \langle 1, 2 \rangle = 1\vec{i} + 2\vec{j}$

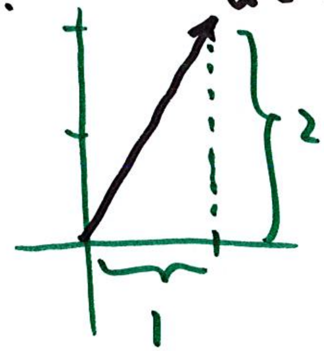
$-\vec{u} = \langle -1, -2 \rangle$



starting point is NOT fixed

all the vectors on the right are $\vec{u}$

magnitude: $\vec{u} = \langle 1, 2 \rangle$



the length of $\vec{u}$

$|\vec{u}| = \sqrt{1^2 + 2^2} = \sqrt{5}$

so, if $\vec{u} = \langle a, b \rangle$

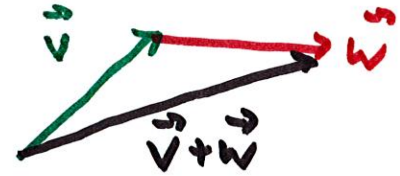
then $|\vec{u}| = \sqrt{a^2 + b^2}$

example: $\vec{v} = \langle 1, 2, 3 \rangle = 1\vec{i} + 2\vec{j} + 3\vec{k}$

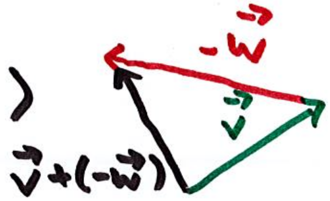
$|\vec{v}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$\vec{v} = \langle 1, 2, 3 \rangle \qquad \vec{w} = \langle 4, 5, 6 \rangle$

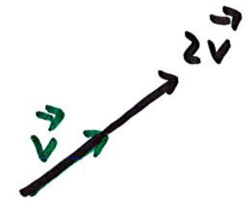
$\vec{v} + \vec{w} = \langle 1+4,\ 2+5,\ 3+6 \rangle = \langle 5, 7, 9 \rangle$



$\vec{v} - \vec{w} = \vec{v} + (-\vec{w})$



$2\vec{v} = \langle 1, 2, 3 \rangle = \langle 2\cdot 1,\ 2\cdot 2,\ 2\cdot 3 \rangle = \langle 2, 4, 6 \rangle$



<u>unit vector</u>: vector with length of 1 ($\vec{i}$, $\vec{j}$, $\vec{k}$, for example)

$\vec{v} = \langle 1, 2, 3 \rangle$ is <u>NOT</u> a unit vector because $|\vec{v}| = \sqrt{14} \neq 1$

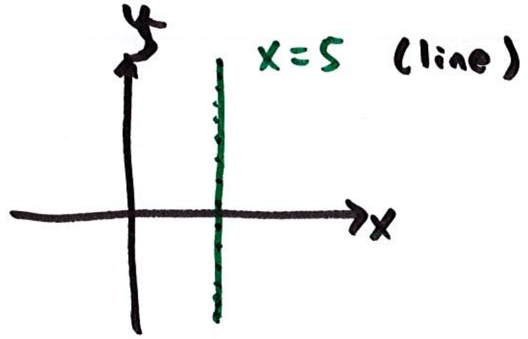
a unit vector in the same direction as $\vec{v}$

$$\frac{\vec{v}}{|\vec{v}|} = \frac{\langle 1, 2, 3 \rangle}{\sqrt{14}} = \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$$

most shapes in 3D ($\mathbb{R}^3$) are similar to their 2D ($\mathbb{R}^2$) counterparts

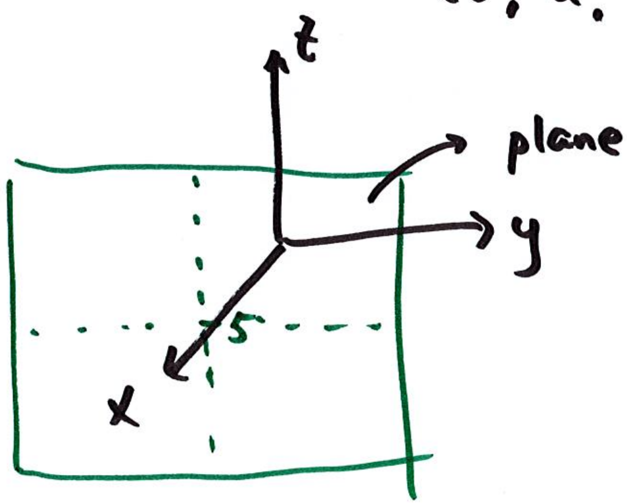
for example, $x=5$ in $\mathbb{R}^2$ is collection of all points

$(5, c)$ where c is some number



in $\mathbb{R}^3$, same idea : $x=5$ is collection of all points

$(5, a, b)$ $\quad -\infty < a < \infty \quad -\infty < b < \infty$



circle becomes sphere

$(x-h)^2+(y-k)^2=r^2$ circle radius r, center (h, k)

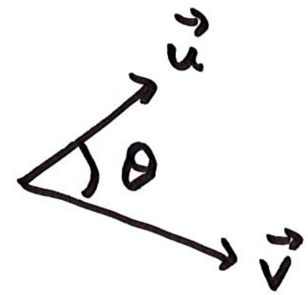
$(x-h)^2+(y-k)^2+(z-l)^2=r^2$ sphere radius r
center (h, k, l)

Dot Product

$\vec{u}=\langle 1,2,3\rangle \quad \vec{v}=\langle 4,5,6\rangle$

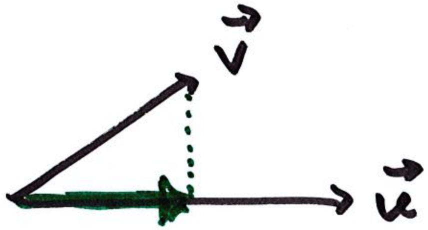
$\vec{u}\cdot\vec{v}=1\cdot 4+2\cdot 5+3\cdot 6=4+10+18=32$ a scalar

another formula: $\vec{u}\cdot\vec{v}=|\vec{u}||\vec{v}|\cos\theta$



note that if $\vec{u}\cdot\vec{v}=0$
then $\vec{u}\perp\vec{v}$

Scalar and vector projections



"shadow" of $\vec{v}$ on $\vec{u}$ → vector projection of $\vec{v}$ onto $\vec{u}$

$$\boxed{\operatorname{proj}_{\vec{u}} \vec{v} = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|^2} \vec{u}}$$

its magnitude is $\operatorname{scal}_{\vec{u}} \vec{v}$ (scalar projection)

$$\boxed{\operatorname{scal}_{\vec{u}} \vec{v} = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|}}$$

Cross Product

$\vec{u} = \langle 2, 1, 1 \rangle \qquad \vec{v} = \langle 5, 0, 1 \rangle$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ 5 & 0 & 1 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & 1 \\ 5 & 1 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & 1 \\ 5 & 0 \end{vmatrix}$$

$$= \vec{i}\,(1 \cdot 1 - 0 \cdot 1) - \vec{j}\,(2 \cdot 1 - 5 \cdot 1) + \vec{k}\,(2 \cdot 0 - 5 \cdot 1)$$

$$= \langle 1, 3, -5 \rangle$$

$\vec{u} \times \vec{v}$ is $\perp$ to <u>both</u> $\vec{u}$ and $\vec{v}$ $\qquad \vec{v} \times \vec{u} = -(\vec{u} \times \vec{v})$