13.6 Quadric Surfaces (part 1)

in $\mathbb{R}^2$ equations like $y = x^2$ are curves

in $\mathbb{R}^3$ equations in terms of x, y, z are surfaces

for example, $x^2+y^2+z^2=1$ is a sphere centered at $(0,0,0)$
radius 1

$(x-2)+2(y+3)-3(z+4)=0$ is a plane
through point $(2,-3,-4)$
normal vector $\langle 1, 2, -3\rangle$ or $\langle -1, -2, 3\rangle$

Sometimes one or more variables is missing

→ that variable is not related or constrained by the others

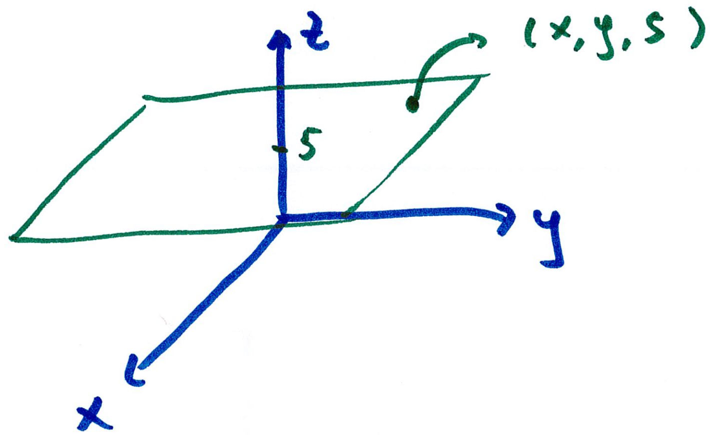
→ "free variable"

it can be any value in its domain

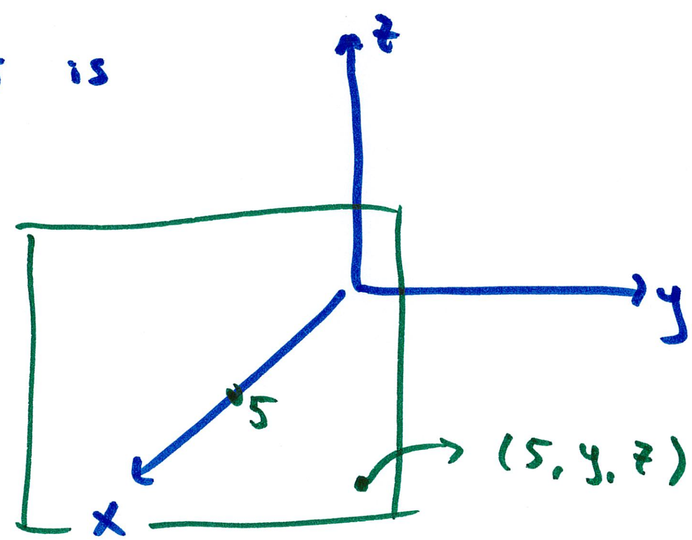
for example, $z = 5$ in $\mathbb{R}^3$ is missing x and y

this means this surface is made up of points $(x, y, 5)$

$-\infty < x < \infty$, $-\infty < y < \infty$

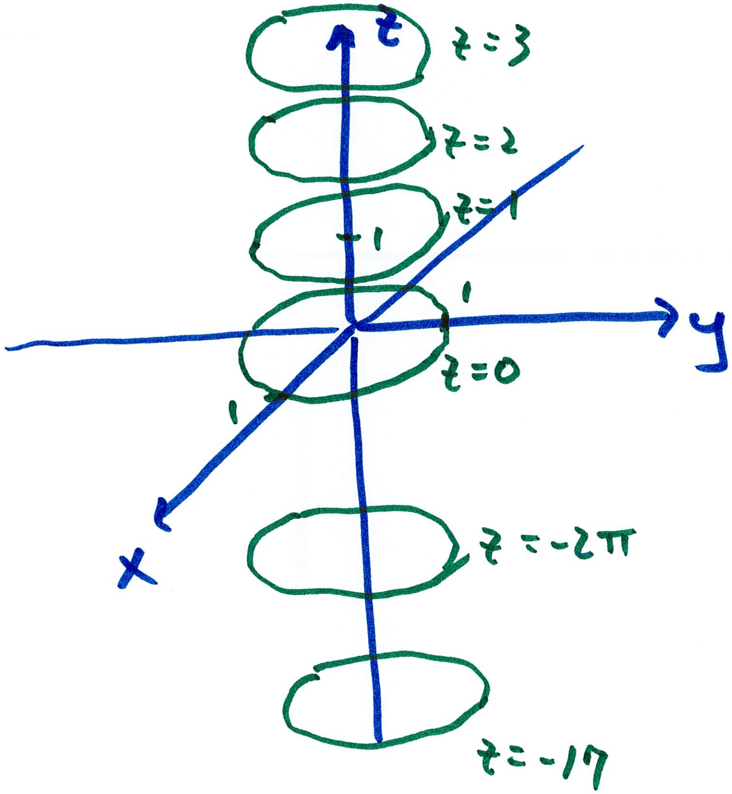


likewise, $x = 5$ is

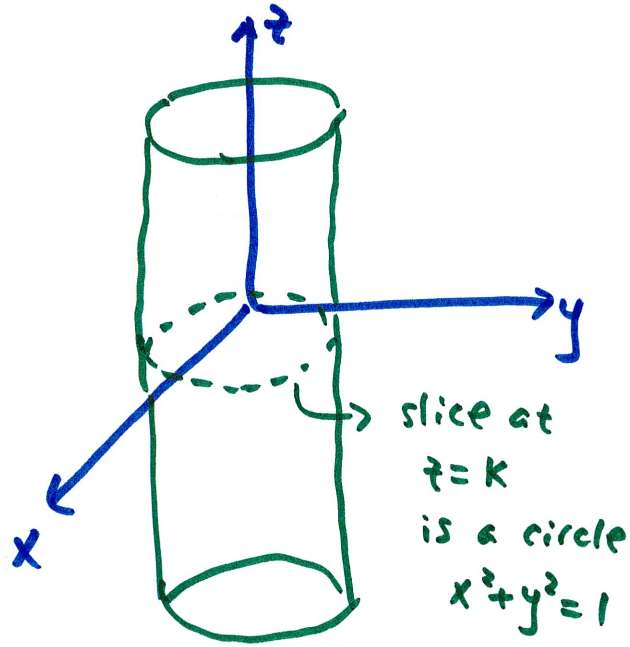


$x^2+y^2=1$ in $\mathbb{R}^3$ has no z

Surface is made up of points (x,y,z) such that $x^2+y^2=1$, $-\infty<z<\infty$

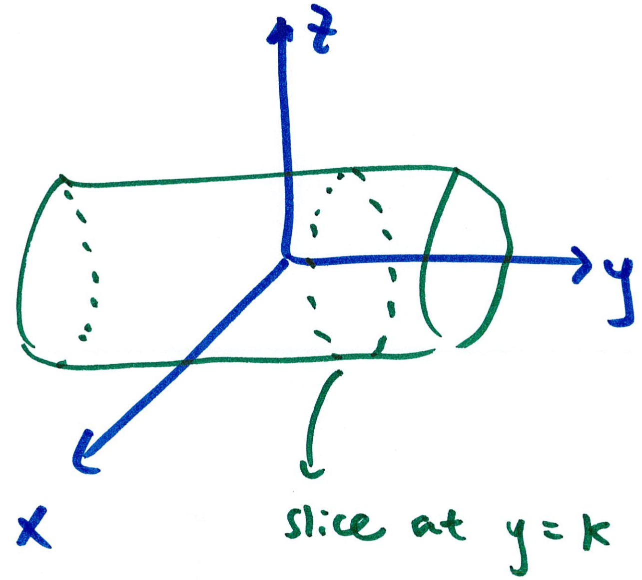


stack them:



circular cylinder

$x^2+z^2=1$ no y. slices at $y=k$ are $x^2+z^2=1$ (circles radius 1, centered at $(0,y,0)$)



these slices (intersections of surface with x, y, z equaling a constant) are called <u>traces</u>

intersection of a surface with the xy-plane is called the xy-trace

" " xz-plane " xz-trace

and so on

cylinders do not have to have circular cross sections

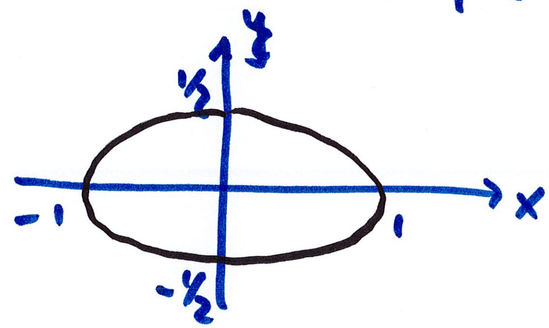
$x^2+4y^2=1$ has z missing

for each $z=k$, the cross section is an ellipse

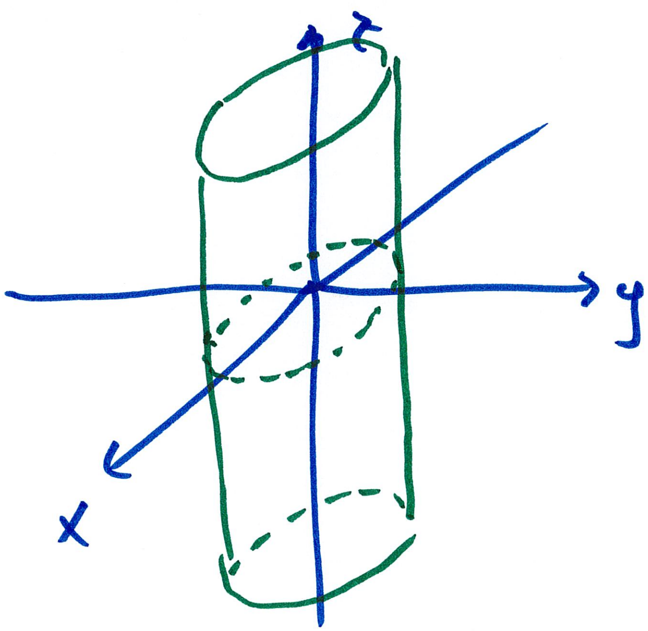
$x^2+4y^2=1$

x-ints: $x=\pm 1$

y-ints: $y=\pm\frac{1}{2}$

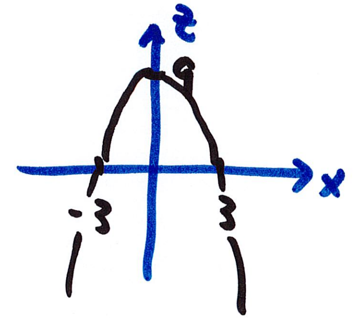


this gives us an ~~elliplit~~ elliptical cylinder

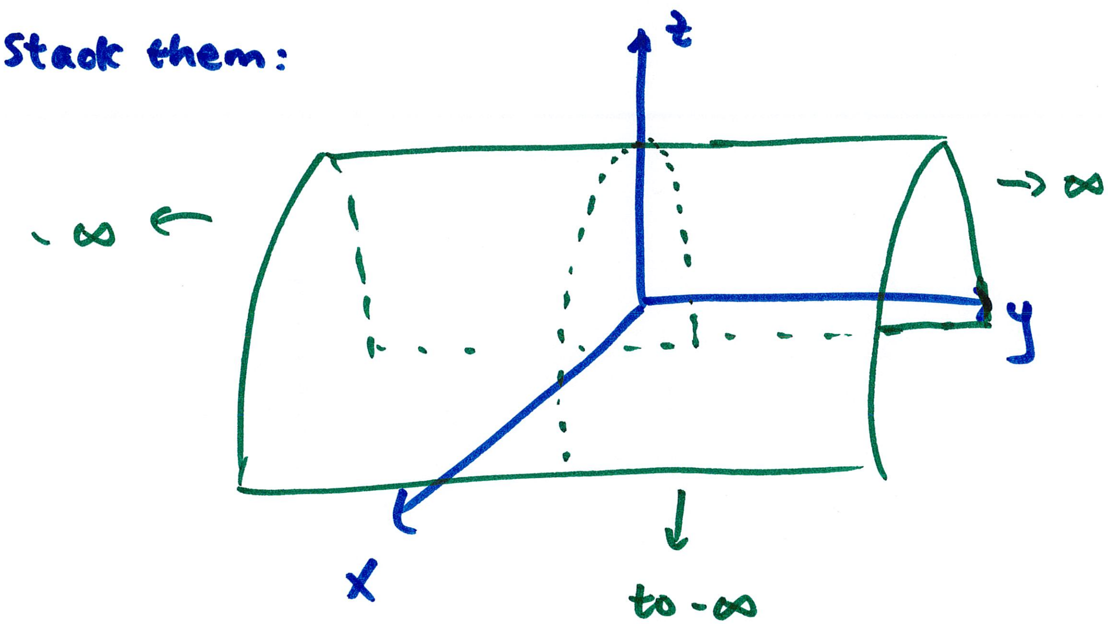


$z = 9 - x^2$ no y

for each $y = k$ the trace is a parabola



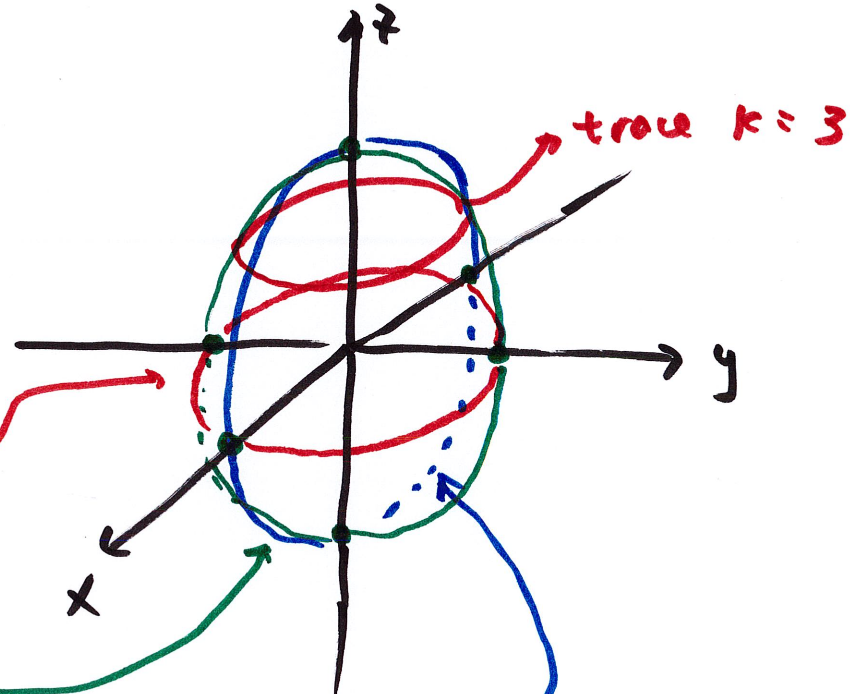
Stack them:



parabolic cylinder

stacking traces is a very good way to sketch quadric surfaces

$x^2+y^2+z^2=16$ (pretend we didn't know this is a sphere)



x-intercepts : $x=\pm 4$

y-intercepts : $y=\pm 4$

z-intercepts: $z=\pm 4$

xy-trace ($z=0$) : $x^2+y^2=16$
circle radius 4

yz-trace ($x=0$): $y^2+z^2=16$
circle radius 4

xz-trace ($y=0$): $x^2+z^2=16$
circle radius 4

trace $z=k$

$x^2+y^2=16-k^2$ circle radius $\sqrt{16-k^2}$

$-4 \leq k \leq 4$

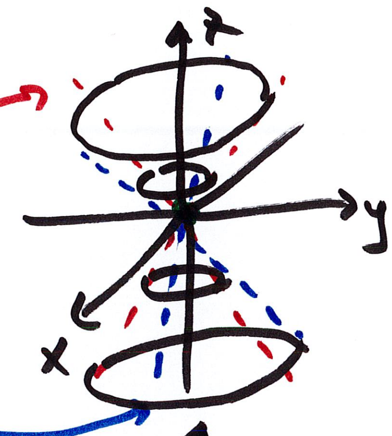
<u>example</u> $x^2+y^2=z^2$

$$\left.\begin{array}{l} x\text{-ints}: x=0 \\ y\text{-ints}: y=0 \\ z\text{-ints}: z=0 \end{array}\right\}\ \text{origin}$$

xy-trace $(z=0)$: $x^2+y^2=0$ point at origin

yz-trace $(x=0)$: $y^2=z^2 \rightarrow y=\pm z$ lines

xz-trace $(y=0)$: $x^2=z^2 \rightarrow z=\pm x$ lines

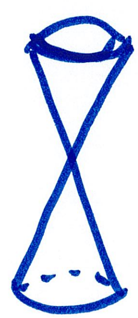


trace with $z=k$

$x^2+y^2=k^2$ circles radius $|k|$ $\quad -\infty<k<\infty$

circle gets bigger as $k\rightarrow\infty$ or $k-\infty$

we get



double cone

example

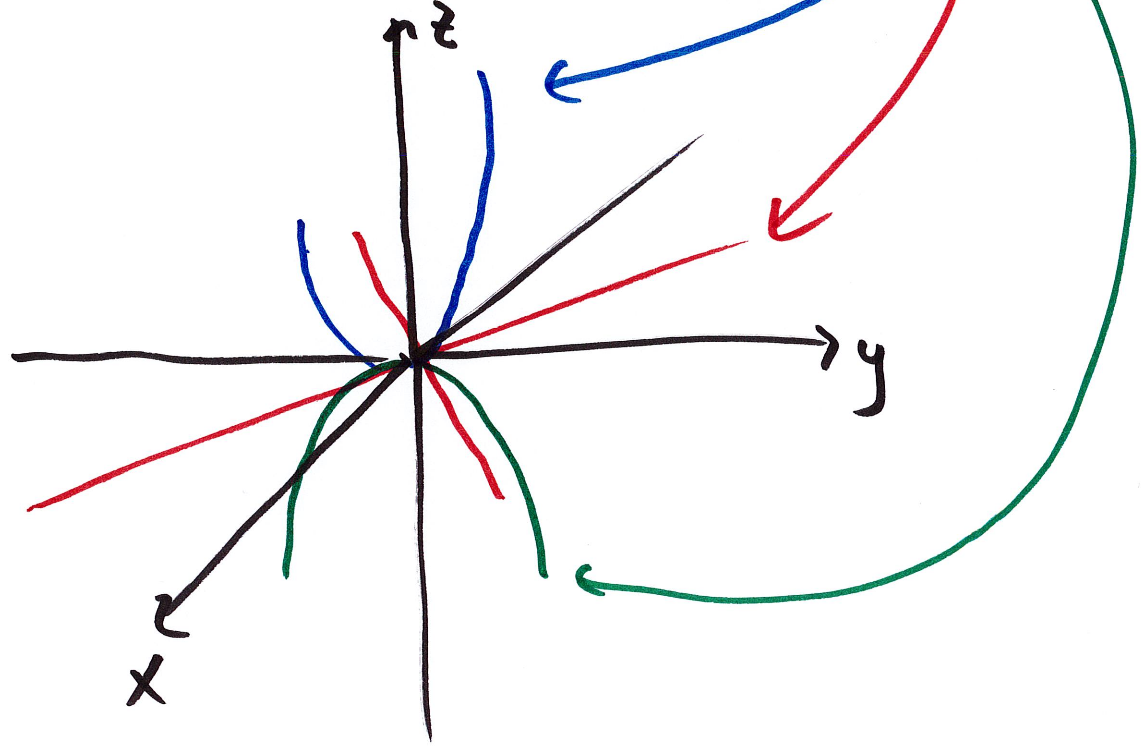
$z = x^2 - y^2$

x, y, z ints are all 0

xy-trace $(z=0)$: $y = \pm x$ lines

yz-trace $(x=0)$: $z = -y^2$ parabola

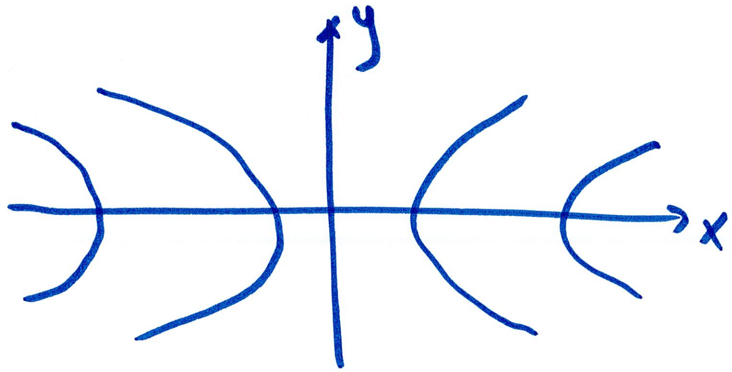
xz-trace $(y=0)$: $z = x^2$ parabola



need more info...

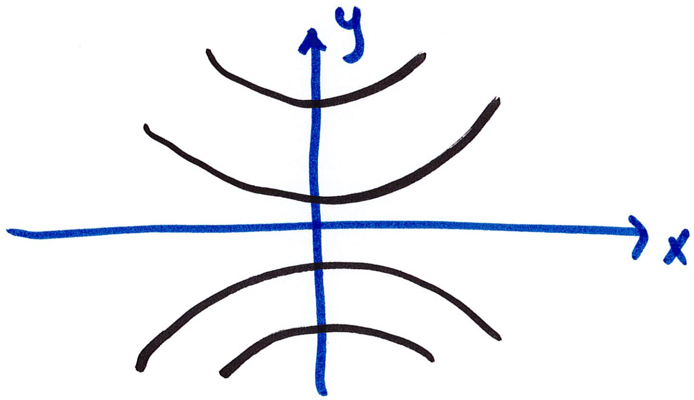
trace w/ $z=k$ $(k>0)$

$x^2-y^2=k$ hyperbola with intercepts on the x-axis



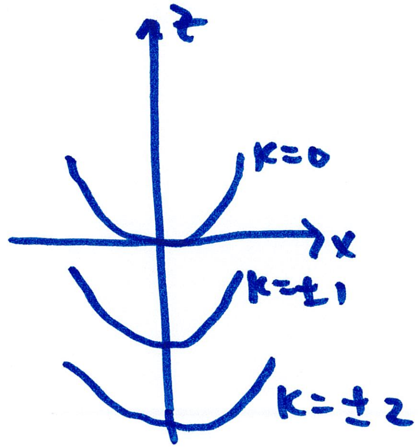
the higher the k, the more they spread out

if $k<0$



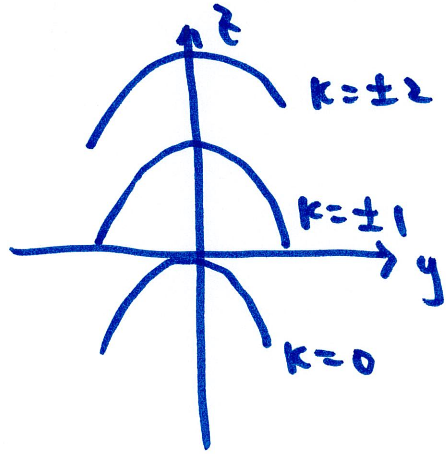
intercepts on y-axis

the smaller k is, the more they spread out trace w/ $y=k$: $z=x^2-k^2$ parabola w/ vertex at $z=-k^2$

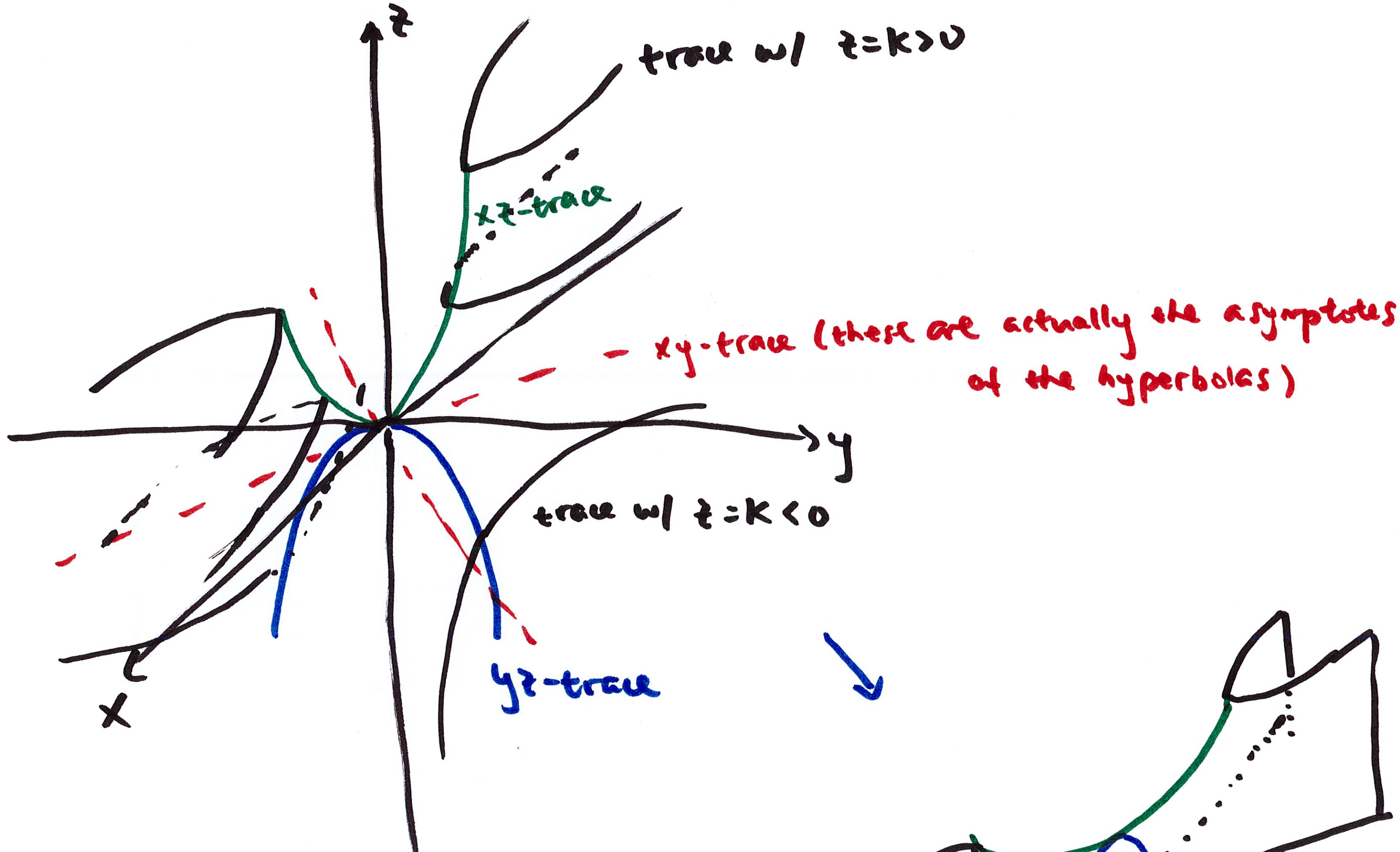


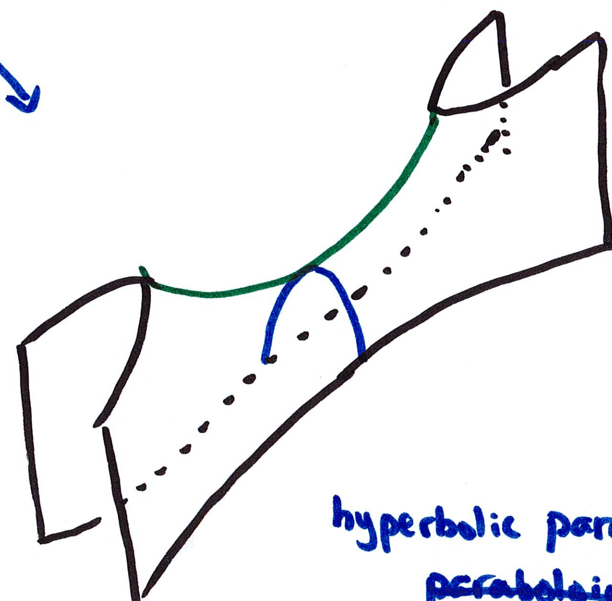
parabola gets lower as $|k|$ increases

trace w/ $x=k$: $z=k^2-y^2$ parabola again



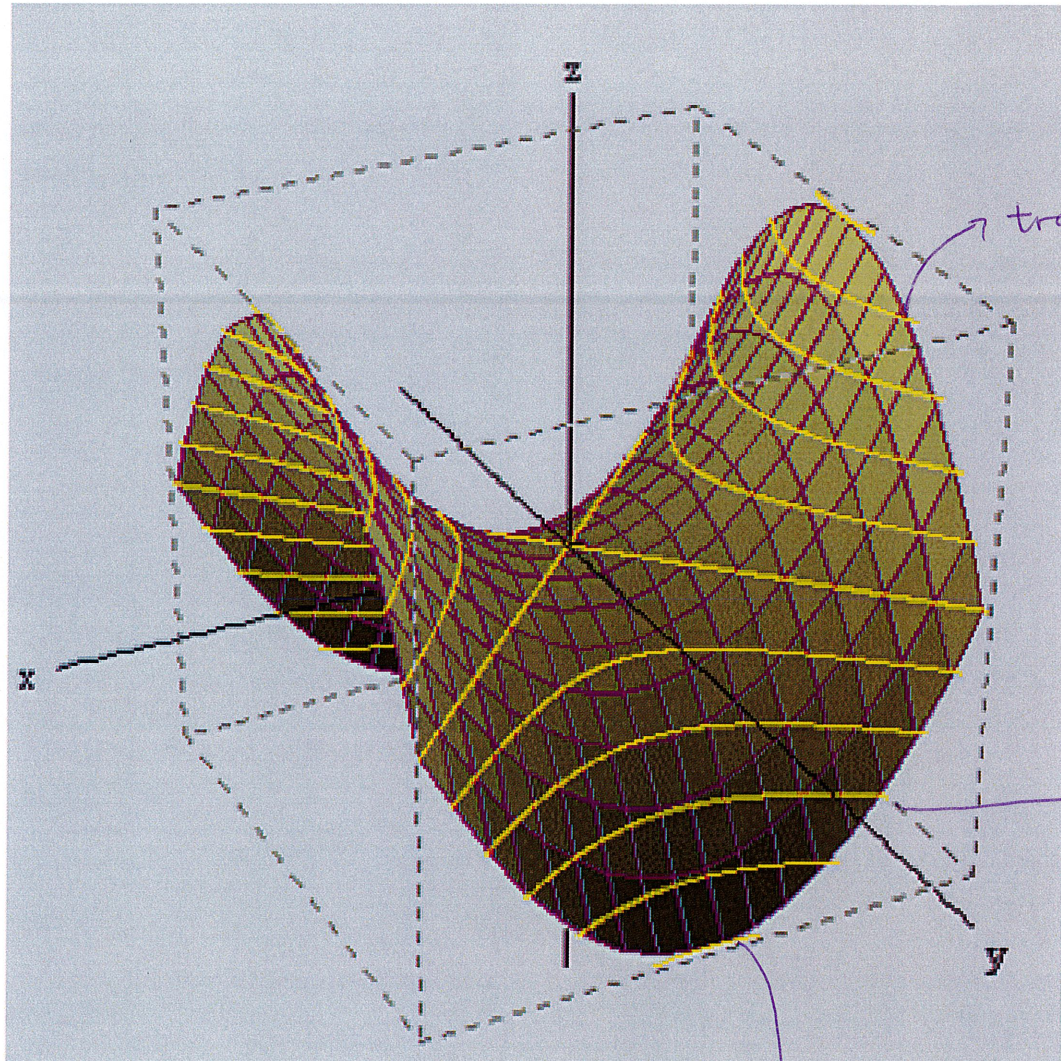
now put them all together





hyperbolic paraboloid

~~paraboloids~~



trace w/ $z = k > 0$
(black on last page)

trace w/ $z = k < 0$

yz-trace (blue on last page)

all of these quadric surfaces can be expressed as

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

and not all A, B, C are zero

we will look at more examples next time